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Viscous Dissipation Effects on Unsteady free Convection and Mass Transfer Flow of Micropolar Fluid Embedded in a Porous Media with Chemical Reaction

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ABSTRACT

Heat and mass transfer effects on the unsteady flow of micropolar fluids embedded in a porous medium are studied by taking into the account of viscous dissipation with a homogeneous chemical reaction of the first order. Using the Galerkin finite element method the expressions for the velocity, microrotation, temperature and concentrations are obtained. The effect of material parameters such as Prandtl number Pr, Grashof number G, Modified Grashof number Gm, Schmidth number Sc, Eckert number Ec, the spin-gradient viscosity γ , the dimensionless viscosity ratio β , the coefficient of gyro-viscosity or vortex viscosity Λ , permeability of the porous medium K, Chemical reaction parameter K1, and time t. Further the results of Skin friction coefficient, the rate of heat transfer and mass transfer at the wall are presented with various values of fluid properties and flow conditions on the velocities, temperature and concentration are plotted in the graph and discussed. In the present analysis we study the effect of viscous dissipation on unsteady free convection flow of a laminar incompressible micropolar fluid with heat and mass transfer past a vertical porous plate embedded in a porous medium with chemical reaction.

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Introduction

The flow and heat transfer behavior of polymeric fluids, colloidal fluids, real fluids with suspensions, fluids containing certain additives, liquids crystals, animal blood, *etc.* cannot be explained by the classical Navier-Stokes theory. Eringen[1] proposed a theory of micropolar fluids and derived constitutive laws for fluids with micro-structure. The theory of thermo-micropolar fluids has been developed by Eringen[2]. The micropolor fluids theory has been applied extensively for studying many complicated fluid motions. Liquid crystal behavior was described by Lee and Eringen[3,4] by using the theory of micropolar fluids. In Refs. [5] and [6] the theory of micropolar fluid was applied in studying a low concentration suspension flow. The presence of dust or smoke, particularly in gas, may also be modelled using micropolar fluid dynamics [7]. Ishak *et al.*[8] studied the boundary layer flow of a micropolar fluid on a continuous moving or fixed surface. The effect of suction/ injection on the boundary layer flow of a micropolar fluid on a continuously moving or fixed surface was investigated by Ishak *et al.*[9].

Also, the porous media heat transfer problems have several practical engineering applications, such as the crude oil extraction, the ground water pollution, and many other practical applications, i.e., in biomechanical problems (e.g., blood, flow in the pulmonary alveolar sheet) and in the filtration transpiration cooling. Hiremath and Patil[10] studied the effect of free convection currents on the oscillatory flow of the polar fluid through a porous medium, which is bounded by the vertical plane surface with a constant temperature. The unsteady hydromagnetic free convection flow of a Newtonian and polar fluid has been investigated by Helmy[11]. El-Hakien *et al.*[12] studied the effects of the viscous and Joule heating on MHD-free convection flows with variable plate temperatures in a micropolar fluid. El-Amin[13] considered MHD free-convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with a constant suction. Kim[14] investigated the unsteady free convection flow of a micropolar fluid past a vertical plate embedded in a porous medium, and extended his work[15] to study the effects of heat and mass transfer in the MHD micropolar fluid flow past a vertical moving plate.





Lukaszewicz [16]. Kim [17] studied the unsteady two-dimensional laminar flow of a viscous incompressible micropolar fluid past a semi-infinite porous plate embedded in a porous medium. Kim [18] investigated transient mixed radiative convection flow of a micropolar fluid past a moving, semi infinite vertical porous plate. Hassanien and Essawy [19] studied the natural convection flow of micropolar fluid from a permeable uniform heat flux surface in porous media. Lok *et al* [20] investigated unsteady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. Mostafa [21] studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Ahmad [22] has studied the effects of thermophoresis on natural convection boundary layer flow of a micropolar fluid.

The combined heat and mass transfer problems with chemical reactions are of importance in many processes, and therefore have received a considerable amount of attention in recent years. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, the heat and mass transfer occurs simultaneously. Chemical reactions can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be the first order if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes take place in numerous industrial applications, such as the polymer production, the manufacturing of ceramics or glassware, the food processing [23] and so on. Das *et al.*[24] considered the effects of a first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumarswamy and Ganesan[25] and Muthucumarswamy[26] studied the first order homogeneous chemical reaction on the flow past an infinite vertical plate. Recently Bala Siddulu Malga and Naikoti Kishan [27] have studied the Viscous Dissipation Effects on Unsteady free convection and Mass Transfer Flow past an Accelerated Vertical Porous Plate with Suction.

Governing Equations

In cartesian coordinate system, we consider the two dimensional unsteady flow of a laminar incompressible micropolar fluid with heat and mass transfer in the presence of chemical reaction past a vertical porous moving plate embedded in a porous medium and subjected to a transverse magnetic field in the presence of a pressure gradient. The analysis is based on the assumption that the viscous and Darcy resistance terms are taken into account with constant permeability porous medium. Under these conditions, the governing equations for the problem are:

Continuity equation:
$$\frac{\partial V^*}{\partial y^*} = 0$$
 (1)

Linear momentum equation:

$$\frac{\partial U^*}{\partial t^*} + V^* \frac{\partial U^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + (v + V_r) \frac{\partial^2 U^*}{\partial y^{*2}} + g\beta_f (T - T_\infty) - v \frac{U^*}{K^*} + 2V_r \frac{\partial \omega^*}{\partial y^*} + g\beta_c (C - C_\infty)$$
(2)

Angular momentum equation:
$$\rho j^* \left(\frac{\partial \omega^*}{\partial t^*} + V^* \ \frac{\partial \omega^*}{\partial y^*} \right) = \gamma \frac{\partial^2 \omega^*}{\partial y^{*\,2}}$$
 (3)

Energy equation:
$$\frac{\partial T}{\partial t^*} + V^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} + \mu \left(\frac{\partial U^*}{\partial y^*}\right)^2$$
 (4)

Diffusion equation:
$$\frac{\partial C^*}{\partial t^*} + V^* \frac{\partial C^*}{\partial y^*} = D^* \frac{\partial^2 C}{\partial y^{*2}} - K_1^* C^*$$
 (5)

where x^* and y^* are the dimensional distances along and perpendicular to the plate respectively, U^*, V^* are the components of dimensional velocities along x^* and y^* respectively, ρ is the fluid density, v is the fluid kinematic viscosity, V_r is the fluid kinematic rotational viscosity, g is the acceleration due to gravity, β_f and β_c are the coefficients of volume expansions for temperature and concentration, K^* is an empirical constant called permeability of the porous medium, j^* is the micro-inertia density, ω^* is the component

of the angular velocity, γ is the spin-gradient viscosity, *T* is the temperature, *C*^{*} is the component of dimensional concentration, α is the fluid thermal diffusivity, *D* is the coefficient of mass diffusivity. μ is the fluid dynamic viscosity, Chemical reaction parameter K_1^* , Dimensionless co-ordinate η . The first term on the RHS of (2) is the pressure term, the second term is the viscous term, the third term is the buoyancy due to temperature difference, the fourth term is the Darcy or porous term and the fifth term is the micropolar term while the last is the mass term.

The boundary conditions for the velocities, temperature and concentration are

$$U^{*} = 0, T = T_{w} + \varepsilon (T_{w} - T_{\infty}) e^{n^{*}t^{*}}, \omega^{*} = -\frac{1}{2} \frac{\partial U^{*}}{\partial y^{*}}, C = C_{w}, \text{ at } y^{*} = 0$$

$$U^{*} \rightarrow U^{*}_{\infty} = U_{0} (1 + \varepsilon e^{n^{*}t^{*}}), T \rightarrow T_{\infty}, \omega^{*} \rightarrow 0, C \rightarrow C_{\infty}, as \quad y^{*} \rightarrow \infty$$

$$V^{*} = -V_{0} (1 + \varepsilon A e^{n^{*}t^{*}})$$
(6)
$$V^{*} = -V_{0} (1 + \varepsilon A e^{n^{*}t^{*}})$$
(7)

Where n^* is the dimensionless exponential index, U_{∞}^* is the free stream velocity, U_0 is a scale of free stream velocity, A is a real positive constant of suction velocity parameter, ε and εA are small less than unity, i.e. $\varepsilon A \ll 1$, V_0 is a scale of suction velocity normal to the plate and is assumed as a function of time only.

Outside the boundary layer, the pressure term in (2) gives

$$-\frac{1}{\rho}\frac{\mathrm{d}p^*}{\mathrm{d}x^*} = \frac{\mathrm{d}U^*_{\infty}}{\mathrm{d}t^*} + \frac{v}{\mathrm{K}^*}\mathrm{U}^*_{\infty} \tag{8}$$

We now introduce the following dimensionless variables as follows:

$$U = \frac{U^*}{U_0}, V = \frac{v^*}{V_0}, y = \frac{y^*V_0}{v}, U_{\infty} = \frac{U_{\infty}^*}{U_0}, \omega = \frac{v}{U_0 V_0} \omega^*, t = \frac{t^*V_0^2}{v}$$

$$\theta = \frac{(T-T_{\infty})}{(T_w - T_{\infty})}, \quad C = \frac{(C^* - C_{\infty})}{(C_w - C_{\infty})}, n = \frac{n^*v}{V_0^2}, K = \frac{K^*V_0^2}{v^2}, j = \frac{V_0^2}{v^2}j^*, K_1 = \frac{K_1^*v}{V_0^2}$$
Sc $= \frac{v}{D^*}$, is the Schmidth number, $P_r = \frac{v}{\alpha}$ is the Prandtl number
$$G = \frac{vg\beta_f(T_w - T_{\infty})}{U_0 V_0^2}, \text{ is the Modified Grashof number for heat transfer}$$

$$G_m = \frac{vg\beta_f(C_w - C_{\infty})}{U_0 V_0^2}, \text{ is the Modified Grashof number for mass transfer}$$

$$\gamma = \mu j^* \left(1 + \frac{1}{2}\beta\right), \beta = \frac{\Lambda}{\mu}, \quad E_c = \frac{U_0^2}{c_P(T_w - T_{\infty})} \text{ is the Eckert Number}$$

$$(9)$$

Where γ is the spin-gradient viscosity, β is the dimensionless viscosity ratio and Λ is the coefficient of gyro-viscosity or vortex viscosity. In view of equation (9), the governing equations (2)–(6) reduce to the following non-dimensional form:

$$\frac{\partial U}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial U}{\partial y} = \frac{dU_{\infty}}{dt} + (1 + \beta) \frac{\partial^2 U}{\partial y^2} + G\theta + \frac{1}{\kappa} (U_{\infty} - U) + 2\beta \frac{\partial \omega}{\partial y} + G_m C$$
(10)

$$\frac{\partial\omega}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial\omega}{\partial y} = \frac{1}{\eta} \frac{\partial^2 \omega}{\partial y^2}$$
(11)

$$\frac{\partial\theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial\theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2\theta}{\partial y^2} + E_c \left(\frac{\partial U}{\partial y}\right)^2 \tag{12}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial C}{\partial y} = \frac{1}{s_c} \frac{\partial^2 C}{\partial y^2} - K_1 C$$
(13)

Where
$$\beta = \frac{V_r}{V}$$
, $\eta = \frac{\mu j^*}{\gamma}$.

The boundary conditions (6) are given by the following dimensionless form:

$$\begin{aligned} U &= 0, \ \theta = 1 + \varepsilon e^{nt}, \ \omega = -\frac{1}{2} \frac{\partial U}{\partial y}, C = 1 \quad on \quad y = 0 \\ U \to U_{\infty}, \ \theta \to 0, \ \omega \to 0, \ C \to 0 \quad as \quad y \to \infty \end{aligned}$$
 (14)

Method of Solution

In order to reduce the above system of partial differential equations to a system of dimensionless form, we may represent the linear and angular velocities, temperature and concentration by applying the Galerkin finite element method for equation (10) over a typical twonoded linear element (e) ($y_i \le y \le y_k$) is

$$u = N. \phi, \text{ where } N = [N_j, N_k], \quad \phi = \begin{bmatrix} u_j \\ u_k \end{bmatrix}, \\ N_j = \frac{y_k - y}{l}, \quad N_k = \frac{y - y_j}{l}, \\ l = y_k - y_j = h, \\ \int_{y_j}^{y_k} N^T \left[\frac{dU_{\infty}}{dt} + (1 + \beta) \frac{\partial^2 U}{\partial y^2} - \frac{\partial U}{\partial t} + (1 + \varepsilon A e^{nt}) \frac{\partial U}{\partial y} + G\theta + \frac{1}{\kappa} (U_{\infty} - U) + 2\beta \frac{\partial \omega}{\partial y} + G_m C \right] dy$$

$$\int_{y_j}^{y_k} \left[(1 + \beta) \frac{\partial N}{\partial y} \cdot \frac{\partial U}{\partial y} - N^T \left((1 + \varepsilon A e^{nt}) \frac{\partial U}{\partial y} - \frac{\partial U}{\partial t} + \frac{dU_{\infty}}{dt} + G\theta + \frac{1}{\kappa} (U_{\infty} - U) + 2\beta \frac{\partial \omega}{\partial y} + G_m C \right) \right] dy = 0$$

$$\int_{y_j}^{y_k} \left[(1 + \beta) \frac{\partial N}{\partial y} \cdot \frac{\partial U}{\partial y} - N^T \left((1 + \varepsilon A e^{nt}) \frac{\partial U}{\partial y} - \frac{\partial U}{\partial t} - \frac{1}{\kappa} U + R \right) \right] dy = 0$$

$$(16)$$

$$where \quad R = \left(\left(\frac{1}{K} + n \right) \varepsilon e^{nt} + \frac{1}{K} + G\theta + G_m C + 2\beta \frac{\partial \omega}{\partial y} \right)$$

The element equation given by

$$\int_{y_{j}}^{y_{k}} (1+\beta) \begin{bmatrix} N_{j}^{'}N_{j}^{'} & N_{j}^{'}N_{k}^{'} \\ N_{k}^{'}N_{j}^{'} & N_{k}^{'}N_{k}^{'} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy - (1+\varepsilon Ae^{nt}) \begin{bmatrix} N_{j}N_{j}^{'} & N_{j}N_{k} \\ N_{k}N_{j}^{'} & N_{k}N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy + \frac{1}{K} \begin{bmatrix} N_{j}N_{j} & N_{j}N_{k} \\ N_{k}N_{j} & N_{k}N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy - R \begin{bmatrix} N_{j} \\ N_{k} \end{bmatrix} dy = 0$$

$$\int_{y_{j}}^{y_{k}} (S+A) dy = \int_{y_{j}}^{y_{k}} R^{*} dy \qquad (17)$$
Where $S = (1+\beta) \begin{bmatrix} N_{j}^{'}N_{j}^{'} & N_{j}^{'}N_{k}^{'} \\ N_{k}N_{j}^{'} & N_{k}N_{k}^{'} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} - (1+\varepsilon Ae^{nt}) \begin{bmatrix} N_{j}N_{j}^{'} & N_{j}N_{k}^{'} \\ N_{k}N_{j}^{'} & N_{k}N_{k}^{'} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} + N \begin{bmatrix} N_{j}N_{j} & N_{j}N_{k} \\ N_{k}N_{j} & N_{k}N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} A = \begin{bmatrix} N_{j}N_{j} & N_{j}N_{k} \\ N_{k}N_{j} & N_{k}N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} and R^{*} = R \begin{bmatrix} N_{j} \\ N_{k} \end{bmatrix}$

Here the prime and dot denote differentiation with respect to y and t. we obtain

$$S = \frac{(1+\beta)}{l} \begin{bmatrix} 1 & -1\\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} - \frac{(1+\varepsilon A e^{nt})}{2} \begin{bmatrix} -1 & 1\\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l}{6K} \begin{bmatrix} 2 & 1\\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix}$$
$$A = \frac{l}{6} \begin{bmatrix} 2 & 1\\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \text{ and } R^* = R \frac{l}{2} \begin{bmatrix} 1\\ 1 \end{bmatrix}$$

We write the element equation for the elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$. Assembling these element equations, we get

$$\frac{(1+\beta)}{l} \begin{bmatrix} 1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} - \frac{(1+\varepsilon A e^{nt})}{2} \begin{bmatrix} -1 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_{i+1} \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}\\ u_i\\ u_i \end{bmatrix} + \frac{l}{6\kappa} \begin{bmatrix} 1 & 0\\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0\\ 0 & 1$$

Now put row corresponding to the node i to zero, from equation (18) the difference schemes with l = h is

$$\frac{h}{6}(\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}) + \frac{(1+\beta)}{h}(-u_{i-1} + 2u_i - u_{i+1}) - \frac{(1+\varepsilon Ae^{nt})}{2}(-u_{i-1} + u_{i+1}) + \frac{h}{6K}(u_{i-1} + 4u_i + u_{i+1}) = R^*$$
(19)

Using the Cranck-Nicolson method to the equation (19), we obtain:

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^j + A_5 u_i^j + A_6 u_{i+1}^j + R^*$$
(20)

Similarly, the equations (11), (12), (13) are becoming as follows:

$$B_1 w_{i-1}^{j+1} + B_2 w_i^{j+1} + B_3 w_{i+1}^{j+1} = B_4 w_{i-1}^j + B_5 w_i^j + B_6 w_{i+1}^j$$
(21)

$$C_1 \theta_{i-1}^{j+1} + C_2 \theta_i^{j+1} + C_3 \theta_{i+1}^{j+1} = C_4 \theta_{i-1}^j + C_5 \theta_i^j + C_6 \theta_{i+1}^j + R^{**}$$
(22)

$$D_1 C_{i-1}^{j+1} + D_2 C_i^{j+1} + D_3 C_{i+1}^{j+1} = D_4 C_{i-1}^j + D_5 C_i^j + D_6 C_{i+1}^j$$

$$A_2 = (1 - 6r(1 + \beta) + 3nV + k/K) \quad A_2 = (4 + 12r(1 + \beta) + 4k/K) \quad A_3 = (1 - 6r(1 + \beta) - 3nV + k/K)$$
(23)

$$A_4 = (1 + 6r(1 + \beta) - 3pV - k/K), A_5 = (4 - 12r(1 + \beta) - 4k/K), A_6 = (1 + 6r(1 + \beta) + 3pV - k/K),$$

$$B_{1} = \left(1 - 6r\frac{1}{\eta} + 3pV\right), B_{2} = \left(4 + 12r\frac{1}{\eta}\right), B_{3} = \left(1 - 6r\frac{1}{\eta} - 3pV\right),$$

$$B_{4} = \left(1 + 6r\frac{1}{\eta} - 3pV\right), B_{5} = \left(4 - 12r\frac{1}{\eta}\right), B_{6} = \left(1 + 6r\frac{1}{\eta} + 3pV\right),$$

$$C_{1} = \left(1 - 6r\frac{1}{p_{r}} + 3pV\right), C_{2} = \left(4 + 12r\frac{1}{p_{r}}\right), C_{3} = \left(1 - 6r\frac{1}{p_{r}} - 3pV\right), \text{ and } R^{**} = E_{c}\left(\frac{U_{i+1} - U_{i-1}}{2h}\right)^{2}$$

$$C_{4} = \left(1 + 6r\frac{1}{p} - 3pV\right), C_{5} = \left(4 - 12r\frac{1}{p}\right), C_{6} = \left(1 + 6r\frac{1}{p} + 3pV\right),$$

$$D_{1} = \left(1 - 6r\frac{1}{s_{c}} + 3pV + K_{1}k\right), D_{2} = \left(4 + 12r\frac{1}{s_{c}} + 4K_{1}k\right), D_{3} = \left(1 - 6r\frac{1}{s_{c}} - 3pV + K_{1}k\right), D_{4} = \left(1 + 6r\frac{1}{s} - 3pV - K_{1}k\right), D_{5} = \left(4 - 12r\frac{1}{s} - 4K_{1}k\right), D_{6} = \left(1 + 6r\frac{1}{s} + 3pV - K_{1}k\right), D_{7} = \left(1 + 6r\frac{1}{s} + 3pV - K_{1}k\right), D_{7} = \left(1 + 6r\frac{1}{s}$$

Here $r = \frac{k}{h^2}$ where k, h is mesh sizes along y direction and time direction respectively. Index i refers to space and j refers to time. The mesh system consists of h=0.4 for velocity profiles and concentration profiles and k=0.125 has been considered for computations. In equation (20)-(23), taking i=1(1) n and using initial and boundary conditions (14), the following system of equation are obtained. $A_i X_i = B_i$, i = 1,2,3... (24)

Where A_i 's are matrices of order n and X_i and B_i 's are column matrices having n-components. The solution of above system of equations are obtained using Thomas algorithm for velocity, angular velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by Mat lab-program. In order to prove the convergence and stability of Galerkin finite element method, the same Mat lab-program was run with slightly changed values of h and k, no significant change was observed in the values of U, w, θ, C . Hence, the Galerkin finite element method is stable and convergent.

Results and Discussion

The numerical computations are carried out for the distribution of steam velocity, angular velocity, temperature and concentration profiles are presented for various values of the flow parameters, such as Prandtl number *Pr*, Grashof number *G*, Modified Grashof number *Gm*, Schmidth number *Sc*, Eckert number *Ec*, index number *n*, the spin-gradient viscosity γ , the dimensionless viscosity ratio β , the coefficient of gyro-viscosity or vortex viscosity Λ , permeability of the porous medium *K*, Chemical reaction parameter *K*₁, Dimensionless coordinate η and time t are chosen over a range as listed in the figures.

Figure.1 (a) and Figure.1 (b) shows the effect of time t, on the steam velocity and angular velocity profiles, it is observed that the steam velocity increases, angular velocity decreases with the values of time t. Figure.2 (a) and Figure.2 (b) displays the effect of n, on the steam velocity and angular velocity profiles shows an accelerating notice as n increases. Figure.3 (a) and Figure.3 (b) displays the effect of Eckert number *Ec* on the steam velocity and angular velocity profiles. It can be seen that the steam velocity profiles increases as *Ec* increases whereas angular velocity profiles decreases with increase of *Ec*. Figure.4 (a) and Figure.4 (b) illustrates the effect of the Chemical Reaction parameter K_1 on the steam velocity and angular velocity profiles. It can be seen that steam wise velocity increases during the generative reaction ($K_1 < 0$) and decreases in the destructive reaction($K_1 > 0$), while the reverse phenomenon is observed

for the microrotation. The effect of Prandtl number Pr is shown in Figure.5 it is obvious that the effect of Pr decreases the steam velocity profiles. Figure.6 illustrates the effect of the Grashof number G, Modified Grashof number Gm on the steam velocity profiles. One notes from this figure that the steam velocity profiles with the increasing of both. Figure.7 displays the effect of η , on the angular velocity profiles it is observed that the angular velocity increases with the increase of η .

The temperature profiles increases with the increase of time t is noticed from Figure.8 and Figure.9 depicts the temperature profiles for different values of Pr, it can be observed that from the figure that the temperature decreases with the increasing Pr. The effect of n on temperature profiles for different values of n is plotted in Figure.10 with the effect of n the temperature profiles increases near the plate while reverse phenomenon is observed far away from the plate. The effect of Eckert number Ec on the temperature profiles are plotted in Figure.11 it can be seen that the effect of Ec accelerates temperature profiles.

Figure.12-15 are presented the concentration profiles for different values of Schmidth number Sc, time t and index number n. Chemical Reaction parameter K_1 . The figure.12 reveals that the concentration profiles increases with the increase of time t. The concentration profiles for different values of Schmidth number Sc is shown in Figure.13 it reveals that an increase in Sc leads to a decrease the concentration distribution. From Figure.14 it can be seen that there is a fall in the concentration profiles due to the increasing the value of n. Figure.15 shows that there is a fall in the concentration due to the increasing the values of chemical reaction parameter K_1 .

Table 1: The values of the Skin friction $\tau_w = \left(\frac{\partial U}{\partial y}\right)$ at y = 0 and the heat transfer coefficient in terms of Nusselt number $N_U = \left(-\frac{\partial \theta}{\partial y}\right)$ at y = 0.

Table 1: Values of Skin friction= τ_W and Nasselt number = N_U										
S.No	А	n	t	3	G	Gm	Pr	Sc	τ_w	NU
1	1	1	1	0.01	1	1	0.7	0.2	2.74525	-1.8395
2	1	1	2	0.01	2	1	0.7	0.2	2.93575	-1.9495
3	1	1	2	0.01	5	2	0.7	0.2	3.59275	-2.3495
4	1	1	2	0.01	5	3	0.7	0.2	3.85625	-2.5495
5	1	2	2	0.01	1	1	0.7	0.2	4.4905	-2.513
6	1	2	2	0.01	5	1	0.7	0.2	5.086	-2.813
7	1	2	2	0.07	1	2	0.7	0.2	19.9825	-2.08775
8	1	2	2	0.07	1	1	1	0.2	19.775	-2.02775
9	1	1	2	0.07	1	1	1	0.2	4.35275	-1.72
10	1	1	1	0.07	1	1	1	0.2	3.246	-1.629



Figure 1(a). Velocity profiles for different values of t Figure 1(b). Angular Velocity profiles for different values of t



Figure 2(a). Velocity profiles for different values of n



Figure 3(a). Velocity profiles for different values of Ec



Δ



Figure 3(b). Angular Velocity profiles for different values of Ec



Figure 4(a). Velocity profiles for different values of K1

Figure 4(b). Angular Velocity profiles for different values of K1



Figure 5. Velocity profiles for dimerent values of Pr



Figure 7. Angular Velocity profiles for different values of η



Figure 9. Temperature profiles for different values of Pr



Figure 6. Velocity profiles for unerent values of G &Gm



Figure 8. Temperature profiles for different values of t



Figure 10. Temperature profiles for different values of n



Figure 11. Temperature profiles for different values of Ec



Figure 13. Concentration profiles for different values of Sc



Figure 12 Concentration profiles for different values of t



Figure 14. Concentration profiles for different values of n



Figure 15. Concentration profiles for different values of K1

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