MHD Effects on Fully Developed Natural Convection Heat and Mass Transfer of a Micropolar Fluid in a Vertical Channel

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ABSTRACT

An analysis is presented for the problem of the fully developed natural convection magnetohydrodynamics micropolar fluid flow of heat and mass transfer in a vertical channel. Asymmetric temperature and convection boundary conditions are applied to the walls of the channel. The cases of double diffusion and Soret-induced connections are both considered. Solutions of the coupled non-linear governing equations are obtained for different values of the buoyancy ratio and various material parameters of the micropolar fluid and magnetic parameters. The resulting non dimensional boundary value problem is solved by the Galerken Finite element method using MATLAB Software. Influence of the governing parameters on the fluid flow as well as heat and solute transfers is demonstrated to be significant.

Introduction

The theory of micropolar fluids, introduced by Eringen [1–2] in order to deal with the characteristics of fluids with suspended particles, has received considerable interest in recent years. Also, as demonstrated by Papautsky et al. [3], Eringen's model predicts successfully the characteristics of flow in microchannels. An excellent review of the various applications of micropolar fluid mechanics was presented by Ariman et al. [4].Many of the non-Newtonian fluid models describe the nonlinear relationship between stress and the rate of strain. But the micropolar fluid model introduced by Eringen [5] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, the micropolar fluid can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and porous media are presented by Lukaszewicz [6]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. Ahmadi [7] studied the boundary layer flow of a micropolar fluid over a semi-infinite plate.

However, it is well known that convection, in a binary mixture, can also be induced by Soret effects. For this situation the species gradients result from the imposition of a temperature gradient in an otherwise uniform-concentration mixture. Two kinds of problems have been considered in the literature concerning the convection of a binary mixture filling a horizontal porous layer. The first kind of problem, called double diffusion, considers flows induced by the buoyancy forces resulting from the imposition of both thermal and solutal boundary conditions on the layer. Early investigations on double-diffusive natural convection in porous media primarily focused on the problem of convective instability in a horizontal layer. To this end Nield [8], Taunton et al. [9] Poulikakos [10], Taslim & Narusawa [11] and Malashetty [12] used linear stability analysis to investigate the onset of thermohaline convection.

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The second kind of problem considers thermal convection in a binary fluid driven by Soret effects. For this situation the species gradients are not due to the imposition of solutal boundary conditions as in the case of double diffusion. Rather, they result from the imposition of a temperature gradient in an otherwise uniform-concentration mixture. Brand & Steinberg [13-14] investigated the influence of Soret-induced solutal buoyancy forces on the convective instability of a fluid mixture in a porous medium heated isothermally.

The first study of the fully developed free convection of a micropolar fluid in a vertical channel was presented by Chamkha et al. [15]. This problem was extended by Kumar et al. [16] to consider the case of a channel with one region filled with micropolar fluid and the other region with a Newtonian fluid. It was found that the effects of the micropolar fluid material parameters suppress the fluid velocity but enhance the microrotation velocity. An analytical solution predicting the characteristics of fluid flow, and heat and mass transfer was derived. It was reported that an increase of the vortex viscosity parameter tends to decrease the fluid velocity in the vertical channel. The same problem was later reconsidered by Bataineh et al. [17]. The problem of the fully developed natural convection heat and mass transfer of a micropolar fluid between porous vertical plates with asymmetric wall temperatures and concentrations was investigated by Abdulaziz and Hashim [18]. Profiles for velocity and that, as the Reynolds number increases, the velocity decreases in the left part of the channel and increases in the right part. The above studies [17-18] are concerned with double-diffusive convection in a vertical channel for which the flows induced by the buoyancy forces result from the imposition of both thermal and solutal boundary conditions on the vertical walls. The first study of Soret-induced convection was described by Bergman and Srinivasan [19], while considering natural convection in a cavity filled with a binary fluid. This flow configuration has also been investigated by R.Krishnan et al.[20].

As pointed out recently by Rawat and Bhargava [21], the study of heat and mass transfer in micropolar fluids is of importance in the fields of chemical engineering, aerospace engineering and also industrial manufacturing effects processes. Sunil et al.[22] studied the Effect of Rotation on Double-Diffusive Convection in a Magnetized Ferrofluid with Internal Angular Momentum; A. A. Bakr et al. [23] studied the Double-Diffusive Convection-Radiation Interaction on Unsteady MHD Micropolar Fluid Flow over a Vertical Moving Porous Plate with Heat Generation and Soret Effects. R. A. Mohamed [24] also analyzed Double-Diffusive Convection-Radiation Interaction on Unsteady MHD Flow over a Vertical Moving Porous Plate with Heat Generation and Soret Effects, A. Bahloul et al. [25] studied Double-diffusive and Soret-induced convection in a shallow horizontal porous layer. Z. Alloui at.al.[26] Double-diffusive and Soret-induced convection of a micropolar fluid in a vertical channel.
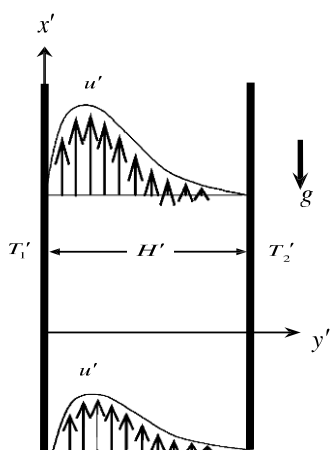

Figure (a). The flow configuration and the coordinate system.

Mathematical Model

We consider a steady fully developed laminar natural convection flow of a micropolar fluid between two infinite vertical plates (see Fig.(a)). The vertical plates are separated by a distance H′. The convection current is induced by both the temperature and concentration gradients. The flow is assumed to be in the x′ direction, which is taken to be vertically upward along the channel walls, while the y′-axis is normal to the plates. The fluid is assumed to satisfy the Boussinesq approximation, with constant properties except for the density variations in the buoyancy force term. The density variation with temperature and concentration is described by the state equation $\rho = \rho_o[1 - \beta_T'(T' - T_o') - \beta_c(C - C_o)]$ where ρ_o is the fluid mixture density at temperature $T' = T_o'$ and mass fraction $C = C_o$, and $\beta'T$ and βC are the thermal and the concentration expansion coefficients, respectively. In the present investigation the Dufour effect is neglected since it is well known that the modification of the heat flow due to the concentration gradient is of importance in gases but negligible in liquids. Under these assumptions, the governing equations can be written as:

$$(1 + K')\frac{d^2 u'}{dy'^2} + K'\frac{dN'}{dy'} = -\rho_0 g[\beta_T'T[(T' - T_o') - \beta_c(C - C_o)] + \sigma B_0^2 u' \tag{1}$$

$$\frac{\gamma}{j}\frac{d^2 N'}{dy'^2} + \frac{K'}{j}\left(2N' + \frac{du'}{dy'}\right) = 0 \tag{2}$$

$$\frac{d^2 T'}{dy'^2} = 0 \tag{3}$$

$$\frac{d^2 C'}{dy'^2} = 0 \tag{4}$$

where u' is the velocity component along the x′ direction, and g is the acceleration due to gravity. Further, μ, κ, j, N' and γ are respectively the dynamic viscosity, vortex viscosity, micro-inertia density, angular velocity and spin gradient viscosity. Following Chamkha et al. [15] it is assumed that γ has the form

$$\gamma = (\mu + \kappa/2)j.$$

The appropriate boundary conditions applied on the walls of the vertical channel are

$$u' = 0, \ N' = -n\frac{du'}{dy'}, \ T' = T_1', \ (1-a)C + a\frac{du'}{dy'} = (1-a)C_1 - a\frac{D'}{D}C_0(1-C_0)\frac{dT'}{dy'} \ on \ y' = 0 \ (5) \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\}$$

$$u' = 0, \ N' = n\frac{du'}{dy'}, \ T' = T_2', \ (1-a)C + a\frac{du'}{dy'} = (1-a)C_2 - a\frac{D'}{D}C_0(1-C_0)\frac{dT'}{dy'} \ on \ y' = H' \ (6)$$

where $0 \le n \le 1$ is a boundary parameter that indicates the degree to which the microelements are free to rotate near the channel walls. The case n = 0 represents concentrated particle flows in which the microelements close to the wall are unable to rotate S.K. Jena & M.N. Mathur [27]. Finally, according to Peddieson [28] the case n = 1 is applicable to the modeling of turbulent boundary layer flows. D and D' are respectively the molecular diffusion coefficient and the thermodiffusion coefficient.

The governing equations are non-dimensionalized by scaling length by H'

$u' = \mu Gr/(\rho_0 H')u,$ is the velocity,

$N' = \mu Gr/(\rho_0 H'^2)N,$ is the microrotation,

$Gr = g\rho_0^2\beta_T'(T_1 - T_0)H'^3/\mu^2$ is the Grashof number ,

$M = \sigma B_0^2 H^2/\mu$ is the Magnetic parameter,

$T = (T' - T_0')/(T_1' - T_0'),$ is the reduced temperature,

$S = (C - C_0)/(C1 - C0),$ is the reduced concentration,

$\Delta T' = T_1' - T_0'$ and $\Delta C = C1 - C0$ for double-diffusive convection,

$\Delta C = -C_0(1 - C_0)\Delta T' D'/D$ for Soret-driven convection.

$\varphi = \beta_c \Delta C/\beta_T' \Delta T'$ is the buoyancy ratio,

$R_T = (T'_2 - T'_0)/(T'_1 - T'_0)$ is the wall temperature ratio,

$R_s = (C_2 - C_0)/(C_1 - C_0)$ is the wall concentration ratio,

$K = \kappa/\mu$ is the vortex viscosity parameter,

$B = H'^2/j$ is the micro-inertia parameter, the subscript $'$ indicates a reference state.

The dimensionless equations governing the present problem then read

$$(1 + K)\frac{d^2u}{dy^2} + K\frac{dN}{dy} = -[T + \varphi S] + Mu \tag{7}$$

$$\left(1 + \frac{K}{2}\right)\frac{d^2N}{dy^2} - BK\left(2N + \frac{du}{dy}\right) = 0 \tag{8}$$

$$\frac{d^2T}{dy^2} = 0 \tag{9}$$

$$\frac{d^2S}{dy^2} = 0 \tag{10}$$

The corresponding boundary conditions in dimensionless form are

$$u = 0, N = -n\frac{du}{dy}, T = 1, (1-a)S + a\frac{dS}{dy} = (1-a) + a\frac{dT}{dy} \text{ on } y = 0 \tag{11}$$

$$u = 0, N = n\frac{du}{dy}, T = R_T, (1-a)S + a\frac{dS}{dy} = (1-a)R_s + a\frac{dT}{dy} \text{ on } y = 1 \tag{12}$$

In the present formulation the particular case a = 0 corresponds to double-diffusive convection for which the solutal buoyancy forces are induced by the imposition of a constant concentration such that S = 1 on y = 0 and $S = R_S$ on y = 1. On the other hand a = 1 corresponds to the case of a binary fluid subject to the Soret effect. For this situation, it follows from Eqs. (11) and (12) that $dS/dy = dT/dy$ on y = 0, 1.

METHOD OF SOLUTION

It can be shown that Eqs. (7)– (10), together with the boundary conditions Eqs. (11)– (12), possess the following Finite Element solution, obtained with the help of the MATLAB software. In order to reduce the above system of differential equations to a system of dimensionless form, we may represent the velocity and microrotation, temperature and concentration by applying the Galerkin finite element method for equation (7) over a typical two-noded linear element (e) $(y_j \leq y \leq y_k)$ is

$$u = N.\emptyset, N = [N_j, N_k], \quad \emptyset = \begin{bmatrix} u_j \\ u_k \end{bmatrix}, N_j = \frac{y_k - y}{l}, \qquad N_k = \frac{y - y_j}{l}, l = y_k - y_j = h,$$

$$\int_{y_j}^{y_k} N^T \left[(1 + K)\frac{d^2u}{dy^2} + K\frac{dN}{dy} - Mu + (T + \varphi S)\right] dy \tag{13}$$

$$\int_{y_j}^{y_k} \left[(1 + K)\frac{\partial^2 u}{\partial Y^2} + Mu - R\right] dy = 0 \qquad where \quad R = K\frac{dN}{dt} + (T + \varphi S)$$

The element equation given by

$$\int_{y_j}^{y_k} (1 + K)\begin{bmatrix} N'_j N'_j & N'_j N'_k \\ N'_k N'_j & N'_k N'_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} dy + M\begin{bmatrix} N_j N_j & N_j N_k \\ N_k N_j & N_k N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} dy - R\begin{bmatrix} N_j \\ N_k \end{bmatrix} dy = 0$$

$$\int_{y_j}^{y_k} S \, dy = \int_{y_j}^{y_k} R^* \, dy \tag{14}$$

$$where \quad S = (1 + K)\begin{bmatrix} N'_j N'_j & N'_j N'_k \\ N'_k N'_j & N'_k N'_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + M\begin{bmatrix} N_j N_j & N_j N_k \\ N_k N_j & N_k N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} dy \quad and \, R^* = R\begin{bmatrix} N_j \\ N_k \end{bmatrix}$$

$$S = \frac{(1 + K)}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{Ml}{6}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} \quad and \, R^* = R\frac{l}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We write the element equation for the elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$. Assembling these element equations, we get

$$\frac{(1+K)}{l}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{M\,l}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = R\frac{l}{2}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{15}$$

Now put row corresponding to the node i to zero, from equation (15) the difference schemes with $l = h$ is

$$\frac{(1+K)}{h}(-u_{i-1} + 2u_i - u_{i+1}) + \frac{Mh}{6}(u_{i-1} + 4u_i + u_{i+1}) = R^* $$

(16)

Using the Cranck-Nicolson method to the equation (16), we obtain:

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^j + A_5 u_i^j + A_6 u_{i+1}^j + R^* \tag{17}$$

Similarly, the equations (8), (9) and (10) are becoming as follows:

$$B_1 N_{i-1}^{j+1} + B_2 N_i^{j+1} + B_3 N_{i+1}^{j+1} = B_4 N_{i-1}^j + B_5 N_i^j + B_6 N_{i+1}^j + R_1^* \tag{18}$$

$$C_1 T_{i-1}^{j+1} + C_2 T_i^{j+1} + C_3 T_{i+1}^{j+1} = C_4 T_{i-1}^j + C_5 T_i^j + C_6 T_{i+1}^j \tag{19}$$

$$D_1 S_{i-1}^{j+1} + D_2 S_i^{j+1} + D_3 S_{i+1}^{j+1} = D_4 S_{i-1}^j + D_5 S_i^j + D_6 S_{i+1}^j$$

(20)

$$A_1 = \left(\frac{Mh}{6} - \left(\frac{1+K}{h}\right)\right), \; A_2 = \left(2\left(\frac{1+K}{h}\right) + \frac{Mh}{3}\right), A_3 = \left(\frac{Mh}{6} - \left(\frac{1+K}{h}\right)\right),$$

$$A_4 = \left(\left(\frac{1+K}{h}\right) - \frac{Mh}{6}\right), \; A_5 =, -\left(2\left(\frac{1+K}{h}\right) + \frac{Mh}{3}\right), A_6 = \left(\left(\frac{1+K}{h}\right) - \frac{Mh}{6}\right),$$

$$B_1 = \left(\frac{hBK}{3} - \frac{1+\frac{K}{2}}{h}\right), \; B_2 = \left(\frac{4hBK}{3} + 2\left(\frac{1+\frac{K}{2}}{h}\right)\right), B_3 = \left(\frac{hBK}{3} - \frac{1+\frac{K}{2}}{h}\right),$$

$$B_4 = \left(\frac{1+\frac{K}{2}}{h} - \frac{hBK}{3}\right), \; B_5 = -\left(\frac{4hBK}{3} + 2\left(\frac{1+\frac{K}{2}}{h}\right)\right), B_6 = \left(\frac{1+\frac{K}{2}}{h} - \frac{hBK}{3}\right),$$

$$C_1 = -\frac{1}{h}, \; C_2 = \frac{2}{h}, C_3 = -\frac{1}{h}, C_4 = \frac{1}{h}, \; C_5 = -\frac{2}{h}, C_6 = \frac{1}{h},$$

$$D_1 = -\frac{1}{h}, D_2 = \frac{2}{h}, D_3 = -\frac{1}{h}, D_4 = \frac{1}{h}, D_5 = -\frac{2}{h}, D_6 = \frac{1}{h},$$

$$R^* = h\left(K\left(\frac{N_{i+1} - N_{i-1}}{h}\right) + T + \varphi S\right), R_1^* = -hK\left(\frac{u_{i+1} - u_{i-1}}{h}\right),$$

Here $r = \frac{k}{h^2}$ where k, h is mesh sizes along y direction and x direction respectively. Index i refers to space and j refers to time. The mesh system consists of h=0.1 for velocity profiles and concentration profiles and k=0.1 has been considered for computations. In equation (8)-(10), taking i=1(1) n and using initial and boundary conditions (11) and (12), the following system of equation are obtained.

$$A_i X_i = B_i, \; i = 1, 2, 3 \dots.$$

(21)

Where A_i's are matrices of order n and $X_i \text{ and } B_i$' s are column matrices having n-components. The solution of above system of equations are obtained using Thomas algorithm for velocity, angular velocity and temperature. Also, numerical solutions for these equations are obtained by MATLAB program. In order to prove the convergence and stability of Galerkin finite element method, the same Mat lab-program was run with slightly changed values of h and k, no significant change was observed in the values of u, N, T, S. Hence, the Galerkin finite element method is stable and convergent.

Results and Discussion

The numerical computations for the velocity u, angular velocity fields N for various governing parameters the buoyancy ratio φ, vortex viscosity parameter K, dimensionless microgyration n and constant a are illustrated in the graphs. The Fig.1 illustrates the

influence of vortex viscosity parameter K on the distribution of velocity u and microrotation N for n=0,a=0 and for φ=5 in Fig.1(a) and φ=-5 in Fig.1(b). It is observed that with the increasing the value of K the intensity of convective velocity u is reduced as compared to the Newtonian fluid situation (K=0). In fact it is found that as K→ ∞, u→ **0**.The influence of parameter K on the microrotation N it is noticed that the variation with K of the value of N evaluated at the position half of the channel also presented in the graphs it can be seen that the intensity of N first increases with increase of K, the reverse phenomenon is observed later.

Fig.1(b) show the results obtained from φ=-5 i.e. when thermal and solutal buoyancy forces are opposing each other for this situation in case of double-diffusive convection indicates that the flow direction is in down wards, since the solutal buoyancy forces predominant. The velocity profiles increases with the increase of K are observed from Fig. 1(b). It is seen that for K=0 when N=0, since no rotation can be occur in the absence of micropolar elements (Newtonian fluid situation). Microrotation N decreases with the increase of K up to half of the channel where as microrotation N flow direction is now downward and the reverse phenomenon is observed.

The effect of buoyancy ratio φ velocity u, microrotation N exemplified in Fig. 2 for the case a=0, n=0, K=5 in the absence of solute concentration effect i.e. when φ=0 the flow is induced solely by the imposed temperature gradients. It is observed from this figure when $\boldsymbol{\varphi} < \boldsymbol{0}$ the thermal and solutal buoyancy forces act in the same direction and the flow is considered to aided thus the magnitude of the fluid of the fluid velocity and microrotation promoted in the vertical channel on the other hand when $\boldsymbol{\varphi} < \boldsymbol{0}$ the solutal and buoyancy forces acts in opposite direction as a result the flow direction is now reversed since it is governed by the predominant solutal effects.

Fig. 3 Depicts the influence of micropolar parameter n velocity u, microrotation N profiles K=5, φ=10 and a=0, it can be seen from this figure, upon increase the value of n,the concentration of the solution becomes weaker such that the particle near the walls are the free to rotate. Which results there is an enhancement of the flow. It also seen that the velocity u increases with the increase of n.

The effect of magnetic filed parameter M on the velocity profiles u and mocrorotation N for K=5, φ=5 when a=0 is shown in Fig.4 (a) and K=5, φ=-5 when a=0 is shown in Fig.4 (b). Here it is observed that the velocity profiles decreases with an increase of M, microrotation profiles increases up to centre of the channel the reverse phenomenon is observed in the other part of the channel. It is indicates that from Fig.4 (b) that the velocity flow direction is now downward for φ=-5, since the solute buoyancy forces are free dominant. It can be seen that the velocity profiles u increases with an increase of M, The microrotation N decreases with the increase of M, up to middle of the channel (flow direction is upward) and it is increases with increase of M, in the other part of the channel is observed.

From Fig.5-10 are shown for the velocity profiles u and microrotation N for different values of flow parameter when a=1. The effect of Magnetic parameter M on the velocity profiles u and mocrorotation N for K=5, φ=5 is shown in Fig.5 (a) and K=5, φ=-5 is shown in Fig.5 (b) is remains same when compared with a=0 in the present case a=1. The effect of vortex viscosity parameter K, on the velocity profiles u and microrotation N are shown in Fig.6 for both φ=5 and φ=-5 in the case of a=1, n=0, the effect of K is same in both the cases a=0 and a=1, where as in the present case (a=1) for φ=-5 from Fig.6 (b) it indicates that the flow direction is upward in the present case where as it is downward case a=0. It also noticed that the effect of K decreases the velocity profiles u, the microrotation N decreases up to half of the channel and decreases the other part of the channel is observed with the effect of K, it is also observed that the reverse phenomenon is observed in the present case (a=1) when compared to a=0.

The buoyancy ratio parameter φ effect on the velocity profiles u and microrotation N are shown in Fig. 7. It is clear that the velocity profiles u increases with the increase of φ from 0 to 10, where as it is decreasing from 0 to -10. However up on increasing φ, considerably the flow pattern is depending on the sign of the parameter up or down in the halves of the channel, it can be also seen that

the microrotation profiles N decreases with the increase of φ from 0 to 10 and   increases with the decrease of φ from 0 to -10, in the first half of the channel is indicating from the Fig.7 and the reverse phenomenon is observed in the other half of the channel.

Fig.8 illustrates the influence of micro-gyration parameter n on velocity u and microrotation N for φ=10, K=5, a=1 it is noticed that the velocity u increases with the increase of n. In the present case for a=1, the results indicates the intensity of convective flow u and that of the angular velocity N are minimum for n = 0. This particular value of n represents the case where the concentration of the microelements is sufficiently large that the particles close to the walls are unable to rotate. Upon increasing the value of n, the concentration of the solution becomes weaker such that the particles near the walls are free to rotate. Thus, as n is augmented the microrotation term is augmented, which induces an enhancement of the flow.

Fig.9 (a) & (b) illustrates the Volume flow rate Q with the buoyancy ration parameter φ, when K=1.5, for the various values of micro-gyration parameter n at a=0 and a=1 for For the case of double-diffusive convection it is observed that when both the thermal and solutal buoyancy forces are aiding ($\phi > 0$), the flow direction is upward ($Q > 0$). The reverse is true ($Q < 0$) when both the thermal and solutal buoyancy forces are opposing ($\phi < 0$). On the other hand, for the case of Soret-induced convection, the flow rate is found to be independent of the buoyancy ratio ϕ. This follows from the fact that, for this situation, the quantity of the solute between the two vertical plates remains constant. The Soret effect acts merely to redistribute the concentration in the system, giving rise to local increase or decrease of the local velocity. However, the global flow rate remains constant. Also, as discussed above, upon increasing the value of n the intensity of the velocity field (and thus of the flow rate Q) is enhanced.

The dimensionless total rate, E, at which heat is added to the fluid is plotted in Fig. 10 (a) and (b) as a function of the buoyancy ratio ϕ and the micro-gyration parameter n, for the case K = 1.5. Fig. 10(a) shows that, in the case of double diffusive convection, for $\phi > 0$, increasing ϕ results in an augmentation of the strength of the convective motion such that E increases. For $\phi < 0$, the results are similar but, since the flow direction is now downward, the value of E is negative. On the other hand the results obtained for Soret-induced convection, Fig. 10(b), are quite different. For this situation, the velocity profiles (not presented here) indicate that for $\boldsymbol{\phi \gg 1}$ the flow is upward near the left hotter wall and downward near the right colder one. Thus, the total rate E at which heat is added to the fluid is promoted upon increasing ϕ as a result of the increase of the flow intensity near the hotter wall.

We now consider the buoyancy ratio. The dimensionless total rate, Φ, at which species are added to the fluid is depicted in Fig. 11 as a function of ϕ and the micro-gyration parameter n, for the case K = 1.5. The Soret-induced convection, represented by a dotted line, indicates that $\Phi = 0$ independently of n. This is expected since for this situation the solid boundaries are impermeable to concentration. The Soret effect is merely to redistribute the originally uniform concentration within the system. However, for double diffusion, the solid lines indicate that increasing ϕ, i.e. increasing the strength of the convective flow, results in an enhancement of the rate of mass transfer through the system. These results are similar to those reported by Cheng [13]. Also, it is observed from Fig. 11 that, for a given value of ϕ, Φ decreases as the value of n is reduced toward n = 0. As already mentioned, a decrease of n corresponds to an increase of the concentration of the solution such that the particles close to the solid boundaries are unable to rotate. This results in a decrease of the flow rate and thus a decrease of Φ. The volume flow rate, Q, and total rate at which heat is added to the fluid, E, are plotted in Fig. 12 as a function of K for $\phi = 2$ and n = 0. Here again, the results obtained for double-diffusive convection and Soret-induced convection are qualitatively similar. In the limit $K \to 0$ both Q and E tend asymptotically to constant values corresponding to the Newtonian fluid situation. On the other hand, in the limit $K \to \infty$, both Q and E become negligible, due to the increase of the vortex viscosity.

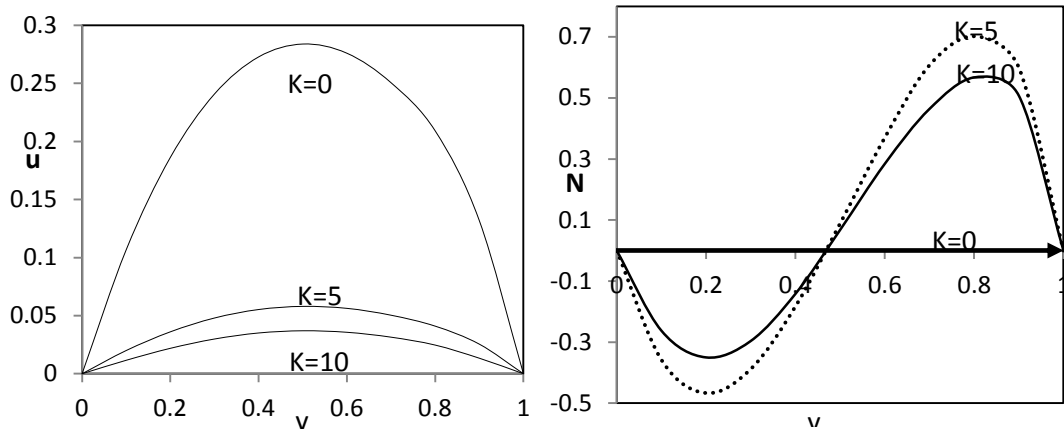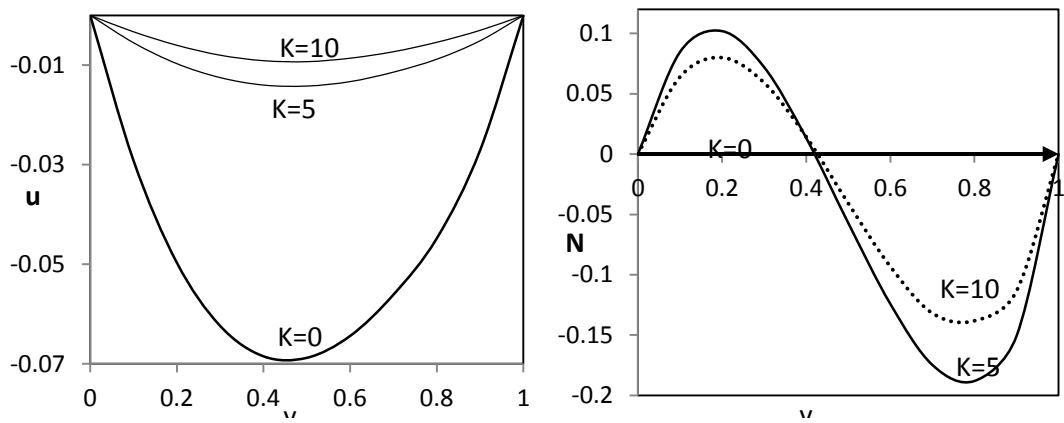Figure 1(a): Effect of Parameter K on the velocity profiles u and the microrotation N for n=0, a=0 and φ=5,

Figure 1(b): Effect of Parameter K on the velocity profiles u and the microrotation N for n=0, a=0 and φ=-5
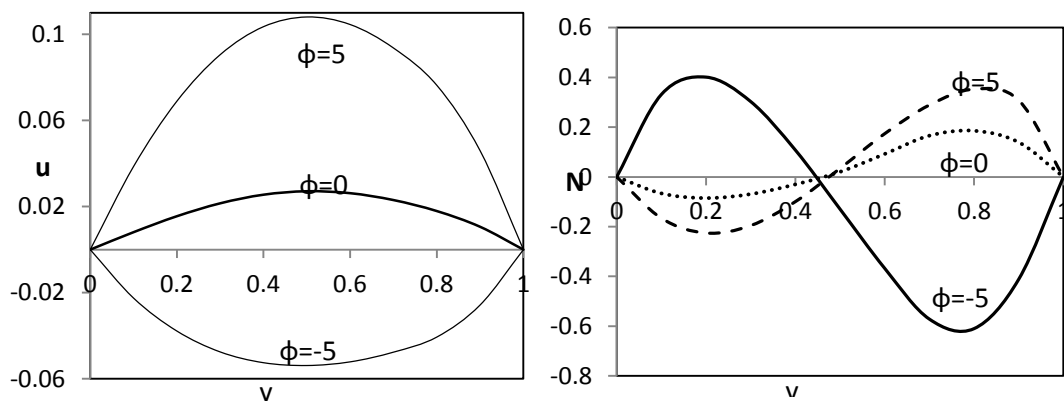

Figure. 2. Effects of buoyancy ratio φ on the velocity profiles u and the microrotation profiles N for K = 5, n = 0 and a = 0.
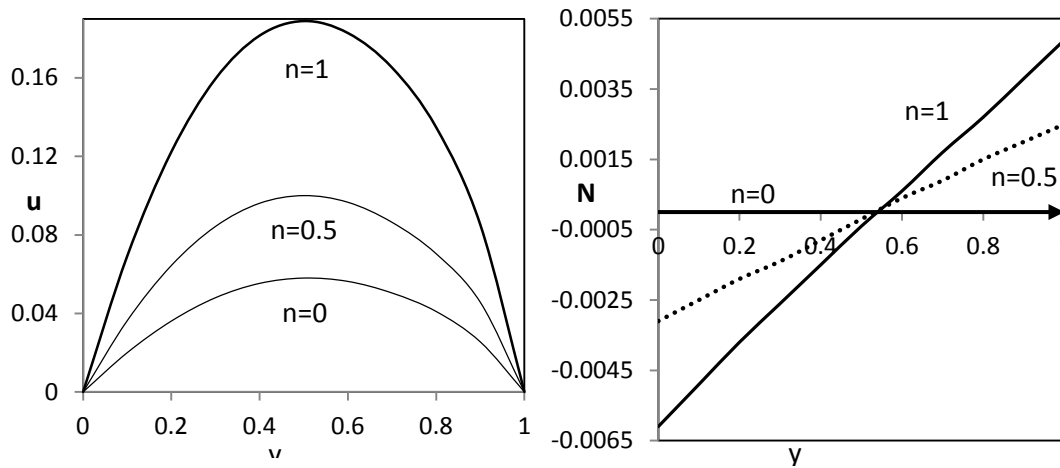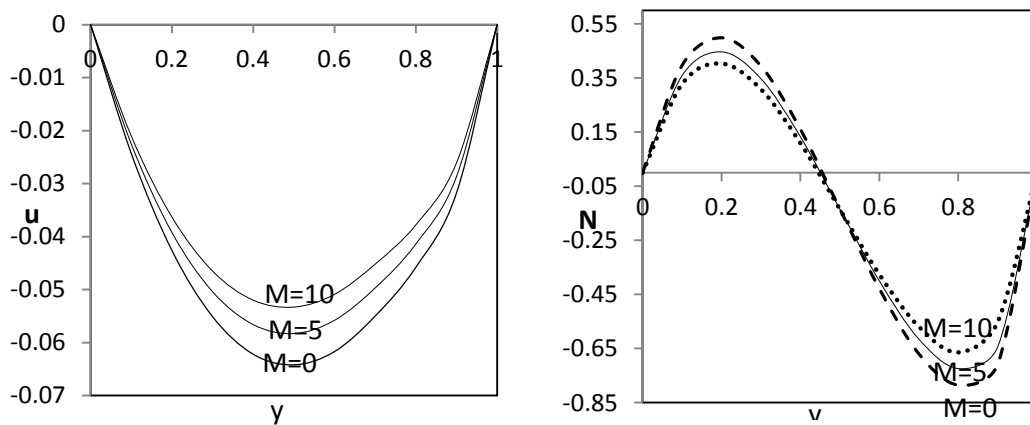
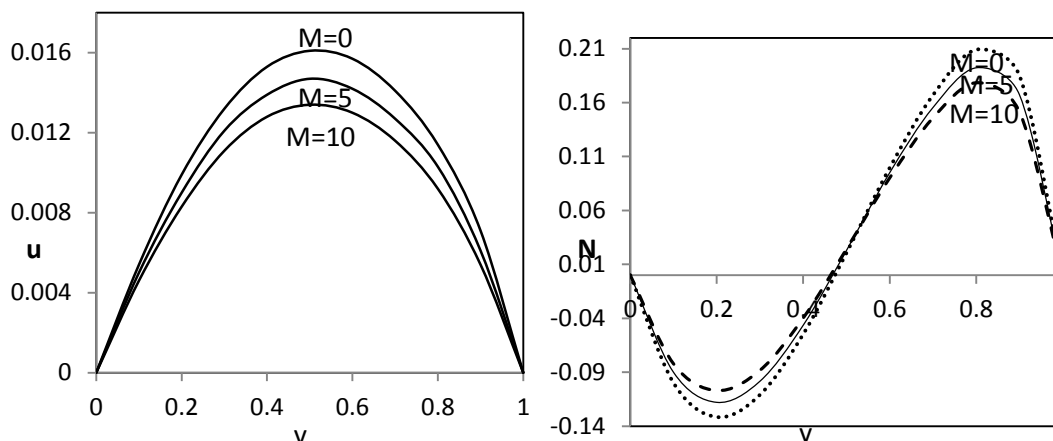Figure 3. Effects of parameter n on the velocity profiles u and the microrotation profiles N for K $=5$, $\phi = 10$ and a $= 0$.
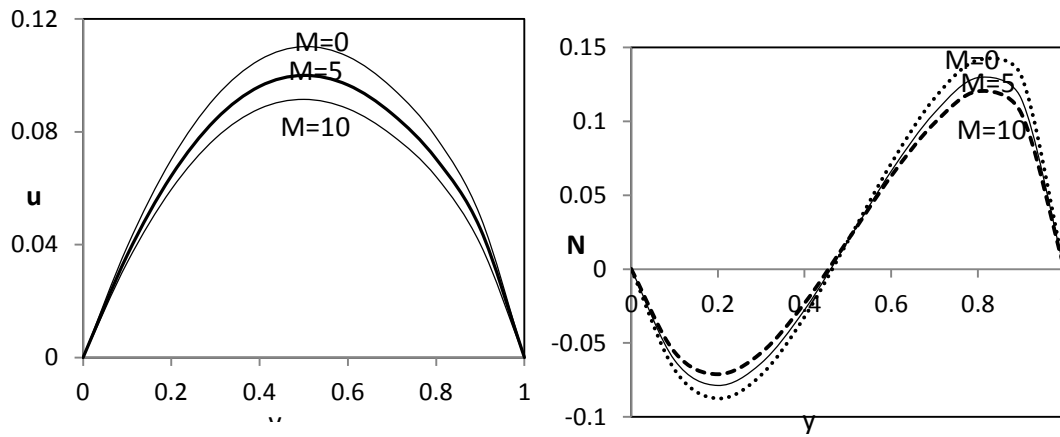
Figure 4(a). Effects of parameter M on the velocity profiles u and the microrotation profiles N for K $=5$, $\varphi=5$ and a$=0$.
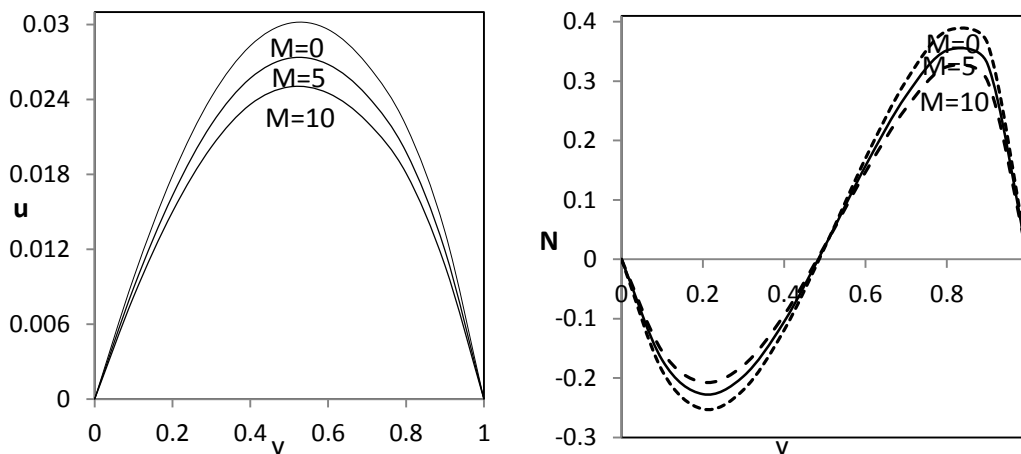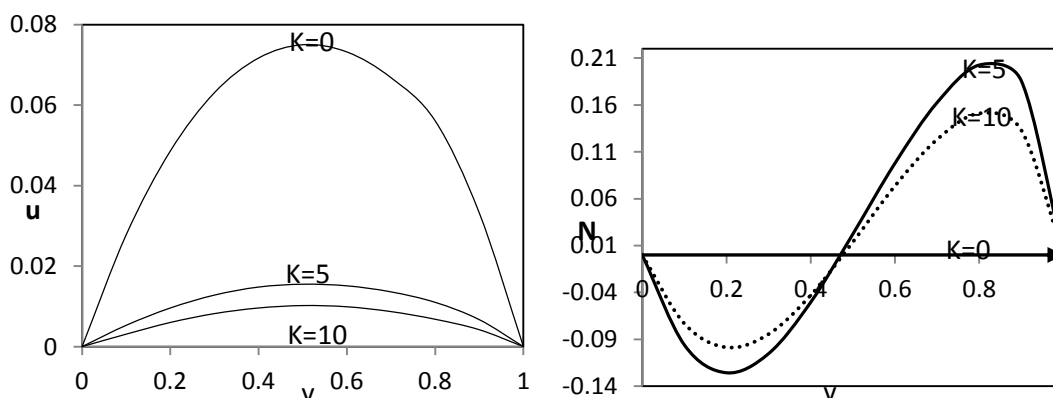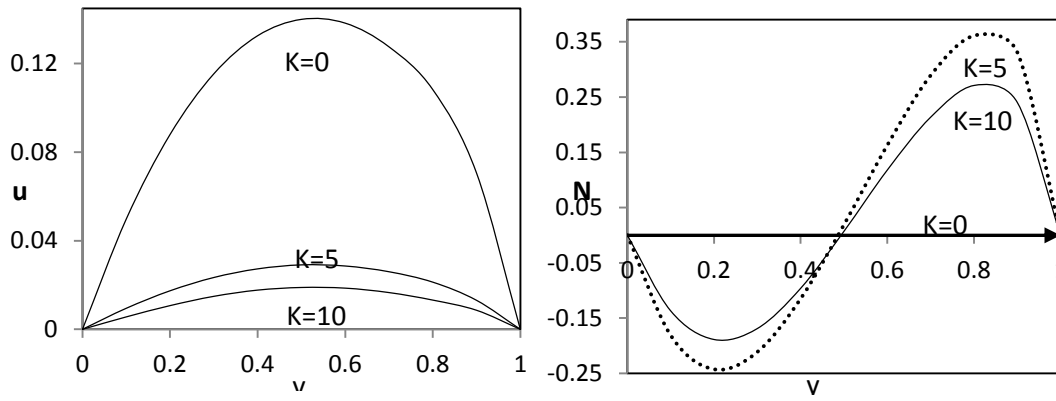

Figure 4(b). Effects of parameter M on the velocity profiles u and the microrotation profiles N for K $=5$, $\varphi=-5$ and a$=0$.

Figure 5(a). Effects of parameter M on the velocity profiles u and the microrotation profiles N for K =5, φ=5 and a=1.

Figure 5(b). Effects of parameter M on the velocity profiles u and the microrotation profiles N for K =5, φ=-5 and a=1.

Figure 6(a): Effect of Parameter K on the velocity profiles u and the microrotation N for n=0, a=1 and φ=5.

Figure 6(b): Effect of Parameter K on the velocity profiles u and the microrotation N for n=0, a=1 and φ=-5.
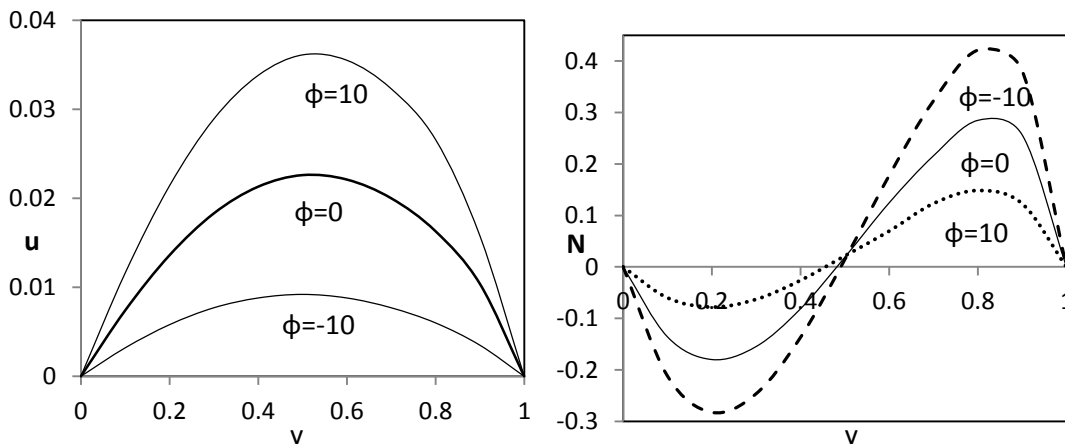

Figure.7. Effects of buoyancy ratio φ on the velocity profiles u and the microrotation profiles N for K = 5, n = 0 and a = 1.
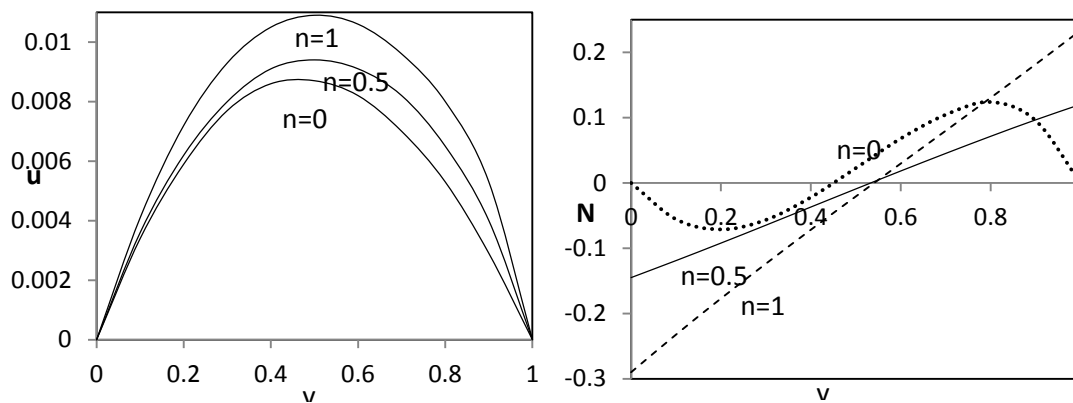

Figure 8. Effects of parameter n on the velocity profiles u and the microrotation profiles N for K=5, φ=10 and a = 1.
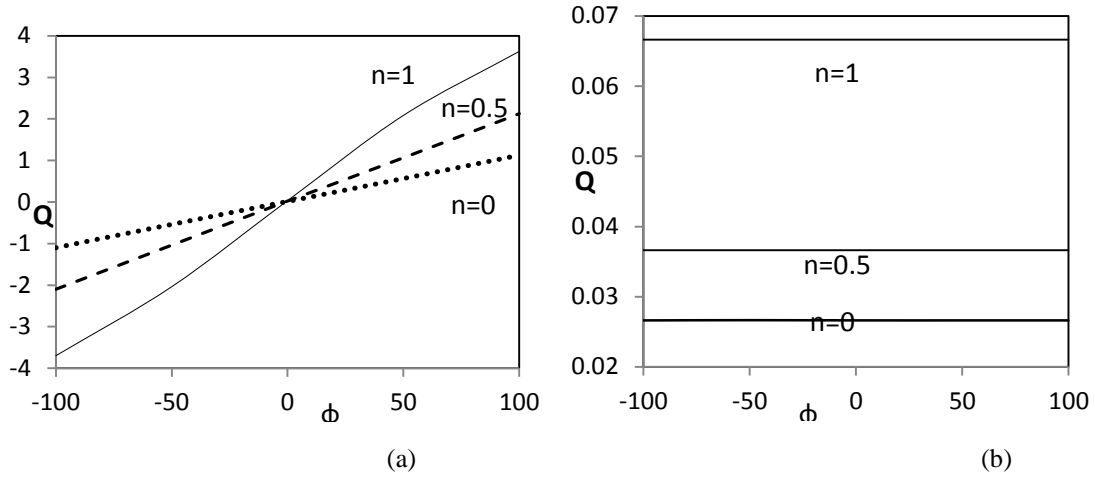
Figure 9. Effects of buoyancy ratio ϕ and parameter n on the volume flow rate Q for K = 1.5, (a) a = 0, (b) a = 1.
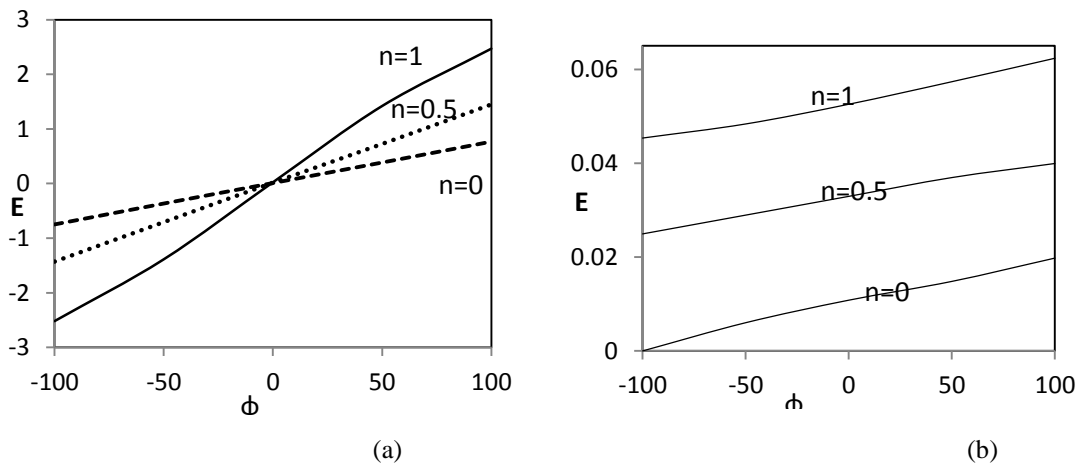

Figure 10.Effects of buoyancy ratio ϕ and parameter n on the total rate at which heat is added to the fluid, E, for K = 1.5,
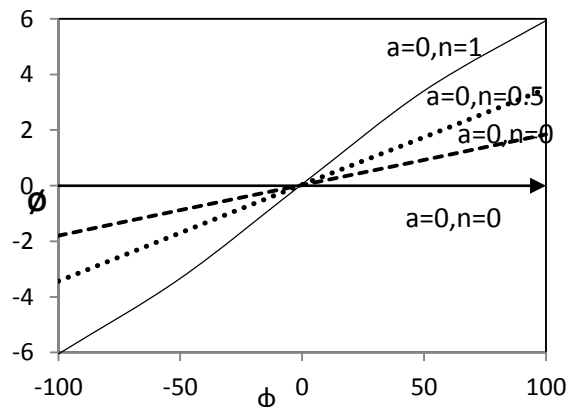
(a) a = 0, (b) a = 1.

Figure 11.Effects of parameter K on the volume flow rate Q and on the total rate at which heat is added to the

fluid for $\phi = 2$ and n = 0

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