



Calculating poverty measures from the generalized burr density function

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ABSTRACT

This paper estimated Foster-Greer-Thorbecke (FGT) poverty indices from the generalized burr density function to further justify the wide flexibility and applicability of the function in fitting many life datasets. It computed estimates of the parameters of the selected density and the goodness of fit statistic from the 2010 Harmonized Nigeria Living Standard Survey (HNLSS) dataset conducted by the National Bureau of Statistics (NBS) of Nigeria. The goodness of fit test indicated that the selected density was appropriate and estimates of the indices obtained from the density were approximately close to the ones obtained through the traditional approach.

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Introduction

Of the planet's half a dozen billion men and women, around 2.8 billion survive lesser than around US \$ 2 each day and 1.2 billion spend less than US \$ 1 on a daily basis (World Bank, 2002). Also, according to (Addison, 2004) excess of one billion individuals are living in alarming poverty today, which have less than US \$ 1 every day to live on. Like in many third world nations, the poverty situation in Nigeria is basically of a rural dimension because a greater number of the impoverished individuals lives in the rural areas, where there job is basically farming (Etim, and Ukoha, 2010). Nigeria is an example of the conundrums of development in that the country is potentially rich but her citizens are not better for it. Records show that poverty has become alarming in Nigeria and is on the rise (Balogun, 2011).

A brief and generally accepted meaning of poverty is difficult to find since it includes different aspects of human conditions, incorporating physical, moral and psychological dimensions (Balogun, 2011). However; World Bank (1990) defines poverty as inability to attain a minimum standard of living. Poverty has been measured in various ways. These include the use of monetary approach, Human Development Index (HDI), Head Count Ratio and Physical Quality of Life Index (PQLI) (Awotide, 2012). Foster et al. (1984) classified the population into two dichotomous groups of poor and non-poor defined in relation to some chosen poverty line based on household expenditure or income. This is the conventional or traditional approach to relative poverty analysis where poverty line is defined as the two-third of the mean value of the per capita expenditure/income. Households with per capita expenditure/income below the poverty line are deemed poor and non-poor otherwise. This approach includes the determination of Foster-Greer-Thorbecke (FGT) class of poverty measures defined as

$$P_{\alpha} = \frac{1}{N} \sum_{i=1}^N \left(\frac{z - y_i}{z} \right)^{\alpha} I(y_i \leq z) \quad \dots \dots \dots (1)$$

where z = the poverty line; N = the total number of households considered; y_i = the per capita expenditure for i^{th} household; and P_{α} = generalized FGT poverty index for $\alpha = 0, 1, \text{ and } 2$ respectively. When, $\alpha = 0$, P_{α} becomes the head count index which is defined as the number of the poor in a given population. When $\alpha = 1$ and 2 in that order, we have the poverty gap and square poverty gap indices. Poverty gap measures the depth of poverty. That is, the total amount of money necessary to raise everyone who is below the poverty line up to the line. The Square Poverty Gap index measures the severity of poverty. Bardhan (1973), Ahluwalia (1976) and Ginneken (1980) noted that poverty could be measured by utilizing the head count proportion which is dependent upon the ratio or percentage of

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the number of people or family units whose earnings are not equivalent to the poverty line to the aggregate number of people or family units. Awotide (2012) equally opined that the Square Poverty Gap index is increasingly being used as a standard poverty measure by the World Bank, the regional development banks, and most United Nations agencies.

A sample of authors who have used the FGT approach in analyzing poverty in Nigeria include Addelkrim and Awoyemi (2006); Awoyemi *et. al.* (2008); Babatunde *et. al.* (2008); Oluwatayo (2009); Fonta *et.al.* (2011); Ben-Chendo *et. al.* (2012); Ezekiel and Olawuyi (2013); and Olawuyi and Adetunji (2013). These past studies have been limited to the analysis of poverty profiles based on FGT poverty indices and identification of poverty correlates. This limitation has allowed further statistical explorative analytical approaches as demonstrated by Osowole and Bamiduro (2012) where the Pearson system of distributions was applied in finding suitable density functions for FGT poverty indices. In the light of this, the study at hand therefore considers the alternative use of the Generalized Burr density function to estimate head count, poverty gap and square poverty gap indices from available household per capita expenditure data. Expenditure is chosen as a proxy for poverty because information provided on it by respondents during surveys is generally more accurate than information on income.

The Generalized Burr Density Function

According to Kibria and Shakil (2011), the generalized burr density function is defined as

$$f(y) = \frac{p(\alpha)^{v-\frac{\mu}{p}} (\beta)^{\frac{\mu}{p}} y^{\mu-1}}{B\left(\frac{\mu}{p}, v-\frac{\mu}{p}\right) (\alpha + \beta y^p)^v} \quad , \text{ for } (y, \alpha, \beta, \mu, v, p) > 0, \text{ and } v > \frac{\mu}{p} \dots\dots\dots (2)$$

They noted that by using certain values of the parameters in (2), a variety of distributions could be obtained as being special or limiting cases. They further defined the cumulative distribution function (c.d.f), moment generating function (m.g.f), characteristic function (c.f.), k^{th} moment about the origin and k^{th} central moment respectively as

$$F(y) = \frac{p(\beta)^{\frac{\mu}{p}}}{B\left(\frac{\mu}{p}, v-\frac{\mu}{p}\right)} \sum_{k=0}^{\infty} \left\{ \frac{(-1)^k \left(\frac{\beta}{\alpha}\right)^k (v)_k y^{(\mu+pk)}}{(k!)} \right\} \dots\dots\dots (3)$$

$$M_Y(t) = E(e^{tY}) = \sum_{k=0}^{\infty} \frac{(t)^k}{k!} E(Y^k) \dots\dots\dots (4)$$

$$M_Y(it) = E(e^{itY}) = \sum_{k=0}^{\infty} \frac{(it)^k}{k!} E(Y^k) \dots\dots\dots (5)$$

$$\alpha_k = E(Y^k) = \left(\frac{\alpha}{\beta}\right)^{\frac{k}{p}} \frac{B\left(\frac{\mu+k}{p}, v-\frac{\mu+k}{p}\right)}{B\left(\frac{\mu}{p}, v-\frac{\mu}{p}\right)} \dots\dots\dots (6)$$

$$\beta_k = E[Y - E(Y)]^k = \sum_{j=0}^k (-1)^j \binom{k}{j} E(Y^j) E(Y^{k-j}) \dots\dots\dots (7)$$

For a couple of reviews on the Burr distributions, the enthusiastic reader will find the following useful: Tadikamalla (1980), Johnson *et. al.* (1994), Nadarajah and Kotz (2006), Chotikapanich (2008), and Marshall and Olkin (2007).

Estimation of the Parameters of the Generalized Burr Density Function

To estimate the parameters of the density function defined in (2), we note that a function with k parameters is generally easier to handle than a similar function with at least $k+1$ parameters. Based on this remark, we shall redefine (2) to have four parameters as follows:

$$\text{Let } Y = \frac{X - \delta}{c} \text{ so that } dy = \frac{dx}{c} \text{ with } g(x) = h(y = \omega(x)) \left| \frac{dy}{dx} \right| \text{ the density of X becomes}$$

$$g(x) = \frac{p(\alpha)^{v-\frac{\mu}{p}} (\beta)^{\frac{\mu}{p}} \left(\frac{x-\delta}{c} \right)^{\mu-1}}{B\left(\frac{\mu}{p}, v-\frac{\mu}{p}\right) \left(\alpha + \beta \left(\frac{x-\delta}{c} \right)^p \right)^v} \left(\frac{1}{c} \right) \dots\dots\dots(8)$$

By choosing α and β such that $\alpha = \beta = 1$, and letting $\mu=p$, (8) becomes

$$g(x) = \frac{p \left(\frac{x-\delta}{c} \right)^{p-1}}{cB(1, v-1) \left(1 + \left(\frac{x-\delta}{c} \right)^p \right)^v} = \frac{p(v-1) \left(\frac{x-\delta}{c} \right)^{p-1}}{c \left(1 + \left(\frac{x-\delta}{c} \right)^p \right)^v} \dots\dots\dots(9)$$

This is a re-parameterization of (2) into four parameters. The Burr density in this form makes the estimation of the parameters less complicated and can be handled by the Easy-fit Distribution Fitting Software (version 5.2).

Description of Data for Study

The dataset used in this study was the 2010 Harmonized Nigeria Living Standard Survey (HNLSS) data comprising the thirty six states and the Federal Capital Territory. ‘The survey was carried out to provide a sound database that will drive government anti-poverty programmes. The welfare component of the survey involved 77400 households while the consumption component involved 38700 households. The number of households having complete information for the consumption component was 33012. The components cover the following scope: demography, health, fertility behavior, education, skills training, employment, housing, social capital, agriculture, household income, consumption and expenditure. Some of the objectives of the survey include in-depth enquiry into the structure and distribution of wages and conditions of work of Nigeria’s labor force, providing valid and reliable data for the development of effective intervention and provision of important tools for designing, implementing and monitoring of economic growth and poverty reduction, provision of a comprehensive analysis for identification and targeting of the poor by different localities, collection of baseline information on the character and nature of poverty for monitoring and evaluating impact of poverty reduction programmes and dissemination of national poverty report highlighting statistical findings derived from in-depth analysis of the survey data’ (NBS, 2012).

Results and Discussion of the Study

The FGT poverty indices were first obtained using the traditional approach as shown in Table1. From the Table, the estimates of the head count, poverty gap and square poverty gap indices were approximately 0.5100, 0.2100 and 0.1100 respectively. The poverty line used was the $2/3 * (\text{mean per capita expenditure})$ with a value of N38297.19. The choice of the Burr density was justified by the Kolmogorov-Smirnov goodness of fit test (Table 2) at $\alpha 5\%$ ($p_{\text{value}}=0.10372$). This fact was further corroborated by the P-P plot (Figure 1) where all the points almost fell on the straight line. The estimates of the parameters (Table 3) of the Burr density (4-parameters) as obtained from the Easy-Fit distribution software showed that $p (= \mu) = 2.392$, $c = 347.51$, $\delta = -5515.4$ and $v = 258.71$. To obtain estimates of the FGT indices from the Burr density, a random sample of size 33000 was generated and from these sample values, a new poverty line ($2/3 * (\text{mean per capita expenditure})$) of N39152 was obtained. This poverty line was used to obtain estimates of the

FGT indices (Table 4) from the Burr density function. The estimates of the head count, poverty gap and square poverty gap indices from the Burr density were approximately 0.5200, 0.2100 and 0.1100 respectively.

Table 1: Estimates of FGT Poverty Indices from the Traditional Approach

Index	Estimate
P ₀ :head count	0.50488~0.5100
P ₁ :poverty gap	0.20717~0.2100
P ₂ :square poverty	0.11046~0.1100
z:poverty line	N38297.19

Table 2: Kolmogorov-Smirnov Goodness of Fit for the Burr Density Function

K-S Statistic	pValue
0.00669	0.10372

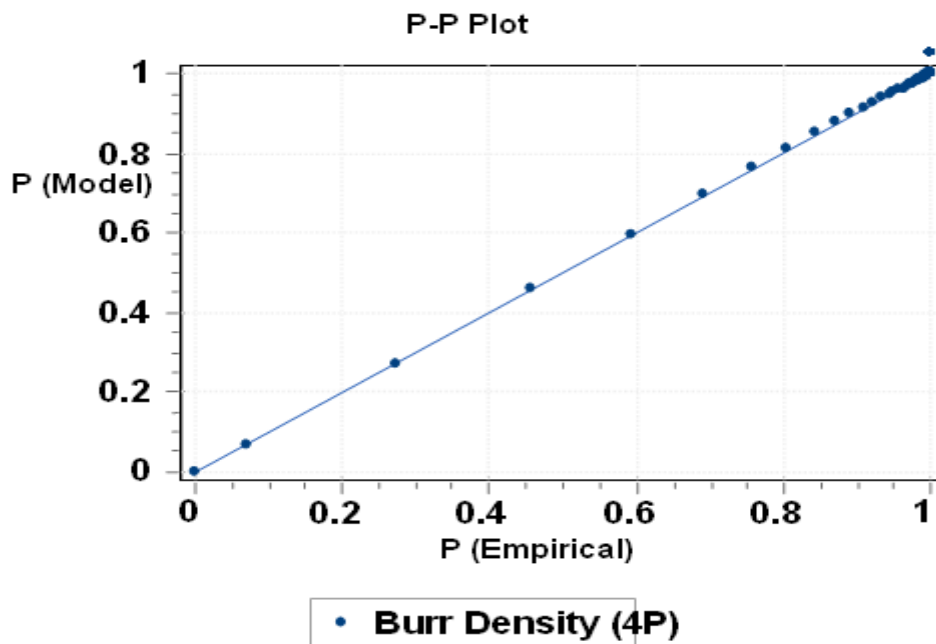


Figure 1: P-P Plot for the Burr Density (4-parameters)

Table 3: Estimates of the parameters of the Burr Density Function (4-parameters)

	2.3
	92
	34
	7.51
	-
	5515.4
	25
	8.71

Table 4: Estimates of FGT Poverty Indices from the Burr Density

Index	Estimate
P ₀ :head count	0.51969~0.5200
P ₁ :poverty gap	0.21290~0.2100
P ₂ :square poverty	0.11357~0.1100
z:poverty line	N39152

Conclusion

The estimation of the FGT poverty indices alternatively from the Burr density, a main objective of this study, has been achieved. The selected density function has been shown to provide good estimates for the FGT poverty indices in Nigeria. The flexibility of the density function in allowing re-parameterization for the purpose of estimating the parameters of the function has also been documented. The proportion of the poor, depth, and severity of poverty in Nigeria were estimated approximately as 50%, 0.2100 and 0.1100 respectively.

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