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Application Groupoids in the algebra

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ABSTRACT

A groupoid is medial if it satis_es the identity $wx \bullet yz = wy \bullet xz$. A groupoid is trimedial if every subgroupoid generated by 3 elements is medial. Medial groupoids and quasigroups have also been called abelian, entropic, and other names, while trimedial quasigroups have also been called triabelian, terentropic, etc. (See [5], especially p. 120, for further background). The notion of variety of algebras having the property (k, n) was given in [6] and equationally defined classes of cancellative groupoids having the property (2, 4) and (2, 5) were considered there. This notion was qeneralized in [7], where it was shown that the condition of the cancellativity is superfluous, that is, any variety of groupoids with the property (2, n) is a variety of quasigroups. Let k and n be two positive integers and k n. An algebra A is said to have the property (k, n) if every subalgebra of A generated by k distinct elements has exactly n elements. We also say that A is a (k, n)-algebra. A class K of algebras is said to be a (k, n)-class if every algebra in K is a (k, n)-algebra. A variety is called a (k, n)-variety if it is a (k, n)-class of algebras. Trivially, the variety of Steiner quasigroups (xx = x, xy = yx, x·xy = y) is a (2, 3)-variety. It is the unique variety of groupoids with the stated property, and the same holds for the (2, 4)-variety (x·xy = yx, xy·yx = x) given by Padmanabhan in [6]. He has also constructed two (2, 5)-varieties. One of them is commutative (xy = yx, x(y·xy) = y, x(x·xy) = y·xy), while the other one (x·xy = y, xy·y = yx)
consists of anticommutative quasigroups. It is an verify the existence of (2, n)-varieties for n≥10

Introduction

Definition (1): A non-empty set G, together with a mapping $*$: G × G −→ G is called a groupoid. The mapping $*$ is called a binary operation on the set G.

Remark: A binary groupoid (G, A) is understood to be a non-empty set G together with a binary operation A.

Often one uses different symbols to denote a binary operation, for example, ◦, ?, •, i.e. we may write x ◦ y instead A(x, y)

An n-ary groupoid (G, A) is understood to be a non-empty set G together with an n-ary operation A.

There exists a bijection (1-1 correspondence) between the set of all bi nary (n-ary, arity is fixed) operations defined on a set Q and the set of all groupoids, defined on the set Q. Really, A ←−→ (Q, A) As usual an $a_1^n = (a_1, a_2, \dots, a_n)$

$\overline{1,n}$ = {1, 2, . . . , n}. We shall say that operations A and B coincide, if $A(a_1^n) = B(a_1^n)$ for all a_i ∈ Q,

i ∈ $\overline{1,n}$

The order of any n-ary groupoid (Q, A) is cardinality | Q | (Q¯) of the carrier set Q. An n-ary groupoid (Q, .) is said to be finite whenever its order is finite. Any finite n-ary groupoid (not a very big size) (Q, A) it is possible to define as a set of (n + 1)-tuples $(a_1, a_2, a_3, \dots, a_n, A(a_1^n))$ In binary case any finite binary groupoid it is possible to define as a set of triplets or with help of square table, for example, as:

Table (1)

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where a • c = b. This table is called Cayley table of groupoid (Q, •), where

Q = {a, b, c}.

Note. Usually it is supposed that elements of carried set Q are arranged. So the groupoid (Q, ◦) defined with help of the following Cayley table.

Cayley Table (2)

is equal (as set of triplets) to the groupoid (Q, •), but (Q, •) = (Q, $*$), where groupoid (Q, $*$) has the following Cayley table

Cayley Table(3)

Definition (2). An n-ary groupoid (Q, A) with n-ary operation A such that in the equality A($x_1, x_2, \dots, x$) = x_{n+1} knowledge of any n elements of $x_1, x_2, \dots, x_n, x_{n+1}$ uniquely specifies the remaining one is called n-ary quasigroup ([14]).

In binary case this definition is equivalent to the following.

Definition(2). Binary groupoid (Q, ◦) is called a quasigroup if for all or- dered pairs (a, b) ∈ Q^2 there exist unique solutions x, y ∈ Q to the equations x ◦ a = b and a ◦ y = b ([2]).

Let (G, •) be a groupoid and let a be a fixed element in G. The so-called translation mapsL_a and R_a can be defined by L_a x = a • x, R_a x = x • a for all x ∈ G. It follows that L_a : G −→ G and R_a : G −→ G for each a ∈ G.

These maps will play a prominent role in much of what we do.

Example of quasigroup and its left and right translations.

Table (4)

For this quasigroup we have the following left and right translations: L_A = (bc);L_B = (ac); L_C = (ab); R_a = (bc); R_B = (ac); R_C = (ab).

It is easy to see that in Cayley table (4) of a quasigroup (Q, •) each row and each column is a permutation of the set Q. So we may give the following definition of a quasigroup.

Theorem. Let G be a right solvableWard groupoid, H a R associative right solvable subward groupoid of G and A is a R associative subset of G. Let G/H denotes the collection of all left cosets of H in G. Let

R_A = {(xH, yH) ∈ G/H × G/H : z ∈ HAH}

Definition (4). A groupoid (G, •) is called a quasigroup if the maps L_a : G −→ G, R_a : G −→ G are bijections for all a ∈ G (95)

These two varieties together with the variety whose defining identities (x•xy = yx, xy•y = x) are dual to the identities of the preceeding variety are the only (2, 5)-varieties of groupoid The non-existences of a (2, 6)-variety can be deduced from the correspondence between the (k, n)-varieties and Steiner systems S(k, n, v) [7].

Here we present (2, n)-varieties of groupoids for n = 7, 8 and 9. Their construction i s given in Sections 2, 3 and 4 respectively.

Result:

(1).A quasigroup satisfying the following three identities must be trimedial.

xx • yz = xy ¢ xz (1) yz • xx = yx • zx (2) (x • xx) • uv = xu • (xx • v) (3)

The converse is trivial, and so these three identities characterize trimedial quasigroups,

(2). Ward groupoid is a quasigroup.

(3). If groupoid satisfying the following identity than every groupoid is trimedial quasigroups:

$[(xy \bullet uu)][(w \bullet ww) \bullet zv] = [(xu \bullet yu)][wz \bullet (ww \bullet v)]$*:*

(Proof. In the result (2).To obtain set z = ww and use right cancellation. To obtain (3) set y = u and use left cancellation).

(4). we present (2, n)-varieties of groupoids for n = 7, 8 and 9. Their construction is given in Sections 2, 3 and 4 respectively. It is an open problem the existence of (2, n)-varieties for n ≥ 10, as well as the answer of the question whether the set of integers {n | There exists a (2, n)-variety of groupoids} is finite.

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