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Fusion of Cauchy and Triangular Fuzzy Numbers: Application in Human Health Risk Assessment

Palash Dutta

Department of Mathematics, Dibrugarh University, Dibrugarh.

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ABSTRACT

Risk assessment is an important and more popular aid in decision making process. Risk assessment is generally performed using *models* and model is a function of some parameters which are usually affected by uncertainty. Here, we consider that the model parameters are affected by epistemic uncertainty. To represent epistemic uncertainty generally triangular fuzzy numbers or trapezoidal fuzzy numbers are used. In this paper, Cauchy fuzzy number together with triangular fuzzy numbers is used to represent epistemic type uncertainty. An effort has been made to fuse Cauchy fuzzy number with triangular fuzzy number and then human health risk assessment is carried out under fuzzy environment.

Introduction

Since the beginning of the earth's existence, human being has always been exposed to radiation from natural sources, which is a continuing and inescapable feature of life on the earth. The world population is too exposed to radiation resulting from releases of radioactive materials from manmade/artificial sources, and from the use of fuels or materials containing naturally occurring radionuclides.

There is a wide range of sources of natural radiation to which we are being continuously exposed. Of these sources, the most familiar to us is the sun which produces infrared radiation that we feel as warmth, visible light, and ultraviolet light. The other sources are cosmic radiation which consists of high energy particles and rays that originate from outside our earth, terrestrial radiation which comes from naturally occurring radionuclides in the earth's crust, and internal radiation from radioactivity that is naturally present in our bodies. Naturally-occurring radioactive materials were discovered in 1896. Naturally-occurring radiation accounts for approximately 80 percent of our exposure.

Man-made/artificial radiation is radiation produced in devices, such as x-ray machines, and artificially produced radioisotopes made in a reactor or accelerator. This type of radiation is used in both medicine and industry. Main users of man-made radiation include: medical facilities, such as hospitals and pharmaceutical facilities; research and teaching institutions; nuclear reactors and their supporting facilities, such as uranium mills and fuel preparation plants; and federal facilities involved in nuclear weapons production. Radiation accidents are the most likely events that threaten population and environment. A radiation accident is a situation in which there is a real or suspected unintentional exposure to ionizing radiation or radioactive contamination. Radiation accidents involved radiation devices, radioisotopes, and criticality incidents. It must be emphasized that radiation accidents could involve either high- or low-level radiation exposures. Since the discovery of radiation, people have benefited from the use of radiation in medicine and industry. Man-made sources of radiation account for about 20 percent of our total exposure to radiation.

When human being is exposed, in general, the amount and duration of radiation exposure affect the severity or type of health effect such as acute radiation sickness, cancer, teratogenic (fetal) damage, hereditary changes etc.

When hazardous substances are released into the environment, an evaluation is necessary to determine the possible impact such substances may have on human health and other biota. For this purpose, risk assessment is performed to quantify the potential detriment to human and evaluate the effectiveness of proposed remediation measures. The assessment is performed using 'models'

Tele:
E-mail addresses: palash.dtt@gmail.com

and a 'model' is a function of parameters which are usually affected by aleatory and epistemic uncertainty. Aleatory uncerta inty and epistemic uncertainty are two distinct facets of uncertainty in risk assessment. Aleatory uncertainty arises from heterogeneity or the random character of natural processes while epistemic uncertainty arises from the partial character of our knowledge of the n atural world. Epistemic uncertainty can be reduced by further study while aleatory cannot be reduced. Here, we have considered that parameters are affected by epistemic uncertainty (fuzzy number).

If the representation of the input parameters of the risk model is in probabilistic sense, the output risk distribution may reflect probabilistic information. However, resalable and sufficient data is required to estimate and characterized the probability d istribution of the input variables. If uncertainty does not arise due to randomness, or if the information are partial, not fully reliable, receive of information from more than one sources or inherent imprecision, then probability theory is inappropriate to represent such kind of uncertainty.

To overcome this limitation of probability theory Zadeh (1965) introduced fuzzy set theory which can be used to incorporate epistemic uncertainty. In general, Triangular or Trapezoidal shape fuzzy membership functions have been widely studied in lit erature to represent uncertainty. However, in practice, there are certain applications where to represent uncertainty, besides Triangular or Trapezoidal shape fuzzy numbers some other types of fuzzy numbers come into picture viz., Gaussian fuzzy number, lognormal shaped fuzzy number, Cauchy shaped fuzzy number etc. In [4] authors discussed Gaussian fuzzy numbers and performed arithmetic operations between Gaussian fuzzy number and triangular fuzzy number and applied it in risk assessment models.

In this paper, we use Cauchy and Triangular fuzzy numbers to represent epistemic type uncertainty. Fusion of Cauchy fuzzy number with triangular fuzzy number is discussed and then human health risk assessment is carried out using proposed fusion technique.

Basic Concept of Fuzzy Set Theory:

To estimate the effects of environmental pollution on human, risk assessment is performed. However, environmental data tends to be vague and imprecise, so uncertainty is associated with any study related with risk assessment. Fuzzy set theory is a tool which is used to characterize imprecisely defined variables, as well as to define relationships between variables based on expert knowledge and use them to compute results. In this section, [2, 3] some necessary backgrounds and notions of fuzzy set theory are reviewed.

Definition: Let X be a universal set. Then the fuzzy subset A of X is defined by its membership function

$$\mu_A : X \to [0,1]$$

which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in X$, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A.

Definition: Given a fuzzy set A in X and any real number $\alpha \in [0, 1]$, the α -cut or α -level or cut worthy set of A, denoted by ${}^{\alpha}A$ is the crisp set

$${}^{\alpha}A = \{x \in X: \mu_A(x) \geq \alpha\}$$

The strong a cut, denoted by ${}^{\alpha+}A$ is the crisp set

$${}^{\alpha+}A = \{x \in X: \mu_A(x) > \alpha\}$$

For example, let A be a fuzzy set whose membership function is given as

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, a \leq x \leq b \\ \dfrac{c-x}{c-b}, b \leq x \leq c \end{cases}$$

To find the α-cut of A, we first set $\alpha \in [0,1]$ to both left and right reference functions of A.

That is, $\alpha = \dfrac{x-a}{b-a}$ and $\alpha = \dfrac{c-x}{c-b}$.

Expressing x in terms of α we have $x = (b-a)\alpha + a$ and $x = c-(c-b)\alpha$, which gives the α-cut of A is

$$^{\alpha}A = [(b-a)\alpha + a, c-(c-b)\alpha]$$

Definition: The support of a fuzzy set *A* defined on *X* is a crisp set defined as

Supp (*A*) = {$x \in X$: $\mu_A(x) > 0$}

Definition: The height of a fuzzy set *A*, denoted by *h*(*A*) is the largest membership grade obtain by any element in the set.

$$h(A) = \sup_{x\in X} \mu_A(x)$$

Definition: A fuzzy number is a convex normalized fuzzy set of the real line *R* whose membership function is piecewise continuous.

Type of fuzzy numbers:

Triangular Fuzzy Number:

A triangular fuzzy number A can be defined as a triplet [*a, b, c*]. Its membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & , a \le x \le b \\ \dfrac{c-x}{c-b} & , b \le x \le c \end{cases}$$

Trapezoidal Fuzzy Number:

A trapezoidal fuzzy number *A* can be expressed as [*a, b, c, d*] and its membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & , a \le x < b \\ 1 & , b \le x \le c \\ \dfrac{d-x}{d-c} & , c < x \le d \end{cases}$$

Gaussian Fuzzy number:

The membership function of a Gaussian Fuzzy number is [4,5]:

$$\mu_A(x) = \exp\left(\frac{-(x-\mu)^2}{2\sigma^2} \right)$$

where μ represents the MFs centre and σ determines the MFs width.

Cauchy Fuzzy number:

The membership function of a Cauchy Fuzzy number is [1,5]:

$$\mu_A(x) = \frac{1}{1+\left(\dfrac{x-p}{q}\right)^2}$$

Where ***p*** represents the MFs centre and ***q*** determines the MFs width.

Fusion of Triangular and Cauchy fuzzy number:

In this section, we perform arithmetic operations between Cauchy and triangular fuzzy numbers.

The general form of the α-cut of triangular fuzzy number is

$$^{\alpha}A_1 = [(b-a)\alpha + a,\ c-(c-b)\alpha] \quad ...(1)$$

Also the general form of the α-cut of Cauchy fuzzy number is

$$^{\alpha}A_2 = \left[p - q\sqrt{\frac{1-\alpha}{\alpha}},\ p + q\sqrt{\frac{1-\alpha}{\alpha}} \right] \quad ...(2)$$

Here, we will relax the convexity condition to combine Cauchy fuzzy number and triangular fuzzy number using α -cut by considering that α corresponds to the α -cut defined in the interval [0.01, 1]. It will not affect the uncertainty involved in the fuzzy number.

Addition:

$$^{\alpha}A_1 + ^{\alpha}A_2 = \left[(b-a)\alpha + a + p - q\sqrt{\frac{1-\alpha}{\alpha}},\quad c-(c-b)\alpha + p + q\sqrt{\frac{1-\alpha}{\alpha}} \right]$$

Subtraction:

$$^{\alpha}A_1 - ^{\alpha}A_2 = \left[((b-a)\alpha + a) - \left(p + q\sqrt{\frac{1-\alpha}{\alpha}} \right), c-(c-b)\alpha - \left(p - q\sqrt{\frac{1-\alpha}{\alpha}} \right) \right]$$

Multiplication:

$$^{\alpha}A_1 {}^{\alpha}A_2 = \left[((b-a)\alpha + a)\left(p - q\sqrt{\frac{1-\alpha}{\alpha}} \right),\quad (c-(c-b)\alpha)\left(p + q\sqrt{\frac{1-\alpha}{\alpha}} \right) \right]$$

Division:

$$\frac{^{\alpha}A_1}{^{\alpha}A_2} = \left[\frac{((b-a)\alpha + a)}{\left(p + q\sqrt{\frac{1-\alpha}{\alpha}} \right)}, \frac{c-(c-b)\alpha}{\left(p - q\sqrt{\frac{1-\alpha}{\alpha}} \right)} \right]$$

Hypothetical Case Study:

To demonstrate and make use of the Cauchy fuzzy number and triangular fuzzy numbers and their combination, a hypothetical case study for health risk assessment is presented here.

Suppose the exposure of a population to chlorinated organic solvent (1, 1, 2-trichloroethene), via the consumption of contaminated drinking water. The general form of a risk assessment model for a lifetime daily dose of exposure as provided by EPA, 2001 is follows [6]:

$$D = \frac{CDI \times IR \times EF \times ED}{BW \times AT} \quad ...(3)$$

Where, D is the dose of exposure (mg/Kg.day), IR is the intake of water (l/d), CDI is the concentration of 1, 1, 2-trichloroethene in the water (mg/l), EF is the exposure frequency (d/yr), ED exposure duration (yr), BW is the body weight (kg), AT is the average time (d).

The excess risk is express as:

$$ER = D \times UER \quad ...(4)$$

UER is the unit excess risk: a probability of excess cancer per unit daily dose of exposure.

In this study, the parameter intake rate of water (*IR)* is considered as triangular fuzzy number and exposure duration (*ED*) is considered to be Cauchy fuzzy number while other parameters are taken to be constant. The parameters values used in the risk assessment are given in table1.

Table 1: Parameter values used in the risk assessment

Variable	Representation	Value
CDI	Deterministic	0.2
IR	Possibilistic	TFN(1, 1.5, 2)
EF	Deterministic	250
ED	Possibilistic	CFN(30,2)
BW	Deterministic	70
AT	Deterministic	70
UER	Deterministic	0.024

As the two input parameters of the health risk assessment model are fuzzy numbers so the resulting risk will be obtained in the form of fuzzy number and its graphical representation is given in the figure1.

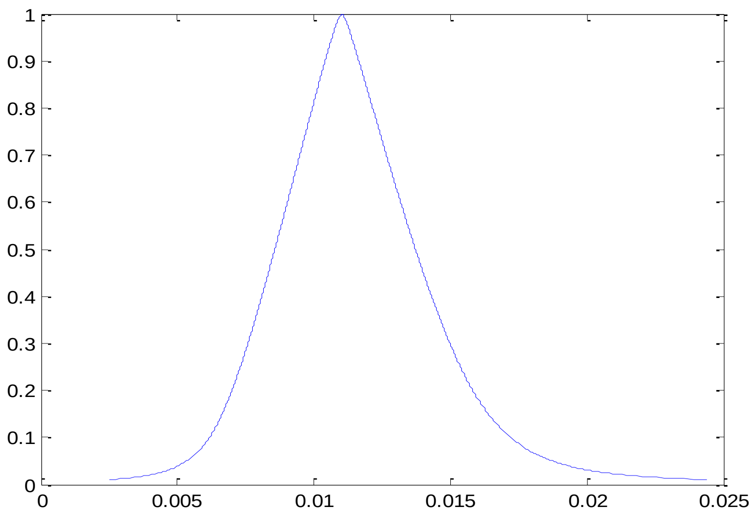

Figure: Human helth risk assessment

Here, the core or modal value of the resulting health risk is 0.01102 while range of risk value is [0.002486, 0.02438].

Conclusion

Risk assessment is a popular and important tool in decision making process. Risk assessment is generally performed using *models* and model is a function of some parameters which are usually affected by uncertainty. Since we have considered that the parameters are tainted with epistemic uncertainty, the same is propagated to the output through the model. In general, Triangular or Trapezoidal shape fuzzy membership functions have been widely studied in literature to represent uncertainty. However, in practice, there are certain applications where to represent uncertainty besides Triangular or Trapezoidal shape fuzzy numbers some other types of fuzzy numbers come into picture viz., Gaussian fuzzy number, lognormal shaped fuzzy number, Cauchy fuzzy number etc. In this paper, fusion of Cauchy fuzzy number with triangular fuzzy number has been discussed and human health risk assessment is carried out using proposed fusion technique. Here, in the risk model one parameter is considered as Cauchy fuzzy numbers and another one is considered as triangular fuzzy number, and some are constant therefore, the resulting risk is also obtained in the form of a fuzzy number. 0.01102 is the core of the output fuzzy risk while the range is [0.002486, 0.02438].

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