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Degrees of q- fuzzy groups over implications operator of [0,1]

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ABSTRACT

In this paper, we introduce the notion of degree to which a Q- fuzzy subset is a Q- fuzzy group by means of the implication operator of [0,1]. A Q-fuzzy subset A in a group G is a Q-fuzzy group if and only if its subgroup degree $d_g(A) = 1$. Some properties of subgroup degree are investigated.

Introduction

The fundamental concept of fuzzy sets was initiated by Zadeh [10] in 1965 and opened a new path of thinking to mathematicians, engineers, physicists, chemists and many others due to its diverse applications in various fields. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analysis, measure theory etc. In 1971, Rosenfeld [5] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structure. Thereafter the notion of different fuzzy algebraic structures such as fuzzy ideals in rings and semirings etc. have seriously studied by many mathematicians. In 1979, Anthony and Sherwood redefined fuzzy groups. Since then many results in group theory has been generalized to fuzzy setting. Some properties of Products, set products, unions, intersections, the homomorphic images and pre images of fuzzy groups are introduces.V.N.Dixit,et.al [4] gave characterizations of fuzzy conjugate subgroups and fuzzy characteristic subgroups by their level subgroups. The philosophy is developing a technique with which we no longer merely decide whether a fuzzy subset is a fuzzy subgroup or not but with which we actually degree to which it is a fuzzy group. A.Solairaju and R.Nagarajan [[6], [7], [8], [9]]introduced the concept of Q- fuzzy groups. Our aim is to introduce the notion of degree to which a Q- fuzzy subset of a group G is a Q- fuzzy subgroup by means of the implication operator of [0,1] and a Q-fuzzy subset A in a group G is a Q- fuzzy group if and only if its subgroup degree $d_g(A) = 1$.

Preliminaries

Throughout this paper , $[0,1]^X$ (or I^X) denote the set of all Q- fuzzy subsets on X .$\hat{a}$ denotes the constant Q-fuzzy sets on X taking the value A. For A $[0,1]^X$ and a [0,1], we can use the notations

$A[a] = \{ x \varepsilon X;\ A(x,q) \geq a\}$

$A[a] = \{ x\varepsilon\ X;\ A(x,q) \geq q\}$

Definition: (T-norm): A function T : $[0,1]\times [0,1] \rightarrow [0,1]$ is called a t-norm if it satisfies, for all x,y,z ε [0,1], the following conditions;

(T1) $T(x,y) = T(y,x)$

(T2) $T(x, T(y,z)) = T(T(y,x),z)$

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(T3) if $y \le z$, then $T(x,y) \le T(x,z)$, that is $T(x,\bullet)$ is increasing,

(T4) $T(x,1) = x$.

Definition: A function $I_T : [0,1] \times [0,1] \rightarrow [0,1]$ is called an R-implication if there exists a t-norm T such that $I_T(x,y) = \sup \{t \, \varepsilon \, [0,1];\ T(x,y) \le y\}$, $x,y \, \varepsilon [0,1]$. I_T is also called the residual of the t-norm T.

Definition: A Q – fuzzy subset A of a group G is said to be Q- fuzzy group of G [7] if for any $x,y \, \varepsilon G$, $A(xy,q) \ge \min \{A(x,q), A(y,q)\}$. If the degree of $a \le b$ is defined by $I_T(a,b)$, then we can naturally introduce the notion of subgroup degree as follows;

Definition: Let A be a Q-fuzzy subset in a group G and T be a t-norm on [0,1]. The subgroup degree $d_g(A)$ of A is defined as

$$d_g(A) = \cap\, I_T(T(A(x,q) , A(y,q)) , A(xy^{-1},q)) , x,y \, \varepsilon G.$$

If T is taken as $T(a,b) = a \cap b$ for any $a,b \, \varepsilon [0,1]$, then obviously A is a Q- fuzzy group of G if and only if $d_g(A) = 1$. In this sequal, we shall take T as $T(a,b) = a \cap b$ for any $a,b \, \varepsilon [0,1]$ and $I_T(a,b)$ is written as $a \dashv b$.

Definition: Let Z be an integer additive group. Define $A : Z \rightarrow [0,1]$ by

$$A(n) = \begin{cases} 0.3 & \text{if n is even} \\ 0.5 & \text{if n is odd.} \end{cases}$$

It is easy to check that $d_g(A) = 0.3$.

Properties of Degrees of Q-fuzzy group

Proposition: Let A be a Q- fuzzy subset in a group G and $x,y \, \varepsilon G$.. Then

$$d_g(A) = \cap ((A(x,q) \cap A(y,q)) \dashv A(xy,q)) \cap \cap(A(x,q) \dashv A(x^{-1},q) \text{ for all } x,y \text{ in G and q in Q.}$$

Proof: It is obvious that

$$d_g(A) = \cap ((A(x,q) \cap A(y,q)) \dashv A(xy^{-1},q))$$
$$\le \cap ((A(e,q) \cap A(y,q)) \dashv A(y^{-1},q)) \cap \cap ((A(x,q) \cap A(x,q)) \dashv A(e,q)) \cap A(x,q))$$
$$\le \cap (A(x,q)) \dashv A(x^{-1},q))$$

Furthur we have

$$d_g(A) = \cap ((A(x,q) \cap A(y,q)) \dashv A(xy^{-1},q)) \cap \cap (A(x,q)) \dashv A(x^{-1},q))$$
$$\le \cap ((A(x,q) \cap A(y,q)) \dashv A(xy^{-1},q)) \cap \cap ((A(x,q) \cap A(y,q)) \dashv A(x,q)) \cap A(y^{-1},q))$$
$$\le \cap (A(x,q)) \cap A(y ,q)) \dashv A(xy,q)).$$

This shows

$$d_g(A) \le \cap ((A(x,q) \cap A(y,q)) \dashv A(xy,q)) \cap \cap (A(x,q)) \dashv A(x^{-1},q))$$

Moreover by

$$\cap ((A(x,q) \cap A(y,q)) \vdash A(xy,q)) \cap (\cap A(x,q)) \dashv A(x^{-1},q))$$
$$\le \cap ((A(x,q) \cap A(y,q)) \dashv A(xy,q)) \cap \cap ((A(x,q) \cap A(y,q)) \dashv A(x,q)) \cap (A(y^{-1},q)$$
$$\le \cap ((A(x,q) \cap A(y,q)) \dashv A(xy^{-1},q)) = dg(A).$$

We obtain

$$d_g(A) = \cap ((A(x,q) \cap A(y,q)) \dashv A(xy,q)) \cap \cap (A(x,q)) \dashv A(x^{-1},q)).$$

The following lemma is obivious.

Lemma: Let A be a Q- fuzzy subset in a group G. Then $d_g(A) \ge a$ if and only if for any $x,y \, \varepsilon$ G. $A(x,q) \cap A(y,q) \cap a \le A(xy^{-1},q))$.

By lemma 3.2 we can easily obtain the following results.

Proposition: Let A be a Q- fuzzy subset in a group G. Then

$$d_g(A) = \cup \{ a \, \varepsilon[0,1] \ / \ A(x,q) \cap A(y,q)) \cap a \le A(xy^{-1},q) \text{ for all } x,y \text{ in G and q in Q.}$$

Proposition: Let A be a Q- fuzzy subset in a group G. Then

$$d_g(A) = \cup \{ a \, \varepsilon[0,1] \ / \text{ for all } b < (0, a) , A_{[b]} \text{ is a subgroup of G}\}.$$

Proof: Suppose that $A(x,q) \cap A(y,q)) \cap a \le A(xy^{-1},q)$,for all x,y in G.Then for any $b \, \varepsilon (0,a]$ and for any $x,y \, \varepsilon A_{[b]}$.

We have

$$A(xy^{-1},q) \ge A(x,q) \cap A(y,q)) \cap a$$

$\geq A(x,q) \cap A(y,q)) \cap b$

$= A(x,q) \cap A(y,q)) \geq b,$

This shows xy^{-1} εA[b] and q ε Q. Therefore $A_{[b]}$ is a subgroup of G. Hence

$d_g(A) =$ Ụ { a ε [0,1] / $A(x,q) \cap A(y,q)) \cap a \leq A(xy^{-1},q)$ for all x,y in G and q in Q.

$\leq$ Ụ { a ε [0,1] / for all b < (0,a] , $A_{[b]}$ is a subgroup of G}.

Conversly, assume that a ε (0,1] and for all bε (0,a]. $A_{[b]}$ is a subgroup of G. For any x,yε G, let $b = A(x,q) \cap A(y,q)) \cap a$, then $b \leq a$ and x,y ε A[b].Thus xy^{-1} ε $A_{[b]}$ that is $A(xy^{-1},q) \geq b = A(x,q) \cap A(y,q)) \cap a$.

This means that

$d_g(A) =$ Ụ{ a ε [0,1] / $A(x,q) \cap A(y,q)) \cap a \leq A(xy^{-1},q)$ for all x,y in G and q in Q.

$\leq$ Ụ { a ε [0,1] / for all b < a , $A_{[a]}$ is a subgroup of G}.

Proposition: Let $\{Ai\}_{i \varepsilon I}$ be a family of Q-fuzzy subsets in a group G. Then

$dg(\cap A_i) \geq \cap dg(A_i)$ for all iε I.

Proposition: Let f be homomorphism from a group G to a group G^1.

(i) If A is Q- fuzzy subset in G, then $d_g(f^{\rightarrow}A) \geq d_g(A)$, where $f^{\rightarrow}(A)$ (y,q) = Ụ{ A(x,q: f(x) = y } ;

(ii) If η is Q- fuzzy subset in G^1, then $d_g(f^{\leftarrow}(\eta)) \geq d_g(\eta)$, where $f^{\leftarrow}(\eta)$ (x,q) =η(f(x,q).

Proof: (i) can be proved from proposition 3.3 and the following fact

$d_g(f^{\rightarrow}A) =$ Ụ{ a ε [0,1] / $(f^{\rightarrow}A)(x,q) \cap (f^{\rightarrow}A)(y,q)) \cap a \leq (f^{\rightarrow}A)\ (x^1y^1,q)$

$\geq$ Ụ { a ε [0,1] / Ụ A(x,q) ∩ Ụ A (y,q)) ∩ a ≤ Ụ A (z,q) and $f(x)=x^1$ and $f(y) = y^1$}

= Ụ { a ε [0,1] / $A(x,q) \cap A(y,q)) \cap a \leq A(xy^{-1},,q)$, for all x,y in G.

(iii) Can be proved from the following fact.

$d_g(f^{\rightarrow}(\eta)) = \cap\ (f^{\rightarrow}(\eta)(x,q) \cap (f^{\rightarrow}(\eta)(y,q)) \dashv (f^{\rightarrow}(\eta)\ (xy^{-1},q)$

$= \cap\ (\eta f(x,q) \cap \eta (f(y,q)) \dashv \eta (f(x,q)\ f(y,q)^{-1})$

$\geq \cap\ (\eta(x^1,q) \cap \eta (y^1,q)) \dashv \eta (x^1y^1,q))$

$= dg(\eta)$.

Conclusion: In this paper, the notion of degree to which a Q- fuzzy subset is a Q- fuzzy group by means of implication operator of [0,1] is discussed and Q-fuzzy subset A in a group G is a Q- fuzzy group if and only if its subgroup degree $d_g(A) = 1$.

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