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Two dimensional flow of micropolar fluid in a porous channel in the light of variable viscosity and thermal conductivity

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ABSTRACT

The effect of variable viscosity and thermal conductivity of two dimensional flow of a micropolar fluid in a porous channel is investigated. The flow is driven by suction or injection at the channel walls and the micropolar model due to Erigen is used to describe the working fluid. An extension of Berman's similarity transform is used to reduce the governing equations to get non-linear coupled differential equations. The equations governing the motion, angular momentum and energy are solved numerically by Runga-Kutta shooting technique. The graphs are plotted for velocity distribution, temperature distribution and microrotation distribution for various values of non-dimensional parameters. It is found that the effects of the parameters giving variable property of viscosity and thermal conductivity are significant.

Introduction

The theory of micropolar fluids was originally formulated by Eringen [3]. In essence, the theory introduces new material parameters, an additional independent vector field-the microrotation-and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents prporous media, turbulent shear flows, and flowing capillaries and microchannels by Lukaszewiez [6].

We analyze the effect of the variable viscosity and the variable thermal conductivity on self-similar boundary layer flow of a micropolar fluid in a porous channel, where the flow is driven by uniform mass transfer through the channel walls. The corresponding Newtonian fluid model was first studied by Berman [1], who described an exact solution of the Navier-Stokes equations by assuming a self-similar solution and reducing the governing partial differential equations to a nonlinear ordinary differential equation of fourth order. The solution is of potential value in understanding more realistic flow in channels and pipes, and study of Berman's exact solution and generalizations of it have attracted numerous studies subsequently, for example Yuan [10], Robinson [8], Zaturska et. al. [11], Desseaux [2].

Through the viscosity and thermal conductivity are assumed as constant properties but in actual these are temperature dependent (Schlichiting [9], Eckert[4]). Therefore, in this paper we consider the effect of variable viscosity and variable thermal conductivity on steady incompressible laminar flow of a micropolar fluid in a porous channel with high mass transfer due to suction or injection which was studied by Kelson et. al. [5] for constant properties of viscosity and the thermal conductivity.

Governing Equations:-

The equation of motion for incompressible viscous micropolar fluid is given by

$$\rho\left\{\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{V}\right\}=-\nabla p+\nabla\left(\mu\nabla.\vec{V}\right)+\kappa\nabla^2\vec{V}+\kappa\left(\nabla\times\vec{N}\right)+\vec{F}, \qquad (1)$$

where ρ is the mass density of the fluid, p is the pressure, μ is the viscosity, $\vec{N}$ is the angular velocity, κ is the material constant and t denotes time.

$\vec{F}$ is the body force per unit volume.

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$$\vec{F} = \vec{F}_g + \vec{F}_e + \vec{F}_d, \tag{2}$$

where $\vec{F}_g$ is the body force per unit volume due to gravity given by , $\vec{F}_g = \rho \vec{g} \beta (T - T_\infty)$ (3)

where g is the acceleration due to gravity, β is the coefficient of expansion and for perfect gas β = 1 and $(T - T_\infty)$ is the temperature difference between a hotter fluid particle and the colder surroundings.

$\vec{F}_e$ is the body force per unit volume due to electric field and magnetic field given by ,

$$\vec{F}_e = \rho_e \vec{E} + \left(\vec{J} \times \vec{B}\right) \tag{4}$$

where ρ_e is the excess charged density, $\vec{E}$ denotes the electrical field components, $\vec{J}$ is the total electric density, $\vec{B}$ is the magnetic induction (also known as magnetic flux density). The term $\left(\vec{J} \times \vec{B}\right)$ is known as the Lorentz force. In electrically neutral field $\rho_e = 0$, hence $\rho_e \vec{E}$ can be omitted from the body force and $\vec{F}_d$ is the body force per unit volume due to flow through porous media given by

$$\vec{F}_d = \frac{\nu}{\lambda^*} \vec{V}, \tag{5}$$

where ν is the kinematic viscosity of the fluid and λ^* is the coefficient of permeability of the porous media.

The equation of angular momentum for incompressible viscous micropolar fluid is given by

$$\rho j \left\{ \frac{\partial \vec{N}}{\partial t} + \left(\vec{V}.\nabla\right) \vec{N} \right\} = -2\kappa \vec{N} + \kappa \left(\nabla \times \vec{V}\right) - \gamma \left\{\nabla \times \left(\nabla \times \vec{N}\right)\right\}, \tag{6}$$

where j is the micro-inertia per unit mass, γ is the material constants.

The equation of heat transfer is given by

$$\rho C_p \left\{ \frac{\partial T}{\partial t} + \left(\vec{V}.\nabla\right) T \right\} = \nabla.(\lambda \nabla T) + (\mu + \kappa)\phi, \tag{7}$$

where C_p is specific heat at constant pressure, T is the temperature of the fluid, λ is the coefficient of thermal conductivity of the fluid and ϕ is the viscous dissipation function and is given by

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2, \tag{8}$$

where u, v, w are the components of the fluid velocity vector in the direction of x, y and z respectively.

Formulation of the problem:-

We consider steady, incompressible, laminar flow of a micropolar fluid along a two-dimensional channel with porous walls through which fluid is uniformly injected or removed with speed q. Using Cartesian coordinate, the channel walls are parallel to the x-axis and located at y = ± h, where 2h is the channel width. The governing equations (1), (6) and (7) under above assumptions become

Mass equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

Momentum equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-p_x+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\kappa\frac{\partial^2 u}{\partial y^2}+\kappa\frac{\partial N}{\partial y}, \tag{10}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-p_y+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\kappa\frac{\partial^2 v}{\partial y^2}-\kappa\frac{\partial N}{\partial x}, \tag{11}$$

Angular momentum equation:

$$\rho j\left(u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}\right)=-\kappa\left(2N+\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\nu_s\frac{\partial N}{\partial y}\right) \tag{12}$$

Energy equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+(\mu+\kappa)\phi, \tag{13}$$

The appropriate physical boundary conditions are

$$u(x,\ \pm h)=0,\quad \mathrm{v}(x,\ \pm h)=\pm q,\quad \mathrm{N}(x,\ \pm h)=-s\left.\frac{\partial u}{\partial y}\right|_{(x,\ \pm h)},\quad T_w=T_0+Bx^2\Big|_{(x,\ \pm h)} \tag{14}$$

and assuming that that the flow is symmetric about y = 0,

$$\frac{\partial u}{\partial y}(x,0)=v(x,0)=0,\qquad T=T_0(x,0), \tag{15}$$

where q > 0 correspondence to suction, q < 0 to injection, and s is a boundary parameter that is used to model the extent to which microelements are free to rotate in the vicinity of the channel walls. For example, the value s = 0 corresponds to the case where microelements close to a wall are unable to rotate, whereas the value s = ½ corresponds to the case where the microrotation is equal to the fluid vorticity at the boundary (Lukaszewiez [6]).

To simplify the governing equations, we generalize Berman's similarity solutions [3] to include micropolar effects by assuming a stream function and microrotation of the form

$$\Psi=-qxF(\eta),\qquad N=\frac{qx}{h^2}G(\eta), \tag{16}$$

where

$$\eta=\frac{y}{h},\quad u=\frac{\partial\Psi}{\partial y}=-\frac{qx}{h}F'(\eta),\quad v=-\frac{\partial\Psi}{\partial x}=qF(\eta) \tag{17}$$

and

$$\theta(\eta)=\frac{T(\eta)-T_0}{T_w-T_0}. \tag{18}$$

In addition we introduce the dimensionless micropolar parameters as

$$N_1=\frac{\kappa}{\rho\nu},\quad N_2=\frac{\nu_s}{\rho\nu h^2},\quad N_3=\frac{j}{h^2} \tag{19}$$

where Re > 0 corresponds to suction, and Re < 0 to injection..

The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [7] as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\alpha(T-T_\infty)\right],\quad \frac{1}{\mu}=a(T-T_r),\quad a=\frac{\alpha}{\mu}\quad \text{and}\quad T_r=T_\infty-\frac{1}{\alpha}, \tag{20}$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general, a > 0 for liquids and a < 0 for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T = T_\infty$

Similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}\left[1+\xi\left(T-T_\infty\right)\right], \quad \frac{1}{\lambda} = b\left(T-T_k\right), \quad \mathrm{b} = \frac{\xi}{\lambda_\infty} \quad \text{and} \quad T_k = T_\infty - \frac{1}{\xi}, \tag{21}$$

where b and T_k are constants and their values depends on the reference state and the thermal property of the fluid. ξ is constant based on thermal property and λ_∞ is the thermal conductivity at $T = T_\infty$.

Using equations (16)and (17), it can be easily verified that the continuity equation is satisfied automatically and using equations (16)-(21) in the equations (10)-(13) become,

$$\mathrm{Re}\left(1+K\frac{\theta_r-\theta}{\theta_r}\right)F''' - \frac{\theta_r-\theta}{\theta_r}FF'' + \frac{\theta'}{\theta_r-\theta}F'' + \frac{\theta_r-\theta}{\theta_r}F'^{\,2} - K\frac{\theta_r-\theta}{\theta_r}G' = 0, \tag{22}$$

$$G_1(FG' - F'G) = -K(F'' - 2G) + G_2G'' \tag{23}$$

and

$$P_rF\theta' = \frac{\theta_k}{\left(\theta_k-\theta\right)^2}\theta'^{\,2} + \frac{\theta_k}{\theta_k-\theta}\theta'' + P_rE_c\left(F''^{\,2} + 4F'^{\,2}\right) \tag{24}$$

where

non-zore cross-flow Reynolds number, $\mathrm{Re} = \frac{qh}{\nu}$

Prandtl number, $P_r = \frac{\rho\nu C_p}{\lambda_\infty}$

Eckert number, $\mathrm{E_c} = \frac{q^2}{C_p\left(T_w - T_0\right)}$

The transformed boundary conditions are

$F^1(x,\pm h)=0$, $F(x,\pm h)=1$, $G(x,\pm h)=0$, $\theta(x,\pm h) = \frac{Bx^2}{T_w - T_0}$

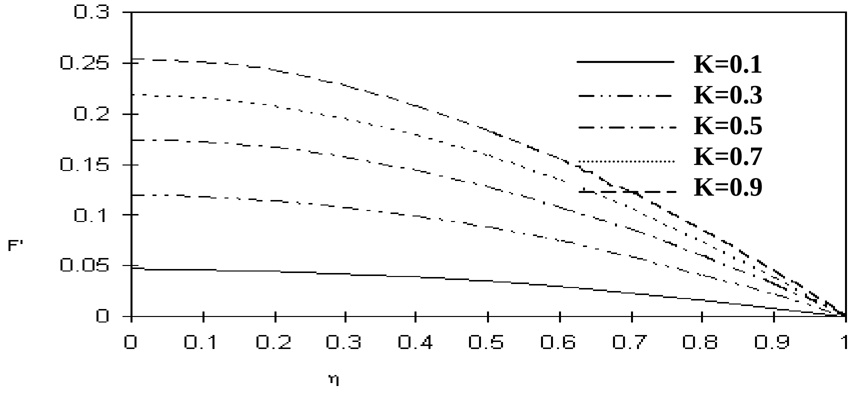

Fig 1: Variation of Velocity distribution of F' against η for various values of Coupling Constant Parameter (K) taking Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2,ϵN_1=.10, θr= - 10, θ_k= - 10

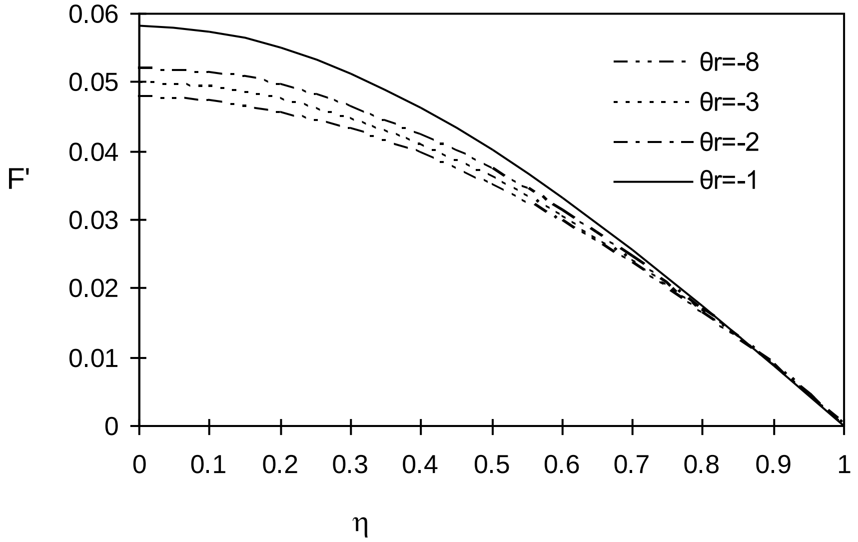

Fig 2: Variation of Velocity distribution of F' against η for various values of temperature corresponding to the viscosity parameter θr taking Pr=0.7, Ec= 0.10, N3=1.0, ϵN2=2 ϵN1=.10, θ_k= - 10

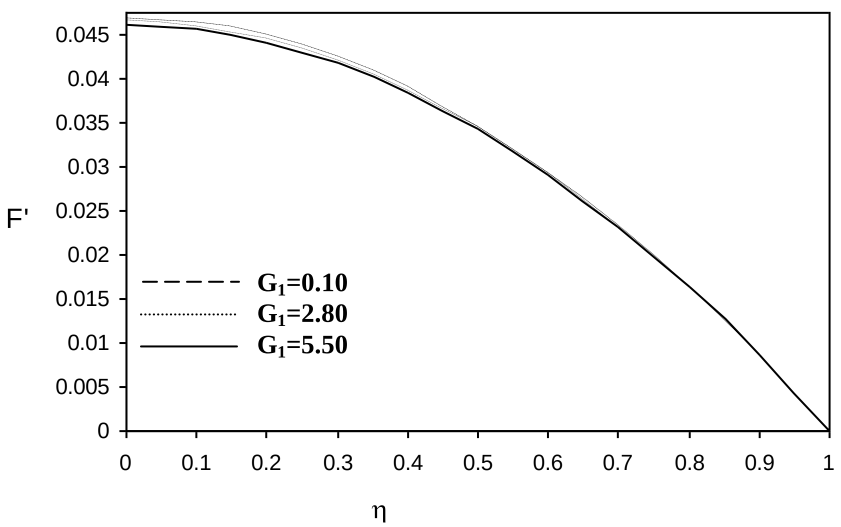

Fig-3 Velocity Distribution profiles F' along the channel for various values of constant parameter (G_1) taking Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2, ϵN_1=.10, θr= - 10

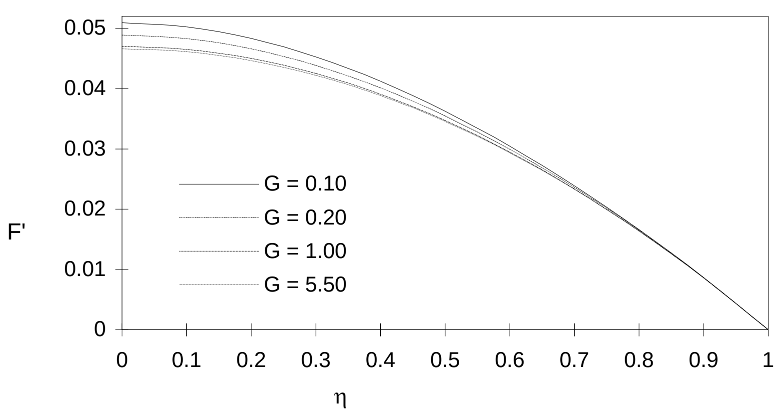

Fig-4 Velocity Distribution profiles F' along the channel for various values of constant (G) taking Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2, ϵN_1=.10, θr= - 10, θ_k= - 10

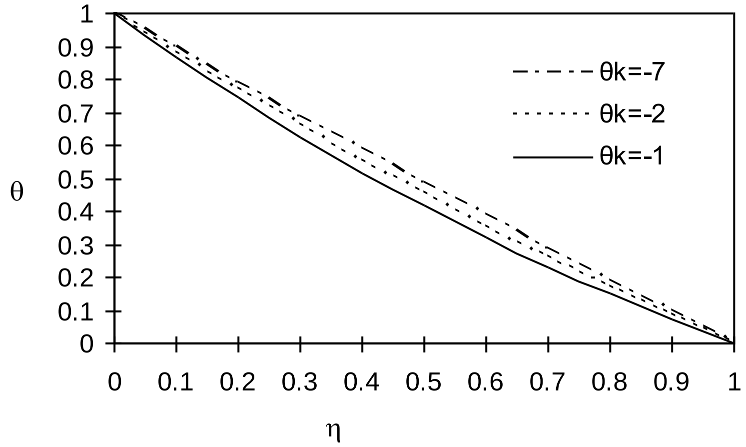

Fig 5:Variation of Temparature Distribution (θ) against η for the various values of temperature corresponding to thermal conductivity parameter (θ_k) taking Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2,ϵN_1=.10, θr= - 10,

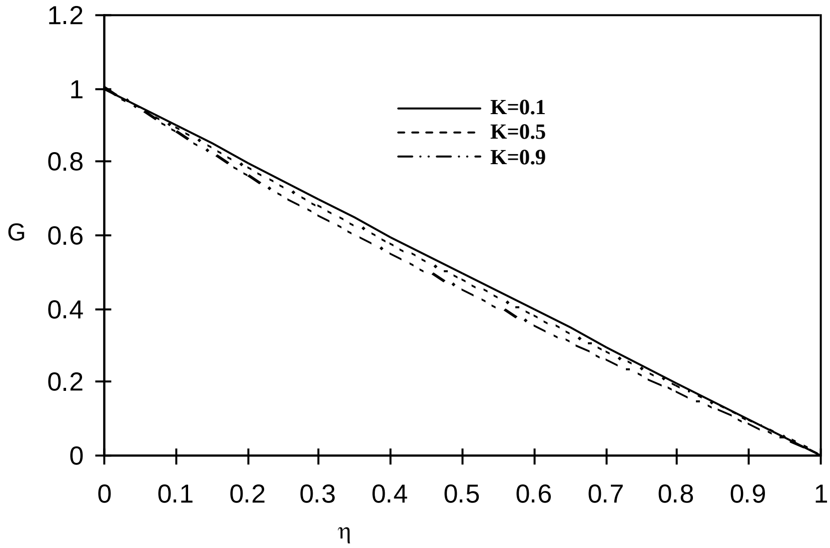

Fig-6 : Variation of Microrotation Distribution(G) against η for various values of Coupling Constant Parameter (K) taking Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2,ϵN_1=.10, θr= - 10, θ_k= - 10

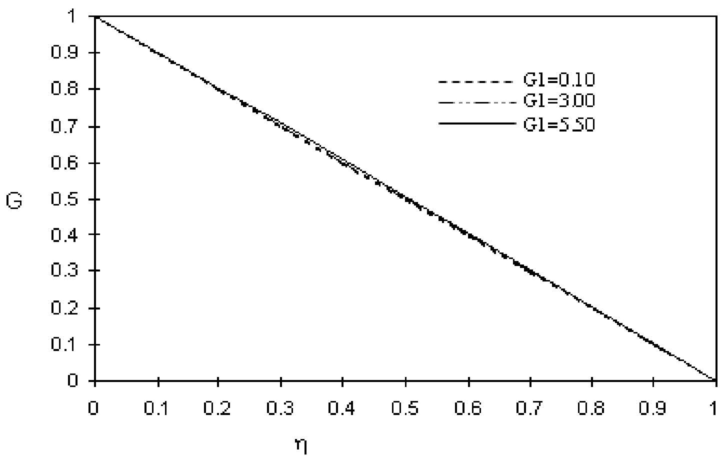

Fig-7 :Variation of Microrotation Distribution (G) against η for various values of Constant Parameter (G_1) taking Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2,ϵN_1=.10, θr= - 10, θ_k= - 10

Results and discussion:-

The equations (10) , (11), (12) and (13) together with the boundary conditions (14) are solved for various condition of the Parameters involved in the equations using an algorithms based on the shooting method (13) and presented results for the distribution of dimensionless velocity distribution, dimensionless micro-rotation distribution and temperature distribution with the variation of different parameters. Initially solution was taken for constant values of Pr=0.7, Ec= 0.10, N_3=1.0, ϵN_2=2, ϵN_1=.10, θr= - 10, θ_k= - 10 with the viscosity Parameter θr ranging from -15 to -1 at the certain value of θ_k= -10. Similarity the solutions have been found with varying the thermal conductivity parameter θ_k ranging from -9 to -1 at the certain value of θr= - 10 keeping the other values remaining same. Solution have been also been found for different values of Coupling Constant Parameter (K), Prandtl number (Pr), Eckert number (Ec). The variation in velocity distribution, micro-rotation distribution and temperature distribution are illustrated in figures (1 – 7). From the equation (17) it is found that the velocity 'u' is dependent on $F'(\eta)$. The figures (1 – 4) represent the variation in velocity (u) distribution with the variation of coupling constant parameter K , viscosity parameter θr ,constant parameter G_1 and G. From fig. (5) It is seen that the variation in temperature distribution with the variation of θ_k. From fig. (6-7) it is found that the variation in micro rotation distribution with variation of Coupling Constant Parameter (K) and parameter G_1 .From figure (1) it is clear that velocity increases as coupling constant parameter (K) increases. From figure (2) and (3), it is seen that velocity distribution decreases with the increasing values of θr and G_1. From (4) it is found that velocity decreases as microrotation G increases. From figure (5) it can be observed that temperature distribution decreases with the increase of θk. From figure (6) it is seen that microrotation distribution decreases as K increases. From (7) it is seen that microrotation distribution increases as constant parameter G_1 increases.

Conclusion:-

In this study, the effect of variable viscosity and thermal conductivity on flow and heat transfer for micropolar flow in a porous channel with large mass transfer through the channel walls is examined. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effect on velocity, temperature and micro-rotation distribution within the boundary layer. The effects of Coupling Constant Parameter (K) is quite significant. Thus the assumption on constant properties may cause a significant error in the flow problems and in the prediction of skin friction while designing fluid machinery.

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