Utilization of photon equation of motion to obtain electromagnetic momentum, time & length in einstein generlazation of special relativity

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ABSTRACT

Special relativity (SR) in the presence of the gravitational field is obtained from the expression of invariant length and a photon equation of motion. Two expressions explain both mass and energy are obtained; one is generalized special relativity (GSR), the other is of Savickas. The (GSR) model is more realistic than Savickas since it is based on the effect of gravity on time which is not recognized by Savickas expression. Moreover (GSR) model predicts pair production phenomena and its equations of motion which are reduced to Newton's second law. Using the equation of motion of the electron in the electromagnetic (E.M) as a simple expression for the total momentum the sum of photon and electron mechanical momentum was found.

Introduction

Light and electromagnetic waves, (E.M.W) play an important role in our daily life [1]. Light is the oldest known form of(E.M.W) Theories about the nature of light can be traced from the writing of ancient authors like Newton, who proposed that light behaves like the smallest of tiny particles. Its particle theory explains the light reflection, but was unable to explain refraction and interference phenomena [2]. This failure motivates Christian Huygens, on 1678, to construct the wave theory of light. The theory succeeds in explaining reflection, refraction, interference and diffraction of light. This theory was confirmed and generally accepted, by the work of Maxwell, who, in 1873, asserted that light is a form of electromagnetic waves [3]. Hertz provided experimental confirmation of Maxwell's theory by producing and detecting electromagnetic waves.

Unfortunately this wave theory faces a real problem in explaining the Black body radiation phenomenon. In 1900, Max Plank explains blackbody radiation, by proposing light as discrete energy quanta called photons with energy proportional to its frequency. This photon theory later on explains Photoelectric Effect, Compton Effect and Pair Production Effect [4].

The nature of light also plays an important role in the formation of special relativity (SR) theory, which was formulated by Einstein in 1905. SR is based mainly on utilizing Lorentz Transformation, where electromagnetic wave equation invariant for inertial frames. It is also based on the fact that the speed of light is constant [5]. The derivation of (SR) laws totally or partially based on the behavior of photons. The theories of light are utilized for wide applications of (E.M.W) in civilization.

These applications include telecommunication [6], laser application in medicine and industry [7, 8, 9, and 10].

Problems of photons within the framework of generalized Special Relativity

The behavior of photons within the framework of (SR) is not consistent within that of general relativity (SR) [11, 12]. In (SR) the energy of the photon is not affected by the gravitational field [13]. This is in direct conflict with the prediction of the gravitational red shift by (GR) [14]. Such prediction was confirmed experimentally by observing the gravitational red shift at the stars [15].Thus the expression of photon energy and momentum in the gravitational field from the (SR) and (GR) stand points need to be modified.

Attempts to unify the photon energy in (SR) and(GR)

Several attempts are made to modify (SR) to include the effect of fields. [16, 17, 18] One of them is that made by M. Dirar et al. It explains nuclear mass defect and the neutrino mass problem. [19, 20]. But its derivation is complicated. A simple derivation is made

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by Savickas [21]. Both derivations are similar to each other, but the former consists of an additional term in the expression of mass and energy.

The aim of the work unifies the expression of energy and mass of the photon in both (SR) and (GR). Relaying on the behavior of the photon in the gravitational field to derive the expression for mass and energy, as well as finding the generalized momentum expression in the presence of electromagnetic fields.

Einstein Generalization

The (GSR) expression for the mass and the energy can be obtained here by using the expression of the invariant proper length in the four dimensional space – time coordinates. The invariant length is given by [19]:

$$d\tau^2 = g_{\mu\gamma} dx_\mu dx^\nu \tag{1}$$

For one dimensional physical system

$$d\tau^2 = g_{0o}(Cdt)^2 + g_{xx}(dx)^2 \tag{2}$$

Where:

$$x_o = ct, x^1 = x$$

The invariant length can be written in terms of the velocity in the in the form

$$d\tau^2 = g_{0o}C^2(dt)^2 + g_{xx}\left(\frac{dx}{dt}\right)^2 (dt)^2$$
$$= g_{0o}C^2(dt)^2 + g_{xx}v_x^2 d\tau^2 \tag{3}$$

In the weak field limit the components of the metric takes the form [19].

$$g_\infty = 1 + \frac{2\phi}{C^2} \qquad g_{xx} = -1 \tag{4}$$

Where ϕ represents the potential per unit mass

For a system at rest in free space:

$$\Phi = 0 \qquad \frac{dx}{dt} = v_x = 0$$
$$g_{0o} = 1 + \frac{2\phi}{C^2} = 1$$
$$g_{xx} = -1 \qquad t = t_o \tag{5}$$

In this case:

$$d\tau^2 = g_{0o}C^2(dt)^2 + (-1)\left(\frac{dx}{dt}\right)^2 (dt)^2$$
$$d\tau^2 = C^2(dt_o)^2 \tag{6}$$

Since $d\tau^2$ is assumed to be invariant, thus equations (3) and (6) give:

$$C^2(dt_o)^2 = d\tau^2 = C^2[g_{0o} + g_{xx}v_x^2]dt^2 \tag{7}$$
$$C^2(dt_o)^2 = C^2\gamma^2 dt^2$$

Where:

$$\gamma = \left[g_{0o} + g_{xx}\frac{v_x^2}{C^2}\right]^{\frac{1}{2}}$$

$$= \left[g_{00} + \frac{g_{xx}v_x^2}{C^2} \right]^{\frac{1}{2}} \tag{8}$$

Where:

$$v_x = v$$

Thus:

$$dt_o = \gamma dt \tag{9}$$

If the photon momentum is P, and the force on it is F, The equation of motion of a photon is given by:

$$\frac{dP}{dt} = F \qquad \frac{d(mc)}{dt} = F = \nabla E = \frac{dE}{dx} \tag{10}$$

Where:

C is the speed of light

E is the energy

$$F = m\frac{dv}{dt} = md = \frac{dx}{dt}\cdot\frac{dv}{dx}$$

$$= mv\frac{dv}{dx} = \frac{d\left(\frac{1}{2}mv^2\right)}{d}\frac{dE}{dx} \tag{11}$$

And

P = mc = momentum

Hence:

$$\frac{d(mc)}{dt} = \frac{dE}{dx}$$

$$\frac{dmc}{dx}\frac{dx}{dt} = \frac{dE}{dx}$$

Since the speed of the photon is constant:

$$\frac{dx}{dt} = v = C \tag{12}$$

Hence:

$$C = \frac{dmc}{dx} = \frac{dE}{dx}$$

But C is constant therefore:

$$\frac{dmC^2}{dx} = \frac{dE}{dx} \tag{13}$$

Integrating both sides yields

$$\int dE = \int dm C^2$$

$$E = mc^2 \qquad (14)$$

Which is the ordinary expression for energy in (SR).

To obtain the expression for the energy in a curved space – time, one can utilize the equation (10) for a frame to get the rest mass:

$$\frac{dm_0 c}{dt_0} = F \qquad (15)$$

Where the rest mass for a system at rest is given by:

$$m = m_0 \emptyset$$

For a frame in which a system is not at rest the mass is denoted by m, and equation (10) reads:

$$\frac{d(mc)}{dt} = F \qquad (16)$$

Equating equation (15) and equation (10) and utilizing equation (9):

$$\frac{d(m_0 c)}{\gamma dt} = \frac{d(mc)}{dt} \qquad (17)$$

Thus:

$$(d(m_1 0 \ c)/(\gamma \ dx) \ dx/dt = d(mc) dx/dx \ dx/dt$$

But:

$$\frac{dx}{dt} = C$$

Hence:

$$\frac{C d(m_0 c)}{\gamma \ dx} = \frac{C d(mc)}{dx}$$

$$\frac{d(m_0 c^2)}{\gamma \ dx} = \frac{d(mc^2)}{dx}$$

Since C is constant:

$$\frac{dm_0 c^2}{\gamma \ dx} = \frac{C^2}{\gamma} \frac{dm_0}{dx} = C^2 \frac{dm}{dx} \qquad (18)$$

Thus:

$$\frac{C^2}{\gamma} dm_0 = C^2 dm$$

Viewing equation (8) γ is independent of m_o thus:

$$\frac{C^2}{\gamma} \int dm_o = C^2 \int dm$$

$$\frac{C^2}{\gamma} m_o = C^2 m \qquad (19)$$

Thus the mass m and energy E in a curved space – time is given by:

$$E = mc^2 \tag{20}$$

$$m = \frac{m_o}{\gamma}$$

With the aid of the equation (8) m is given by:

$$m = \frac{m_o}{\sqrt{g_{0o} + g_{xx}\dfrac{v^2}{C^2}}} \tag{21}$$

For Quai – Minkowskian Space:

$$g_{xx} = -1 \qquad g_{0o} = 1 + \frac{2\phi}{C^2} \tag{22}$$

Thus:

$$m = \frac{m_o}{\sqrt{1 + \dfrac{2\varphi}{C^2} - \dfrac{v^2}{C^2}}} = m = \frac{m_0}{\sqrt{g_{00} - \dfrac{v^2}{C^2}}} \tag{23}$$

The corresponding energy is given according to equation (14) and equation (23) by:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{g_{0o} - \dfrac{v^2}{c^2}}} = \frac{m_o c^2}{\sqrt{1 + \dfrac{2\phi}{c^2} - \dfrac{v^2}{c^2}}} \tag{24}$$

This expression resembles the energy found by Savickas [12]. If one considers the expression of time in a curved space time

The time is thus given by [19]:

$$dt_c = \sqrt{g_{0o}}\, dt \tag{25}$$

In this case equation (17) can be rewritten as:

$$\frac{d(m_o c)}{\gamma\, dt} = \frac{d}{dt_c}\left[m\frac{dx}{dt_c}\right]$$

$$\frac{C}{\gamma}\frac{dm_o}{dx}\frac{dx}{dt} = \frac{1}{g_{0o}}\frac{d}{dt}\left[m\frac{dx}{dt}\right] = \frac{1}{g_{0o}}\frac{d(mc)}{dt}$$

$$\frac{C^2}{\gamma}\frac{dm_o}{dx} = \frac{1}{g_{0o}}\frac{d(mc)}{dx}\frac{dx}{dt} = \frac{1}{g_{0o}}c\frac{d(mc)}{dx}$$

$$= \frac{c^2}{g_{0o}}\frac{dm}{dx} \tag{26}$$

Thus:

$$\frac{g_{0o}c^2}{\gamma}\int dm_o = c^2\int dm$$

The energy is thus given by:

$$E = mc^2 = \frac{g_{00} m_0 c^2}{\gamma} \tag{27}$$

Thus the mass is given by:

$$m = \frac{g_{00} m_0^2}{\gamma} = \frac{g_{00} m_0}{\sqrt{g_{00} - \dfrac{v^2}{c^2}}} \tag{28}$$

The corresponding energy is given with the aid of equation (14) as:

$$E = mc^2 = \frac{g_{00} m_0 c^2}{\sqrt{g_{00} - \dfrac{v^2}{c^2}}} \tag{29}$$

Which resembles the expression of energy derived by M. Dirar.

Generalized Momentum in the presence of electromagnetic field

The total momentum of a charged particle in an electromagnetic field can be found by considering the equation of motion of a charged particle like the electron in an electromagnetic field. If the election is affected by a magnetic field of flux density B, the equation of motion becomes:

$$\frac{d(mv)}{dt} = Bev \tag{29}$$

But, since the velocity v and B are given by [19]:

$$B = \frac{\partial A}{\partial x} = \frac{dA}{dx} \qquad v = \frac{dx}{dt} \tag{30}$$

Inserting equation (30) in equation (29) yields:

$$\frac{d(mv)}{dt} = e\frac{dA}{dx}\frac{dx}{dt} = e\frac{dA}{dt} \qquad \frac{d(mv - eA)}{d} = 0 \tag{31}$$

This equation looks like the equation of momentum conservation:

$$\frac{dP}{dt} = 0 \tag{32}$$

Comparing equation (32) and equation (31) yields:

$$P = mv - eA \tag{33}$$

To throw light on the term eA, one can use the definition of the electric field intensity E in terms of A to get:

$$E = -\frac{\partial A}{\partial t} = -\frac{dA}{dt} = -\frac{dA}{dx}\frac{dx}{dt} = -c\frac{dA}{dx} \tag{34}$$

Where the speed of the photon is constant. Thus the work done by the photon field is given by:

$$W = +e\int E.dx = eC\int\frac{dA}{dx}dx = -eC\int\frac{dA}{dx}dx$$

$$W = -eCA \tag{35}$$

The photon energy is given also in terms of the momentum P_p as:

$$E_p = P_p C \tag{36}$$

Where the mass term is neglected by assuming m_0 to be very small. But W is equal to E_p Thus:

$$E_p = W$$
$$P_p C = -eAC$$

Thus the photon momentum is given by:

$$P_p = -eA \tag{37}$$

Thus equation (33) represents the sum of mechanical and photon momentum, i.e.:

$$P = P_{mec} + P_p \tag{38}$$

Where:

$$P_{mec} = mv \qquad\qquad P_p = -eA \tag{39}$$

The total momentum can also be found from the equation of motion of the electron in the electric field, where:

$$\frac{d(mv)}{dt} = -eE \tag{40}$$

With the aid of the equation (34) one, gets:

$$\frac{d(mv)}{dt} = \frac{edA}{dt} = \frac{d(eA)}{dt}$$

$$\frac{d(mv - eA)}{dt} = 0 \tag{41}$$

Using the same procedures as in equation (31) and equation (32) again the total momentum P is given by:

$$P = mv - eA = P_{mec} + P_p \tag{42}$$

This equation resembles that obtained in the standard texts of Savickas [12].

Conclusion &Discussion

Using the invariant length expression in a curved space time, equation (3) together with the equation of motion of the photon (10) a useful expression of mass and energy are obtained one (see equation (17)) resembling Savickas, energy expression [12] and equation(18) resembling M. Dirar expression [17].

The difference between the two expressions can be explained easily from the fact that when the effect of the field on (t) is ignored as in equation (17) Savickas energy is obtained. But when the system in which the mass is (m) is permeated by a field the time is given by equation (15).it is clear that M.Dirar energy expression found from really the mass and time are affected by gravity to be More accurate and realistic.

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