



A new method for PAPR reduction in OFDM using Embedded Transform techniques

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ABSTRACT

High speed research in communication trends demands high data rates with reliable transmission system and has thus led to many new emerging modulation techniques. Orthogonal Frequency Division Multiplexing (OFDM) is one of the most promising technologies for high data rate wireless communications due to its robustness, high spectral efficiency, frequency selective fading, and low computational complexity and easy implementation of Very Large Scale Integrated technology (VLSI) now. Because the OFDM system effectively provides numerous parallel narrowband channels, OFDM is considered a key technology in emerging high-data rate systems such as 4G, IEEE 802.16, and IEEE 802.11n. But there is one main disadvantage of OFDM that is the high peak-to-average power ratio (PAPR) of the transmitter's output signal on different antennas. High Peak to Average Power Ratio (PAPR) for OFDM system is still a demanding area and difficult issue. By now, for reducing PAPR, numerous techniques have been recommended. In this paper the embedded technique which is a combination of Selective level Mapping (SLM), square root and Discrete Cosine Transform (DCT) based PAPR reduction technique has been analyzed and discussed. The results have been verified in terms of various graphs and plots and have been compared with earlier results. The performance comparison of the proposed scheme with conventional technique has also been presented in the paper.

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Introduction

The increasing demand on high data rates and reliable wireless systems has led to many new emerging modulation techniques. Orthogonal Frequency Division Multiplexing (OFDM) is a form of multicarrier modulation that can be seen as a modulation technique or a multiplexing scheme. OFDM is an FDM modulation technique for transmitting large amounts of digital data over a radio wave. OFDM works by splitting the radio signal into multiple smaller sub-signals that are then transmitted simultaneously at different frequencies to the receiver. OFDM reduces the amount of crosstalk in signal transmission. 802.11a WLAN and 802.16 and WiMAX protocols use OFDM.

OFDM has many advantages but one of the major challenges associated with OFDM is higher peak-to-average power ratio than single carrier signals do, which causes poor efficiency and system degradation due to inherent non-linearity in power amplifiers [1]. The reason for high PAPR in an OFDM signal is that in time domain, a multicarrier signal is sum of many narrowband signals. At some time instances, this sum is large and at other times it is small, which means that the peak value of the signal is substantially larger than the average value. The use of Selective level mapping (SLM) technique allows the transmitter to generate a set of sufficiently different candidate data blocks all representing the same information as original data block and selects the most favorable one having minimum PAPR for transmission. Selection of proper phase sequences to achieve good PAPR reduction is very important in SLM technique. Phase sequence set is chosen from set $\pm 1, \pm j$ as proposed by Bauml [2] who first introduced SLM technique. Discrete cosine transform based SLM technique is based on precoding the constellation symbols with DCT precoder after the multiplication of phase rotation vector in SLM_OFDM System. DCT_SLM along with combination of square root of coded OFDM signal has been proposed which obtains a better PAPR reduction.

Hadamard transform is an effective technique to reduce PAPR. The main advantage of using Hadamard transform over clipping process is that the latter technique results both in in-band distortion and out-band distortion

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which results in performance degradation of OFDM systems [3][4]. Hadamard transform reduces the PAPR while the error probability or BER of the system is not increased [5].

PAPR problem

The key challenge for OFDM signal is large peak-to-average power ratio (PAPR) due to non-linearity of power amplifier, phase noise problems of local oscillator, frequency offset due to Doppler shift or difference between transmitter and receiver. Large PAPR distorts the signal if transmitter contains non-linear components like power amplifiers. The non-linear effects on transmitted OFDM symbols are spectral spreading, inter modulation and changing signal constellation. The PAPR of OFDM signal is defined as ratio between maximum power and the average power. The peak to average power ratio (PAPR) for a signal $x(t)$ is mathematically equal to :

$$PAPR = \frac{\max[x(t)x^*(t)]}{E[x(t)x^*(t)]} \tag{1}$$

Where * corresponds to the conjugate operator.

Expressing in decibels,

$$PAPR(db) = 10 \log_{10}(PAPR) \tag{2}$$

PAPR of a single tone

Consider a sinusoidal signal $x(t) = \sin(2\pi ft)$, the peakvalue of signal is

$$\max[x(t)x^*(t)] = +1 \tag{3}$$

The mean square value of signal is

$$E[x(t)x^*(t)] = \frac{1}{T} \int_0^T \sin^2(2\pi ft) dt = \frac{1}{2} \tag{4}$$

Given so, the PAPR of a single tone is

$$PAPR = \frac{1}{\frac{1}{2}} = 2 \tag{5}$$

PAPR of a complex sinusoidal

Consider a complex sinusoidal signal $x(t) = e^{j2\pi ft}$, the peakvalue of signal is

$$\max[x(t)x^*(t)] = +1 \tag{6}$$

The mean square value of signal is given as

$$E[x(t)x^*(t)] = \frac{1}{T} \int_0^T \exp^{j2\pi ft} \exp^{-j2\pi ft} dt = 1 \tag{7}$$

Given so the PAPR of a single complex tone is

$$PAPR = 1$$

Maximum expected PAPR from OFDM waveform

OFDM signal is the sum of multiple sinusoidal having frequency separation $1/T$ where each sinusoidal gets modulated by independent information. Mathematically, the transmitted signal is

$$x(t) = \sum_0^{N-1} a_N e^{j2\pi Nt/T} \tag{8}$$

For simplicity, let us assume that $a_N = 1$ for all N sub carriers. The peak value of signal is

$$\max[x(t) x^*(t)] = \max[\sum_0^{N-1} e^{j2\pi Nt/T} \sum_0^{N-1} e^{-j2\pi Nt/T}] \tag{9}$$

$$= \max[a_N a_N^* \sum_0^{N-1} 1 \sum_0^{N-1} 1] = N^2 \tag{10}$$

$$= N^2 \tag{11}$$

The mean square value of the signal is

$$E[x(t) x^*(t)] = E[\sum_{l=0}^{N-1} a_l N e^{j2\pi Nt/T} \sum_{l=0}^{N-1} a_l N^* e^{-j2\pi Nt/T}] \tag{12}$$

$$= E[a_l N a_l N^* \sum_{l=0}^{N-1} \sum_{l=0}^{N-1} e^{j2\pi Nt/T} e^{-j2\pi Nt/T}] \tag{13}$$

$$= N \tag{14}$$

The peak to average power ratio for an OFDM system with N

Subcarriers given the same modulation is,

$$PAPR = \frac{N^2}{N} = N \tag{15}$$

The above value corresponds to maximum value of PAPR when all sub-carriers are equally modulated and peak value hits maximum. As Per IEEE 802.11a specification, we have N= 52 used subcarriers. The maximum expected PAPR is thus 52(~ 17 dB).However, due to scrambling SLM techniques PAPR is reduced considerably.

Review of different PAPR reduction techniques in OFDM

Selective Level Mapping (SLM) technique

Selective Level Mapping (SLM) is a probabilistic technique in which the probability of occurrence of high PAPR is reduced by modifying the signal [6]. In selective level mapping technique transmitter generates a set of sufficiently different candidate data blocks, all representing the same information as original data blocks and one with minimum PAPR is selected for transmission. The CCDF (complementary cumulative distribution function) of the original signal sequence $PAPR$ above threshold $PAPR_0$ is written as $Pr(PAPR > PAPR_0)$. Accordingly for K statistical independent waveforms, CCDF can be written as $[[Pr(PAPR > PAPR_0)]]^K$ so that the probability of PAPR that exceeds the same threshold will drop to a small value. After applying the IFFT, an OFDM signal x_n of N subcarriers can be expressed as [7],[8]

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad 0 \leq n \leq N-1 \tag{16}$$

Where $K = 0, 1, 2, 3 \dots N-1$ are the input symbols modulated by QPSK and n is the discrete line index.

Figure.1. Shows block diagram of SLM technique [6].

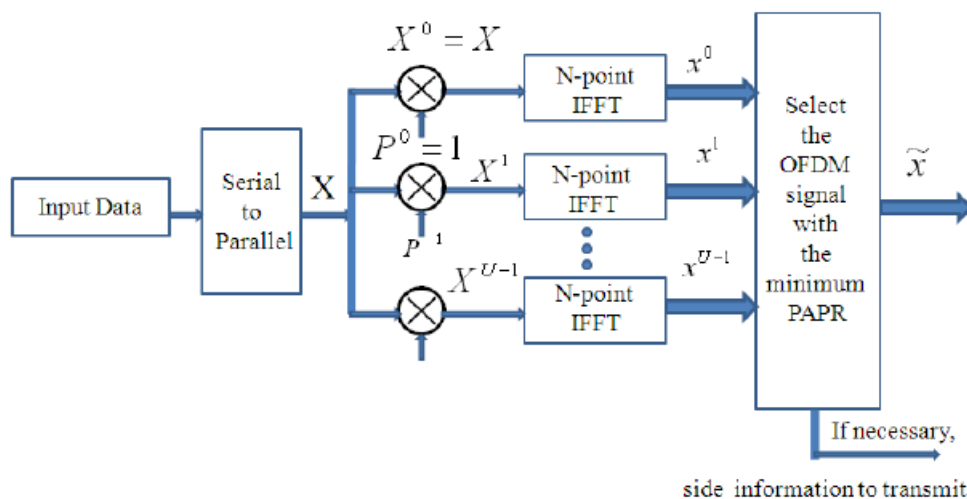


Figure.1. A block diagram of Selective Level Mapping Technique

The data blocks after multiplying by U different phase sequences each of length N are represented as X(U), U=1,2,.....,u, the one with lowest PAPR is selected for transmission after undergoing inverse fast Fourier transform. Information about the selected phase sequence should be transmitted to receiver as side information. At the receiver reverse

operation is performed to recover original data block. The amount of PAPR reduction for SLM depends on the number of phase sequences U and design of phase sequence. The SLM technique implementation needs U IFFT operations and number of required side information bits is $\log_2 U$ for each data block [10].

Discrete Cosine Transform (DCT)

The basic idea to use DCT transform in OFDM is to reduce the autocorrelation of the input sequence to reduce the peak to average power problem and it requires no side information to be transmitted to the receiver. DCT_SLM combined with square root of coded OFDM signal obtain a better PAPR reduction. The combination of the DCT matrix and square root of the coded OFDM signal is used after applying inverse fast Fourier transform to reduce the PAPR of the signal. In this technique the autocorrelation of the signal which has been processed by SLM is reduced by DCT

Hadamard Transform

The Hadamard transform reduces the occurrences of high peaks comparing the original OFDM signal and requires no side information to be transmitted [9]. The Hadamard transform H_n is a $2^m \times 2^m$ matrix, that transforms 2^n real or complex number x_n into 2^n real numbers x_n^* . Every element of Hadamard matrix is 1 or -1. The Hadamard matrix of order n is stated by

$$H_n = \frac{1}{\sqrt{2}} \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{bmatrix} \quad (17)$$

BY definition $H_0 = \mathbf{1}$ thus,

$$H_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (18)$$

After the sequence $X = [X_1, X_2, \dots, X_n]$ is transformed by Hadamard matrix of order N , the new sequence obtained is,

$$Y = HX \quad (19)$$

The Hadamard code is given after applying inverse fast Fourier transform.

3.4 Proposed Algorithm for PAPR reduction using embedded scheme

The algorithm for Embedded method proposed in this work is as follows:

- i. Choose the no of sub-carriers 'N' and oversampling factor 'of'
- ii. Multiply both to obtain K (in this work $k=512$)
- iii. Select QPSK constellation symbols and define the rotation factor value range
- iv. Generate the OFDM symbols in frequency domain as an array of 0's and 1's for different paths
- v. Take the IFFT for different paths
- vi. Calculate the PAPR for each path
- vii. Calculate the Complementary Cumulative Distribution Function (CCDF) of original signal
- viii. Define the different route number used in proposed SLM for $M=2, 4, 8, 16$
- ix. Calculate the PAPR value of OFDM signal using SLM method with different M
- x. Calculate signal's Complementary Cumulative Distribution Function (CCDF with $M=2, 4, 8, 16$)
- xi. Plot the CCDF curves with different M value

The basic algorithm for other three proposed techniques is same, however few changes are made for each technique

For SQRT_SLM;

Take the square root of IFFT generated signal after PAPR evaluation and rest of algorithm is same

For SQRT_SLM+DCT_SLM;

The DCT of IFFT signal is obtained and added with Square root of OFDM signal and rest of algorithm being same

For Hadamard_SLM

The Hadamard code is obtained for IFFT generated signal using Hadamard transform $Y=HX$ with rest of algorithm being same

Simulation and Results

In this scheme embedded method in which combination of selective level mapping (SLM), Discrete Fourier Transform (DFT), square root method have been used. Sixty four carriers have used in this scheme and the over sampling factor used is eight. The specifications used in this work has been made as per the International Telecommunication Union (ITU). The following four simulation results illustrate the effect of implementing basic SLM, Square of coded OFDM based SLM(sqrt_SLM), combination of DCT based SLM along with square root of coded OFDM signal based SLM(DCT+sqrt_SLM),and Hadamard transform based SLM techniques for $M=2,4,8,16$ and compares it with original OFDM signal. The graphs shown are plotted between CCDF and PAPR0 (db). The simulation result for basic SLM has been shown in figure 2 with $CCDF(Pr[PAPR>PAPR0])$ equal to max(i.e. equal to 1), it can be seen that the original signal has the PAPR of $\cong 10.6$ db and shows the significant decrease with increasing values for M, for $M=16$, the reduction is nearly $\cong 3.59$ db.

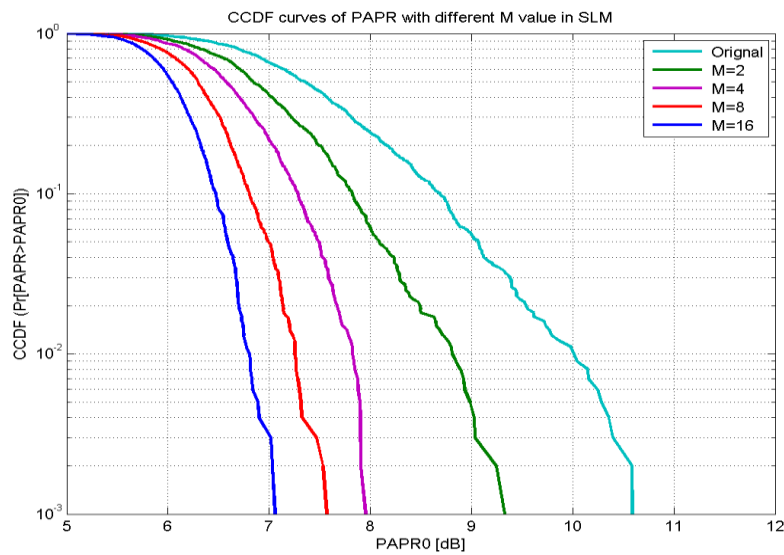


Figure.2. Plot of Basic_SLM

Figure.3 shows the simulation result for Sqrt_SLM, the PAPR reduction obtained for $M=16$ is $\cong 3.3$ db as compared to original OFDM signal.

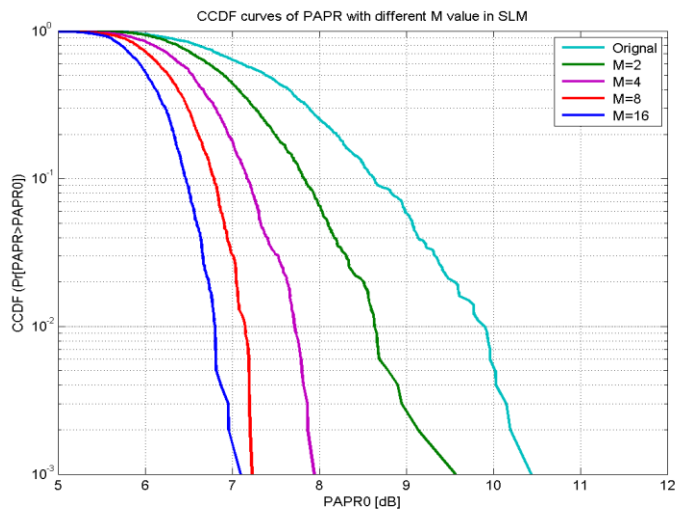


Figure.3. Plot of Sqrt_SLM

Figure.4 shows the result for combination of DCT_SLM and sqrt_SLM (DCT+sqrt_SLM), the reduction in PAPR obtained in this result is ≈ 3.6 db for M=16 as compared to original signal. The value of PAPR for M=16 is lowest (≈ 6.91 db) with the PAPR reduction of ≈ 3.6 db.

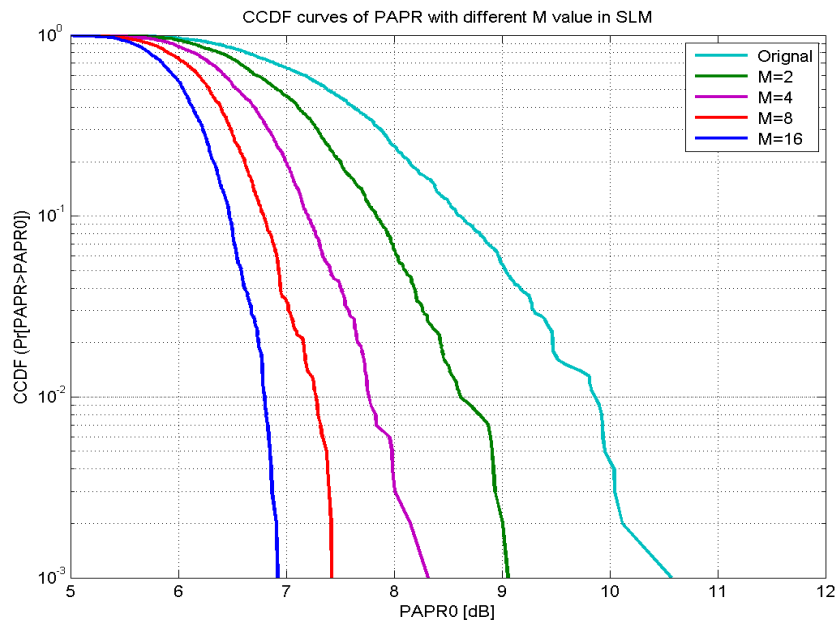


Figure.4. Plot of SQRT_SLM+ DCT_SLM

Figure.5 shows the simulation result for Hadamard code (Hadamard_SLM). The PAPR decreases for increasing values of M. The PAPR for M=16 reduces to ≈ 6.97 db as compared to the original signal of ≈ 10.8 db, thus there is a significant decrease in PAPR reduction of ≈ 3.73 db.

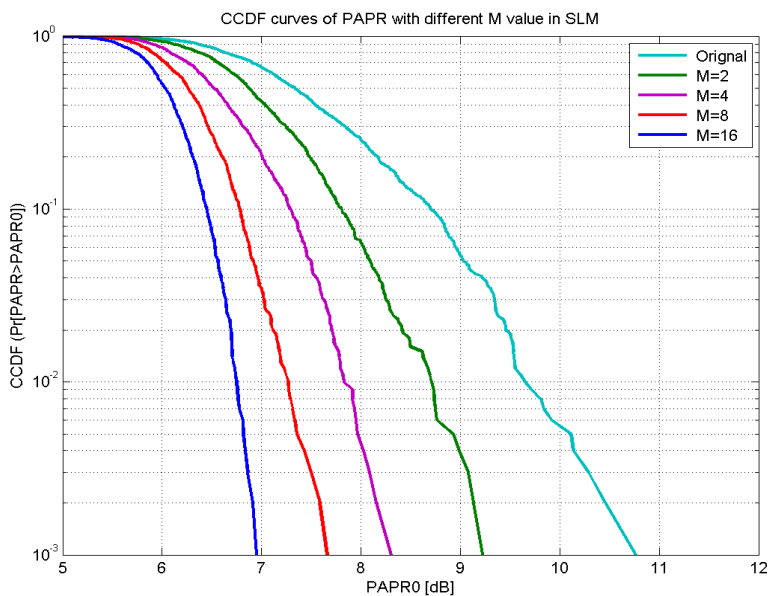


Figure.5. Plot of Hadamard_SLM

The comparison results have also been given in tabular form for different M values and is shown in table 1. The PAPR of the original OFDM signal is calculated using the proposed techniques. Subsequently, the PAPR for different values of M is calculated using the suggested techniques. For higher values of M there is significant reduction in PAPR as compared to the original signal.

Table 1. PAPR comparison for various M-ary Phase modulations and original OFDM signal

M	PAPR due to basic SLM (db)	PAPR due to SQRT_SLM(db)	PAPR due to DCT+SQRT_SLM (db)	PAPR due to hadamard_SLM (db)
Original signal	10.6	10.4	10.55	10.8
M=2	9.35	9.55	9.05	9.2
M=4	7.9	7.84	8.3	8.3
M=8	7.6	7.2	7.4	7.7
M=16	7.05	7.05	6.91	6.97

Conclusion

In this paper SLM, SQRT_SLM, combination of DCT_SLM and SQRT_SLM and Hadamard_SLM techniques are proposed. The results are evaluated using computer simulation. The results show a reduction in PAPR with increasing value of M as compared to the original signal. The combination of DCT_SLM and SQRT_SLM show the significant reduction in PAPR for M=16 with the PAPR of $\cong 6.91$ db as compared to original OFDM signal of $\cong 10.55$ db, besides there is no degradation of system performance. The proposed scheme has a lot of scope in next generation network systems. Moreover with this improvement it can be considered as a potential candidate for high speed data transmission systems.

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