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Edge LICT Domination in Graphs

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ABSTRACT

For any graph G, the lict graph $n(G)=J$ of a graph G is the graph whose vertex set is the union of the set of edges and set of cut vertices of G in which two vertices are adjacent if and only if corresponding members are adjacent or incident .A set F' of edges in a graph $n(G)$ is called edge dominating set of $n(G)$ if every edge in $E[n(G)]-F'$ is adjacent to atleast one edge in F',denoted as $\gamma_n'(G)$ and is the minimum cardinality of edge dominating set in $n(G)$. In this paper, many bounds on $\gamma_n'(G)$ were obtained in terms of vertices , edges and other different parameters of G but not in terms of elements of J. Further we develop its relation with other different domination parameters.

Introduction

In this paper, all the graphs considered here are simple, finite, non-trivial, undirected and connected. As usual p and q denote, the number of vertices and edges of a graph G .In this paper, for any undefined terms or notations can be found in Harary [2].

The study of domination in graphs was begun by Ore [5] and Berge [1].The domination in graphs discussed by S.L.Mitchell and S.T.Hedetineimi [4].

As usual, the maximum degree of a vertices in $V(G)$ is denoted by Δ(G) and the maximum edge degree of edges in $E(G)$ is denoted by $\Delta'(G)$. A vertex v is called a cut vertex if removing it from G increases the number of components of G .For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x.

A subdivision of an edge $e=uv$ of a graph G is the replacement of the edge by a path uwv .The graph obtained from G by subdividing each edge of G exactly once is called the subdivision graph of G and is denoted by $S(G)$.

A Line graph $L(G)$ is the graph whose vertices corresponds to the edges of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent.

We begin by recalling some standard definitions from domination theory.

A set $D \subseteq V$ of a graph $G=(V,E)$ is a dominating set, if every vertex not in D is adjacent to atleast one vertex in D.The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set in G.

A set $D' \subseteq V'$ is said to be a dominating set of $L(G)$, if every vertex not in D' is adjacent to atleast one vertex in D'.The domination number of $L(G)$ is denoted by $\gamma[L(G)]$ and is the minimum cardinality of a dominating set in $L(G)$.

A set F of edges in a graph $G=(V,E)$ is called an edge dominating set of G if every edge in $E-F$ is adjacent to at least one edge in F. The edge domination number $\gamma'(G)$ of a graph G is the minimum cardinality of an edge dominating set in G. Edge domination number was studied by S.L. Mitchell and Hedetniemi [4].

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A set I of edges in a graph $L(G)$ is called an edge dominating set of $L(G)$ if every edge in $E[L(G)]-I$ is adjacent to at least one edge in I. The edge domination number of $L(G)$ is denoted as $\gamma'[L(G)]$ and is a minimum cardinality of an edge dominating set in $L(G)$.

Analogously, we define edge domination number in lict graph .

A set F' of edges of Lict graph $J=n(G)$ is called edge dominating set of $n(G)$ if every edge in $E[n(G)]-F'$ is adjacent to atleast one edge in F'.The edge domination number $\gamma_n'(G)$ of a graph $n(G)$ is the minimum cardinality of a edge dominating set in $n(G)$.

The edge dominating set F' is called connected edge dominating set of $n(G)$, if induced subgraph $\langle F' \rangle$ is also connected. The connected edge domination number $\gamma_{nc}'(G)$ of a connected graph is the minimum cardinality of a connected edge dominating set..

We need the following Theorems to establish our further results.

Theorem A[3]: If G is a non trivial connected (p,q) graph whose vertices have degree d_i and l_i be the number of edges to which cutvertex C_i belongs in G, then lict graph $n(G)$ has $q+\sum C_i$ vertices and $-q+\sum\left(\frac{d_i^2}{2}+l_i\right)$ edges.

Theorem B[2] : If G is a (p,q) graph whose vertices have degree d_i, then $L(G)$ has q vertices and q_L edges where $q_L=-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2$.

Results

Initially we begin with edge domination number of Lict graph of some standard graphs , which are straight forward in the following theorem.

Theorem 1: (i) For any cycle C_p with $p \geq 3$ vertices,

$$\gamma_n'(C_p)=\left\lceil \frac{p}{3} \right\rceil.$$

(ii) For any path P_p with $p>2$ vertices,

$$\gamma_n'(P_p)=\left\lfloor \frac{p}{2} \right\rfloor.$$

(iii) For any star $K_{1,p}$ with $p \geq 2$ vertices,

$$\gamma_n'(K_{1,p})=\left\lceil \frac{p}{2} \right\rceil$$

(iv) For any wheel W_p with $p \geq 4$ vertices,

$$\gamma_n'(W_p)=p-2.$$

In the following theorem we obtain the relation for $\gamma_n'(G)$ in terms of number of edges and maximum edge degree Δ' of G .

Theorem 2: For any connected (p,q) graph G with $\Delta'(G) \leq q-1$ and $G \neq P_p$ $(p>6)$, $G \neq C_p$ $(p>4)$. Then $\gamma_n'(G) \geq q-\Delta'(G)$. Equality holds if G is P_3, P_5, P_6, C_3, C_4 and W_p $(p \geq 4)$.

Proof: To the contrary, suppose G is a path P_p with $p>6$ vertices .Let $G=P_p$ with $p>6$ vertices ; $P_p = v_1, e_1, v_2, e_2, v_3, e_3, ..., v_{p-1}, e_{p-1}, v_p$. In $n(G)$, $n[P_p] = e_1, e_2, e_3, ..$ $......, e_{p-1}$, gives induced path with $p-1$ vertices. Let F' be a minimal edge dominating set of $n(G)$, such that $|F'| = \left\lfloor \frac{p}{2} \right\rfloor$. For any path P_p with $p>6$ vertices, $\Delta' = 2$ and $p = q+1$. Then it is clear that $|F'| < p-1-\Delta'$ and $|F'| < q - \Delta'(G)$, which gives $\gamma'_n(G) <$ $q - \Delta'(G)$.

Now if $G = C_p$ with $p>4$ vertices. Let $G=C_p$ with $p>4$ vertices ; $C_p = v_1, e_1, v_2, e_2, ...v_p, e_p, v_1$. In $n(G)$, $V[n(C_p)] = e_1, e_2, e_3, ...,e_p, e_1$, which is isomorphic to C_p. Let F' be the minimal edge dominating set of $n(G)$.By Theorem 1, $|F'| = \left\lceil \frac{p}{3} \right\rceil$ and $\Delta' = 2$. Thus it is clear that $|F'| < p - \Delta'(G)$, which gives $\gamma'_n(G) < q - \Delta'(G)$.

For $\gamma'_n(G) \geq q - \Delta'(G)$, let $E(G) = \{e_1, e_2, e_3, ..., e_n\}$ be the edge set of G which gives $V[n(G)]$. We consider the of $E(G)$ such as subsets $E_1(G) = \{e_i\}; 1 \leq i \leq n$ and $E_2(G) = \{e_j\}$, where $i \neq j; 1 \leq j \leq n$ .Every element of $E_1(G)$ has maximum edge degree in G and every element of $E_2(G)$ has minimum edge degree in G. Let $E_3(G) =$ $\{e_k\}$; $1 \leq k \leq n$, such that $E_3(G) = E(G) - \{E_1(G) \cup E_2(G)\}$ be the set of edges which have neither maximum edge degree nor minimum edge degree in G .Let $C = \{c_1, c_2, c_3, .\cdots$ $......., c_n\}$ be the set of cutvertices in G .Denote $E[n(G)] = \{q_1, q_2, q_3, ..., q_n\}$, the edge set of $n(G)$ and $V[n(G)] = E(G) \cup C(G)$. Suppose F' be the minimal edge dominating set of $n(G)$, such that $\forall\, q_i \in F'; 1 \leq i \leq n$ is incident with atleast one of combination of $E_1(G)$, $E_2(G)$, $E_3(G)$ and $C(G)$. For this combination we consider the following cases.

Case 1: Suppose $\{q_i\}; 1 \leq i \leq n$ be the set of edges joining some of the vertices of $E_1(G)$ and $C(G)$ in $n(G)$.Then $\{q_i\} = F'$ be a minimal edge dominating set of $n(G)$. By the Theorem A , $n(G)$ has $-q + \sum\left(\frac{d_i^2}{2} + l_i\right)$ edges , which gives $F' \geq E(G) - \Delta'(G)$ and $|F'| \geq |E(G)| - \Delta'(G)$. Clearly $\gamma'_n(G) \geq q - \Delta'(G)$.

Case 2: Suppose the edge set $\{q_i\}; 1 \leq i \leq n$ forms a minimal edge dominating set in $n(G)$.Then the edges $\{q_i\}$ are joining the some vertices of $E_2(G)$ and $E_3(G)$ in $n(G)$. Since every edge of $E[n(G)] - \{q_i\}$ are adjacent to atleast one edge of $\{q_i\}$ .Then $F' = \{q_i\}$ be a minimal edge dominating set of $n(G)$. Hence $|F'| = |E(G)| - \Delta'(G)$, generates $\gamma'_n(G) \geq q - \Delta'(G)$.

Case 3: Suppose the elements of $\{q_i\}; 1 \leq i \leq n$ are incident with the vertices of $E_1(G)$ and $E_2(G)$ in $n(G)$. Then $\{q_i\} = F'$ be a minimal edge dominating set of $n(G)$.Clearly $|F'| \geq |E(G)| - \Delta'(G)$. Hence $|F'| \geq q - \Delta'(G)$.Which implies that $\gamma'_n(G) \geq q - \Delta'(G)$.

Case 4: Suppose $\{q_j\}$ and $\{q_k\}$ are edge sets such that $\{q_i\}, \{q_k\} \in E[n(G)]$ and $\{q_j\} \subset \{q_i\}$; $\{q_k\} \subset \{q_i\}$. Since every edge of $\{q_j\}$ are adjacent with the vertices of $E_2(G)$ and $C(G)$, also every edge of $\{q_k\}$ are incident with the vertices of $E_3(G)$ and

$C(G)$ in $n(G)$. Then $\{q_j\} \cup \{q_k\} = \{q_i\} = F'$ be a minimal edge dominating set of $n(G)$. Then clearly $|F'| \geq |E(G)| - \Delta'(G)$, which gives $\gamma'_n(G) \geq q - \Delta'(G)$.

For the remaining combination of the vertex sets $E_1 \cup E_3, E_2 \cup C, E_3 \cup C$, denote the edge set $\{q_{l_1}\}$ joining the vertices of $E_1(G)$ and $E_3(G)$, the edge set $\{q_{l_2}\}$ joining the vertices of $E_2(G)$ and $C(G)$, the edges of $\{q_{l_3}\}$ joining the vertices of $E_3(G)$ and $C(G)$ in $n(G)$.Since $\{q_{l_1}\} \supset \{q_i\}$, $\{q_{l_2}\} \supset \{q_i\}$ and $\{q_{l_3}\} \supset \{q_i\}$.Then the edge set $\{q_i\}$
is a minimal edge dominating set of $n(G)$.Clearly the combination of $[E_1 \cup E_3, E_2 \cup C,$
$E_3 \cup C] \notin F'$.

For the equality

i) If G is isomorphic to P_3, then $n(P_3) = C_3$ and $\gamma'_n(P_3) = 1$. Since $\Delta' = 2$. Hence it follows that $\gamma'_n(P_3) = q - \Delta'(G)$.

ii) If G is isomorphic to P_5, then $\gamma'_n(P_5) = 2$ .Since $\Delta' = 2$ and $p = q + 1$. Then $\gamma'_n(P_5) = p - 1 - \Delta'(G)$. It follows that $\gamma'_n(P_5) = q - \Delta'(G)$.

iii) If G is isomorphic to P_6, then $\gamma'_n(P_6) = 3$. Thus $\gamma'_n(P_6) = p - 1 - \Delta'(G)$ .Clearly $\gamma'_n(P_6) = q - \Delta'(G)$.

iv) If G is isomorphic to C_3 or C_4, then $n(G) = C_3$ or C_4, by definition .Since for any cycle C_p with p vertices, we have $p = q$ and also $\Delta'(G) = 2$. Hence clearly it follows that for $p = 3$ or 4 $\gamma'_n(C_p) = q - \Delta'(G)$.

v) If G is isomorphic to W_p with $p \geq 4$ vertices, then by Theorem 1, we have $\gamma'_n(W_p) = p - 2$. For any wheel W_p with $p \geq 4$ vertices, $\Delta' = p$ and $q > p$. Thus $\gamma'_n(W_p) = q - p$, which gives $\gamma'_n(W_p) = q - \Delta'(G)$.

In the following theorem we established the result on lower bound for $\gamma'_n(G)$.

Theorem 3: If G is connected (p,q) graph, then $\gamma'_n(G) \leq \left\lfloor \frac{p}{2} \right\rfloor$.

Proof: We consider the following cases.

Case 1: If $C(G) = \phi$.Now we partition edge set $E(G) = \{e_1, e_2, e_3, ..., e_n\}$, such as set $E_1(G) = \{e_i\}$, where $1 \leq i \leq n$ has maximum edge degree in G and set $E_2(G) = \{e_j\}$;where $1 \leq j \leq n$ has minimum edge degree in G and $E_3(G) = E(G) - \{E_1(G) \cup E_2(G)\} = \{e_k\}$ where $1 \leq j \leq n$ be the edge set with neither maximum nor minimum edge degree in G.

We obtain $\gamma'_n(G)$ set with respect to the edges joining the $V[n(G)]$, where $E_1(G)$, $E_2(G), E_3(G) \in V[n(G)]$. Further we consider $E[n(G)] = \{q_i\} \cup \{q_j\} \cup \{q_k\}$; $\forall q_n$; $1 \leq n \leq i$; $\forall q_m$; $1 \leq m \leq j$ and $\forall q_t$; $1 \leq t \leq k$ are the set of edges joining the vertices of $E_1(G), E_2(G)$; $E_1(G), E_3(G)$ and $E_2(G), E_3(G)$ in $n(G)$,where $E_1(G), E_2(G)$,

$E_3(G) \in V[n(G)]$.The remaining set $\{q_j\}$ and $\{q_k\}$ are the edges joining the elements of $E_1(G), E_2(G)$ and $E_3(G)$ in all other combinations. Since $q_n \subset q_i$, if $|q_n| =$ minimum, then $|q_n| = \gamma_n'(G)$.By the Theorem A, $n(G)$ has $-q+\frac{1}{2}\sum\left(\frac{d_i^2}{2}+l_i\right)$ edges which gives $|q_n| \subset E[n(G)]$. Clearly $\gamma_n'(G) \le \frac{p}{2}$.

Case 2: If $C(G) \ne \phi$, let $C = \{c_1, c_2, c_3, ..., c_n\}$ the set of cutvertices of G. Since $E[n(G)] = \{q_i\} \cup \{q_j\} \cup \{q_k\}$.We consider the subset $\{q_i\}$, such that some of $q_i's$ are incident with the elements of $E_1(G)$ and $E_2(G)$ in $n(G)$.For the subset $\{q_j\}$, some of q_j''s are incident with the elements of $E_2(G)$ and $E_3(G)$ in $n(G)$.Further for the subset $\{q_k\}$, for some of q_k''s are incident with the elements of $E_1(G)$ and $E_3(G)$ in $n(G)$.For the set $\{C_j\}$,some of elements of C_j are adjacent with either the elements of $E_1(G)$ or $E_2(G)$ or $E_3(G)$, we consider the following subcases.

Subcase 2.1: If $F' = \{q_i' \cup q_k'\}$,where $q_i' \subset q_i$ and $q_k' \subset q_k$ be the edge dominating set of $n(G)$ formed by joining the elements of $E_1(G)$ and $E_2(G)$ in $n(G)$,such that $|F'| = \gamma_n'(G) \le \left\lfloor \frac{p}{2} \right\rfloor$.

Subcase 2.2: If $F' = \{q_j' \cup q_k'\}$, where $q_j' \subset q_j$, be the edge dominating set of $n(G)$, formed by joining the elements of $E_2(G)$ and $E_3(G)$ in $n(G)$, such that $|F'| = \gamma_n'(G)$.

Since $\{q_i' \cup q_k'\} \subset E[n(G)]$. Which gives the result as $\gamma_n'(G) \le \left\lfloor \frac{p}{2} \right\rfloor$.

Subcase 2.3: Let $F' = \{q_i' \cup c_m'\} \subset E[n(G)]$ be the edge dominating set of $n(G)$, where c_m' is the set of edges joining the elements of $E_1(G)$ or $E_2(G)$ or $E_3(G)$ in $n(G)$ and $c_m' \subset c_m$, such that $|F'| = \gamma_n'(G) \le \left\lfloor \frac{p}{2} \right\rfloor$.

Further we deal with the edges of $\{q_i\}$, $\{q_j\}$,$\{q_k\}$ with $\{c_m\}$, $c_m \in V[n(G)]$ in the following subcase.

Subcase 2.4: Let the edge dominating set F' of $n(G)$ as $F' = \{q_i\} \cap \{c_m'\} \cup \{q_j'\}$ or $\{q_j\} \cap \{c_m'\} \cup \{q_k'\}$ or $\{q_k'\} \cap \{c_m'\}$, by Theorem A we have $E[n(G)] = -q + \sum\left(\frac{d_i^2}{2}+l_i\right)$

then $|F'| = \gamma_n'(G) \ge \left\lfloor \frac{p}{2} \right\rfloor$.Which gives the result.

Theorem 4: If G is (p,q) connected graph with $p > 2$ vertices then $\frac{q}{\Delta'(G)+1} \le \gamma_n'(G)$.

Proof: For a connected (p,q) graph G , with $p > 2$ vertices ,$n(G)$ be the Lict graph with p' vertices and q' edges . Now we consider a set F', be an edge dominating set of $n(G)$, such that $|F'| = \gamma_n'(G)$.

Then

$$|F'|\ \Delta'(G) \le \sum_{q' \in F'} d(q') = \sum N(q')$$

$$\le \left|\bigcup_{q' \in F'} N(q')\right|$$

$\leq \left|E\left[n(G)\right]-F'\right|$

$\leq \left|E\left[n(G)\right]\right|-\left|F'\right|$

$\left|F'\right| \Delta'(G) + \left|F'\right| \leq \left|E\left[n(G)\right]\right|$

$\left|F'\right| \left(\Delta'(G)+1\right) \leq \left|E\left[n(G)\right]\right|$

$\left|F'\right| \left(\Delta'(G)+1\right) \leq q'$.

By the definition of Lict graph .Since $V\left[n(G)\right]=E(G)\cup C(G)$ and also $E\left[n(G)\right]$ $\geq E(G)$.

Thus

$\left|E\left[n(G)\right]\right| \geq \left|F'\right| \left(\Delta'(G)+1\right) \geq \left|E(G)\right|$.

$\left|F'\right| \left(\Delta'(G)+1\right) \geq \left|E(G)\right|$.

$\left|F'\right| \left(\Delta'(G)+1\right) \geq q$, where $q \in E(G)$.

Hence

$$\left|F'\right| \geq \frac{q}{\left(\Delta'(G)+1\right)}.$$

Which gives $\gamma'_n(G) \geq \dfrac{q}{\left(\Delta'(G)+1\right)}$.

The following theorem provides the relation between $\gamma'_n(G)$ and $\gamma\left[L(G)\right]$.

Theorem 5: For any connected (p,q) graph G with $p>2$ vertices, $\gamma'_n(G)\geq\gamma\left[L(G)\right]$.

Equality holds if G is isomorphic to P_3, P_5 and $C_p\ (p\geq 3)$.

Proof: Let $E(G)=\{e_1,e_2,e_3,........,e_n\}$ be the edge set of G and let $F\subset E(G)$, such that , $\{e_i\};1\leq i\leq n$ be a minimal edge dominating of G. By definition of Line graph , let $D' =$ $\{v_1,v_2,v_3,............,v_n\}$,with $e_i \leftrightarrow v_i$, is a minimal dominating set of $L(G)$. Hence $|D'| =$ $\gamma\left[L(G)\right]$. Further we consider the edge set $F'=\{q_1,q_2,q_3,.............,q_n\}$, such that $F'\subset$ $E\left[n(G)\right]$ be a minimal edge dominating set of $n(G)$. Hence $|F'|=\gamma'_n(G)$ .Since each edge in G is adjacent to atleast one edge of F and $D' \in V\left[L(G)\right]\subseteq V\left[n(G)\right]$ .Clearly it follows that $|F'|\geq|D'|$. Hence $\gamma'_n(G)\geq\gamma\left[L(G)\right]$.

For the equality,

i) If $G\cong P_3$,then $\gamma'_n(G)=\gamma\left[L(P_3)\right]=1$

ii) If $G\cong P_5$, then $\gamma'_n(P_5)=\gamma\left[L(P_5)\right]=2$

iii) If $G\cong C_p$ with $p\geq 3$ vertices.For any cycle C_p ,we have $n(C_p)=L(C_p)\cong C_n$.
Hence $\gamma'_n(C_p)=\gamma'\left[L(C_p)\right]=\gamma(C_p)$.

The following theorem provides the analogous relation between $\gamma'_n(G)$ and $\gamma'\left[L(G)\right]$.

Theorem 6: For any connected (p,q) graph G with $p>2$ vertices , $\gamma'_n(G)\geq\gamma'\left[L(G)\right]$.

Equality holds if G is a block.

Proof : We consider the following cases.

case1: Suppose G is not a block .Then $\exists$ $C(G) \neq \phi$ in G. We partition the $E(G) = E_1(G) \cup E_2(G) \cup E_3(G)$. Now we consider $E_1(G) = \{e_1, e_2, e_3, ..., e_k\}$ be the edge set with maximum edge degree in G and $E_2(G) = \{e_1', e_2', e_3' \cdots\cdots, e_j'\}$ be the edge set with minimum edge degree in G .Let $E_3(G) = E(G) - \{E_1(G) \cup E_2(G)\}$ be the edges with neither maximum nor minimum edge degrees in G. $V[n(G)] = E_1(G) \cup E_2(G) \cup E_3(G)$ $\cup C(G)$, where $C(G)$ is the set of cutvertices in G. Denote $E[n(G)] = \{q_i\} \cup \{q_j\} \cup \{q_k\} \cup \{q_l\}$. For the set $\{q_i\}$, let $q_m \subset q_i; 1 \leq m \leq i$ are incident with the elements of $E_1(G)$ and $E_2(G)$ in $n(G)$. For the set $\{q_j\}$, let $q_n \subset q_j; 1 \leq n \leq j$ are incident with the elements of $E_2(G)$ and $E_3(G)$ in $n(G)$.For the set $\{q_k\}$, let $q_p \subset q_k$ $; 1 \leq p \leq k$ are incident with the elements of $E_1(G)$ and $E_3(G)$ in $n(G)$. Further for the set $\{q_l\}$, let $q_t \subset q_l$; $1 \leq t \leq l$ are incident with the elements of $E_1(G)$ or $E_2(G)$ or $E_3(G)$ in $n(G)$.

Let F' be a minimal edge dominating set of $n(G)$; $F' = \{q_m\}$ or $\{q_n\}$ or $\{q_p\}$ or $\{q_t\}$; such that $|F'| = \gamma_n'(G)$. But in line graph , $E_1(G) \cup E_2(G) \cup E_3(G) = V[L(G)]$. Here we consider $E[L(G)] = \{q_i'\} \cup \{q_j'\} \cup \{q_k'\}$. For the set $\{q_i'\}$, let $q_m' \subset q_i'; 1 \leq m \leq i$ are incident with the elements of $E_1(G)$ and $E_2(G)$ in $L(G)$. For the set $\{q_j'\}$, let $q_n' \subset q_j'; 1 \leq n \leq j$ are incident with the elements of $E_2(G)$ and $E_3(G)$ in $L(G)$.Further for the set $\{q_k'\}$, let $q_p' \subset q_k'$ $; 1 \leq p \leq k$ are incident with the elements of $E_1(G)$ and $E_3(G)$ in $L(G)$. Let I be a minimal edge dominating set of $L(G); I = \{q_m'\}$ or $\{q_n'\}$ or $\{q_p'\}$, such that $|I| = \gamma'[L(G)]$. By Theorem A and Theorem B , $E[L(G)] \subseteq E[n(G)]$.

Clearly it follows that $|F'| > |I|$. Thus $\gamma_n'(G) > \gamma'[L(G)]$.

Case 2: Suppose G is a block .Clearly $C(G) = \phi$. Then $V[n(G)] = V[L(G)] = E(G)$.

Then $E_1(G) \cup E_2(G) \cup E_3(G) \subseteq E(G)$ and $\{E_1(G), E_2(G), E_3(G)\} \in V[n(G)] = V[L(G)]$. Denote $E[n(G)] = \{q_i\} \cup \{q_j\} \cup \{q_k\}$; $\forall$ q_n $; 1 \leq n \leq i$; $\forall$ q_m $; 1 \leq m \leq j$; $\forall$ q_t ; $1 \leq t \leq k$ are the set of edges joining the elements of $E_1(G), E_2(G)$ and $E_3(G)$ in $n(G)$. For the set $\{q_i\}$, we consider q_n elements , for the set $\{q_i\}$; $\forall$ $q_i \subset E[n(G)]$ are joining the elements of $E_1(G)$ and $E_2(G)$ in $n(G)$. The remaining sets $\{q_j\}$ and $\{q_k\}$ are the edges joining the elements of $E_1(G), E_2(G)$ and $E_3(G)$ in the remaining combination. Since $q_n \subseteq q_i$ then $|q_n| =$ minimum and $|q_n| = \gamma_n'(G) = \gamma'[L(G)]$.Which gives the required result.

In the next result we obtain the relation of $\gamma_n'(G)$ with $\gamma'[L(G)]$ and $\gamma[L(G)]$.

Theorem 7: For any connected (p,q) graph G with $p > 2$ vetices , $\gamma_n'(G) \leq \gamma'[L(G)] + \gamma[L(G)]$, Equality holds if and only if $G \cong P_4$.

Proof : Let γ' be a minimal edge dominating set of G, which is also a γ set of $L(G)$. If

G has no cutvertex then $E[n(G)] = E[L(G)]$ and $E(G) = V[n(G)]$. If G has atleast one cutvertex then $E[n(G)] > E[L(G)]$ which gives $\gamma'_n(G) > \gamma'[L(G)]$ and also

$E[n(G)] = E[L(G)] \cup E_1(G)$; where $E_1(G) \subset E[n(G)]$; $\forall\ e_i \in E_1(G)$ are incident to each cutvertex of G in $n(G)$. By Theorem 5 and Theorem 6 we have the above result.

For equality, if $G \cong P_4$, then $L(P_4) = P_3$ with $\gamma'(P_3) = \gamma(P_3) = 1$ and $n(P_4) = C_3.C_3$ with $\gamma'_n(P_4) = \frac{p}{2} = 2$ .Which gives the equality result.

In the following theorem we obtain the relation between $\gamma'_n(G)$ and a domination number of G.

Theorem 8: For any connected (p,q) graph G, with $p > 2$ vertices $\gamma'_n(G) \geq \gamma(G)$.

Equality holds if G is isomorphic to P_p with $p = 3,7$, $C_p\ (p \geq 3)$ and spider or a wounded spider with exactly one or two wounded legs.

Proof: Let $D = \{v_1, v_2, v_3, ..., v_n\}$ be the minimal dominating set of G, such that $|D| = \gamma(G)$. Further let $F' = \{q_1, q_2, q_3, ..., q_n\}$ be the minimal edge dominating set of $n(G)$, where some of the $q_i\ 's \subseteq F'$; where $1 \leq i \leq n$ joining the elements of $V[n(G)]$ with $C(G) = V[n(G)] \cap V(G)$ in G and $\{C\} \subseteq D$, which gives $|F'| > |D|$. Thus $\gamma'_n(G) > \gamma(G)$.

For equality,

i) If $G = P_3, P_7$, then $\gamma(P_3) = \gamma(P_7) = \left\lfloor \frac{p}{2} \right\rfloor$ and also by Theorem1,we have $\gamma'_n(P_p) = \left\lfloor \frac{p}{2} \right\rfloor$.

which gives the result.

ii) If $G = C_p$ with $p \geq 3$ vertices .For any cycle C_p, $n(C_p) \cong C_p$, $\gamma'_n(C_p) = \gamma(C_p) = \left\lceil \frac{p}{3} \right\rceil$.

iii) If G is a spider, then we partition $E(G)$ into $E_1(G) = \{e_1, e_2, e_3, ..., e_i\}$; $E_2(G) = \{e_1, e_2, e_3, ..., e_j\}$, where each e_i is incident to v, where $\deg(v) = \Delta(G)$. For the set $E_2(G)$; $\forall\ e_j \in E_2(G)$ are adjacent to the elements of $E_1(G)$ with $e_i \leftrightarrow e_j$, with preserving 1-1 correspondence .Let $D = \{c_1, c_2, c_3, ..., c_i\}$ be a minimal dominating set of G and $D = C(G) - v$, where $\{C(G) - v\} \subset V(G)$ a cutest of G. Hence $|D| = \gamma(G)$. In $n(G)$, $E_1(G)$ together with v forms a complete subgraph K_{e_i+1}. Since $V[n(G)] = E(G) \cup C(G)$, then $\{(c_1, c_2, c_3, ..., c_i) \cup E_2(G) \cup E_1(G)\}$ forms c_i number of copies of K_3.

In $n(G)$, edges joining the elements of $E_1(G)$ and the elements of C_i or edges joining the elements of $E_2(G)$ and the elements of $E_1(G)$ will be denoted as $\{q_1, q_2, q_3, ...$ $....., q_i\}$ and $\{q_1, q_2, q_3, ..., q_j\}$. Let $F' = \{q_1, q_2, q_3, ..., q_i\}$ or $\{q_1, q_2, q_3, ..., q_j\}$ be a minimal edge dominating set of $n(G)$ obtained by above sets with $q_i \leftrightarrow q_j$. Hence $|D| = |F'| = \gamma'_n(G)$.

iv) For the wounded spider with exactly one or two wounded legs .Then the set $E_2(G) = \{e_2, e_3, ..., e_j\}$ or $\{e_3, e_4, ..., e_j\}$; the cutvertex set $C = \{c_2, c_3, ..., c_i\}$ or $\{c_3, c_4, ..., c_i\}$.

Let D be a minimal dominating set of G, such that $|D| = \{v, c_2, c_3, ..., c_i\}$ or $\{v, c_3, c_4, ...$

$\ldots, c_i\}$. Then $|D| = \gamma(G)$. In $n(G)$, since G is a wounded spider with one leg , then $F' = \{e, q_2, q_3, \ldots\ldots, q_i\}$ or $\{e, q_2, q_3, \ldots\ldots\ldots, q_j\}$, where $e = v\, q_1$. Further if G is a wounded spider with two wounded legs ,then $F' = \{e, q_3, q_4, \ldots\ldots\ldots, q_i\}$ or $\{e, q_3, q_4, \ldots\ldots\ldots., q_j\}$. Clearly $|D| = |F'| = \gamma'_n(G)$.

The following Theorem gives the relation between $\gamma'_n(G)$ and its connected edge domination number $\gamma'_{nc}(G)$.

Theorem 9: For any connected (p,q) graph G, $\gamma'_n(G) \leq \gamma'_{nc}(G)$.

Proof: Let $F' = \{q_1, q_2, q_3, \ldots\ldots\ldots, q_n\}$ be a minimal edge dominating set of $n(G)$, such that $|F'| = \gamma'_n(G)$. If $H = E[n(G)] - F'$ and let $F'_1 = \{a'_1, a'_2, a'_3, \ldots\ldots\ldots, a'_i\}$; $\forall\ a'_i \in E[n(G)]$ be the set of edges of $n(G)$, such that $F'_1 \in N(F')$ and $H \subset F'_1$ in $n(G)$. Then $\langle F' \cup H \rangle$ is a minimal connected edge dominating set of $n(G)$. Clearly $|F' \cup H| \geq |F'|$. Thus $\gamma'_{nc}(G) \geq \gamma'_n(G)$.

The following Theorem provides the result for $\gamma'_n(G)$ in terms of edge domination in subdivision of Lict graph of a graph G.

Theorem 10: For any connected (p,q) graph G, $\gamma'_n(G) \leq \gamma'_n[S(G)]$.

Proof : The above result is obvious.

Finally we obtain the Nordhous – Gaddum type result,

Theorem 11: Let G be a connected (p,q) graph, such that both G and $\overline{G}$ are connected, then

i) $\gamma'_n(G).\gamma'_n(\overline{G}) \leq p+2$.

ii) $\gamma'_n(G) + \gamma'_n(\overline{G}) \leq p$.

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