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A study on (q, l)-fuzzy ideals of a ring

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ABSTRACT

In this paper, we study some of the properties of (Q, L)-fuzzy ideal of a ring and prove some results on these.

Introduction

After the introduction of fuzzy sets by L.A.Zadeh[19], several researchers explored on the generalization of the notion of fuzzy set. Azriel Rosenfeld[4] defined a fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[16, 17, 18] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of (Q, L)-fuzzy ideal of a ring and established some results.

Preliminaries:

Definition: Let X be a non-empty set and $L = (L, \leq)$ be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q, L)-fuzzy subset** A of X is a function $A : X\times Q \to L$.

Definition: Let (R, +, ·) be a ring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a **(Q, L)-fuzzy ideal (QLFI)** of R if the following conditions are satisfied:

(i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,

(ii) $A(-x, q) \geq A(x, q)$,

(iii) $A(xy, q) \geq A(x, q) \vee A(y, q)$, for all x and y in R and q in Q.

Definition: Let A and B be any two (Q, L)-fuzzy subsets of sets R and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{ \langle ((x, y), q), A\times B((x, y), q) \rangle / \text{ for all } x \text{ in } R \text{ and } y \text{ in } H \text{ and } q \text{ in } Q \}$, where $A\times B((x, y), q) = A(x, q) \wedge B(y, q)$.

Definition: Let A be a (Q, L)-fuzzy subset in a set S, the **strongest (Q, L)-fuzzy relation** on S, that is a (Q, L)-fuzzy relation V with respect to A given by $V((x, y), q) = A(x, q) \wedge A(y, q)$, for all x and y in S and q in Q.

Properties of (Q, L)-fuzzy idealS:

Theorem: If A is a (Q, L)-fuzzy ideal of a ring (R, +, ·), then $A(x, q) \leq A(e, q)$, for x in R, the identity e in R and q in Q.

Proof: For x in R, q in Q and e is the identity element of R. Now, $A(e, q) = A(x-x, q) \geq A(x, q) \wedge A(-x, q) = A(x, q)$. Therefore, $A(e, q) \geq A(x, q)$, for x in R and q in Q.

Theorem: If A is a (Q, L)-fuzzy ideal of a ring (R, +, ·), then $A(x-y, q) = A(e, q)$ gives $A(x, q) = A(y, q)$, for x and y in R , e in R and q in Q.

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Proof: Let x and y in R, the identity e in R and q in Q. Now, A(x, q) = A(x–y+y, q) $\geq$ A(x–y, q)$\wedge$A(y, q) = A(e, q)$\wedge$A(y, q) = A(y, q) = A(x–(x–y), q) $\geq$ A(x–y, q)$\wedge$ A(x, q) = A(e, q)$\wedge$A(x, q)= A(x, q). Therefore, A(x, q) = A(y, q), for x and y in R and q in Q.

Theorem: Let A be a (Q, L)-fuzzy subset of a ring (R, +, ·). If A(e, q) =1 and A(x–y, q) $\geq$ A(x, q) $\wedge$ A(y, q), A(xy, q) $\geq$ A(x, q) $\vee$ A(y, q), then A is a (Q, L)-fuzzy ideal of R, for all x and y in R and q in Q, where e is the identity element of R.

Proof: Let x and y in R, e in R and q in Q. Now, A(–x, q) = A(e–x, q)$\geq$ A(e, q)$\wedge$A(x, q) = 1$\wedge$A(x, q) = A(x, q). Therefore, A(–x, q) $\geq$ A(x, q), for all x in R and q in Q. Now, A(x+y, q) = A(x– (–y) , q) $\geq$ A(x, q) $\wedge$ A(–y, q) $\geq$ A(x, q) $\wedge$ A(y, q). Therefore, A(x+y, q) $\geq$ A(x, q) $\wedge$ A(y, q), for all x and y in R and q in Q and clearly A(xy, q) $\geq$A(x, q)$\vee$A(y, q), for all x and y in R and q in Q. Hence A is a (Q, L)-fuzzy ideal of R.

Theorem: If A is a (Q, L)-fuzzy ideal of a ring (R, +, ·), then H = { x / x$\in$R: A(x, q) = 1} is either empty or is a ideal of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then A(x–y, q) $\geq$ A(x, q) $\wedge$ A(–y, q) $\geq$ A(x, q) $\wedge$ A(y, q) = 1$\wedge$ 1= 1. Therefore, A(x–y, q) = 1.

We get x–y in H. And A(xy, q) $\geq$ A(x, q) $\vee$ A(y, q) = 1$\vee$ 1= 1. Therefore, A(xy, q) = 1. We get xy in H. Therefore, H is a ideal of R. Hence H is either empty or is a ideal of R.

Theorem: If A is a (Q, L)-fuzzy ideal of a ring (R, +, ·), then H = { x$\in$R: A(x, q) = A(e, q) } is a ideal of R.

Proof: Let x and y be in H. Now, A(x–y, q) $\geq$ A(x, q) $\wedge$ A(–y, q) $\geq$ A(x, q) $\wedge$ A(y, q) = A(e, q) $\wedge$ A(e, q) = A(e, q). Therefore, A(x–y, q) $\geq$ A(e, q) ---------- (1). And, A(e, q) = A((x–y) – (x–y), q) $\geq$ A(x–y, q) $\wedge$ A(– (x–y), q) $\geq$ A(x–y, q) $\wedge$ A(x–y, q) = A(x–y, q).

Therefore, A(e, q)$\geq$ A(x–y, q) --------- (2). From (1) and (2), we get A(e, q) = A(x–y , q).

Therefore, x–y in H. Now, A(xy, q) $\geq$ A(x, q) $\vee$ A(y, q) = A(e, q) $\vee$ A(e, q) = A(e, q). Therefore, A(xy, q) $\geq$ A(e, q) ------------- (3). And clearly, A(e, q) $\geq$ A(xy, q) --------- (4).

From (3), (4), we get A(e, q) = A(xy , q). Therefore, xy in H. Hence H is a ideal of R.

Theorem: Let A be a (Q, L)-fuzzy ideal of a ring (R, +, ·). If A(x–y, q) = 1, then A(x, q) = A(y, q), for x and y in R and q in Q.

Proof: Let x and y in R and q in Q. **N**ow, A(x, q) = A(x–y+y , q) $\geq$ A(x–y, q) $\wedge$ A (y, q) = 1$\wedge$ A(y, q) = A(y, q) = A(–y, q) = A(–x+x–y, q) $\geq$ A(–x, q) $\wedge$ A(x–y, q) = A(–x, q) $\wedge$ 1= A(–x, q) = A(x, q). Therefore, A(x, q) = A(y, q), for x and y in R, q in Q.

Theorem: Let A be a (Q, L)-fuzzy ideal of a ring (R, +, ·). If A(x–y, q) = 0, then either A(x, q) = 0 or A(y, q) = 0, for all x and y in R and q in Q.

Proof: Let x and y in R and q in Q. By the definition A(x–y, q) $\geq$ A(x, q) $\wedge$ A(y, q) which implies that 0 $\geq$ A(x, q) $\wedge$ A(y, q). Therefore, either A(x, q) = 0 or A(y, q) = 0.

Theorem: Let (R, +, ·) be a ring and Q be a non-empty set. If A is a (Q, L)-fuzzy ideal of R, then A(x+y, q) = A(x, q) $\wedge$ A(y, q) with A(x, q) $\neq$ A(y, q), for each x and y in R and q in Q.

Proof: Let x and y belongs to R and q in Q. Assume that A(x, q) > A(y, q). Now, A(y, q) = A(–x + x + y, q) $\geq$ A(–x, q)$\wedge$ A(x + y, q) $\geq$ A(x, q) $\wedge$A(x + y, q) $\geq$ A(y, q) $\wedge$A(x + y, q) = A(y, q). And A(y, q) = A(x, q) $\wedge$ A(x+y, q) = A(x+y, q). Therefore, A(x+y, q) = A(y, q) = A(x, q)$\wedge$A(y, q), for all x and y in R and q in Q.

Theorem: If A and B are two (Q, L)-fuzzy ideals of a ring R, then their intersection A$\cap$B is a (Q, L)-fuzzy ideal of R.

Proof: Let x and y belong to R and q in Q, A = { $\langle$ (x, q), A(x, q) $\rangle$ / x in R and q in Q } and B = { $\langle$ (x, q), B(x, q) $\rangle$ / x in R and q in Q }. Let C =A$\cap$B and C = {$\langle$ (x, q), C(x, q) $\rangle$ / x in R and q in Q }. (i) C(x+y, q) = A(x+y, q) $\wedge$ B(x+y, q) $\geq$ {A(x, q) $\wedge$ A(y, q)} $\wedge$ { B(x, q) $\wedge$ B(y, q) }$\geq$ {A(x, q) $\wedge$ B(x, q) }$\wedge${A(y, q) $\wedge$ B(y, q)}= C(x, q) $\wedge$C(y, q). Therefore, C(x+y, q) $\geq$ C(x, q) $\wedge$ C(y, q), for all x and y in R and q in Q. (ii) C(–x, q) = A(–x, q)$\wedge$B(–x, q) $\geq$A(x, q)$\wedge$ B(x, q) = C(x, q). Therefore, C(–x, q) $\geq$ C(x, q) , for all x in R and q in Q. (iii) C(xy, q) = A(xy, q) $\wedge$ B(xy, q) $\geq$ {A(x, q) $\vee$ A(y, q) }$\wedge${ B(x, q) $\vee$ B(y, q) }$\geq$ {A(x, q) $\wedge$ B(x, q)} $\vee${ A(y, q) $\wedge$ B(y, q)}=

C(x, q) $\vee$ C(y, q). Therefore, C(xy, q) $\geq$ C(x, q) $\vee$ C(y, q), for all x and y in R and q in Q. Hence A$\cap$B is a (Q, L)-fuzzy ideal of the ring R.

Theorem: The intersection of a family of (Q, L)-fuzzy ideals of a ring R is a (Q, L)-fuzzy ideal of R.

Proof: Let $\{A_i\}_{i\in I}$ be a family of (Q, L)-fuzzy ideals of a ring R and A = $\bigcap_{i\in I} A_i$. Then for x and y belongs to R and q in Q, we have (i) A(x+y, q) = $\inf_{i\in I} A_i(x+y,q) \geq \inf_{i\in I} \{ A_i(x, q) \wedge A_i(y, q)\} \geq \inf_{i\in I} (A_i(x,q)) \wedge \inf_{i\in I} (A_i(y,q))$ = A(x, q) $\wedge$ A(y, q). Therefore, A(x+y, q) $\geq$ A(x, q) $\wedge$ A(y, q), for all x and y in R and q in Q. (ii) A(–x, q) = $\inf_{i\in I} A_i(-x,q) \geq \inf_{i\in I} A_i(x,q)$ = A(x, q). Therefore, A(–x, q) $\geq$ A(x, q) , for all x in R and q in Q. (iii) A(xy, q) = $\inf_{i\in I} A_i(xy,q) \geq \inf_{i\in I} \{ A_i(x, q) \vee A_i(y, q) \} \geq \inf_{i\in I} (A_i(x,q)) \vee \inf_{i\in I} (A_i(y,q))$ = A(x, q) $\vee$ A(y, q). Therefore, A(xy, q) $\geq$ A(x, q) $\vee$ A(y, q), for all x and y in R and q in Q. Hence the intersection of a family of (Q, L)-fuzzy ideals of the ring R is a (Q, L)-fuzzy ideal of R.

Theorem: Let A be a (Q, L)-fuzzy ideal of a ring R. If A(x, q) < A(y, q), for some x and y in R and q in Q, then A(x+y, q)=A(x, q)= A(y+x, q), for all x and y in R and q in Q.

Proof: Let A be a (Q, L)-fuzzy ideal of a ring R. Also we have A(x, q) < A(y, q), for some x and y in R and q in Q, A(x+y, q) $\geq$ A(x, q) $\wedge$ A(y, q) = A(x, q); and A(x, q) = A(x + y – y, q) $\geq$ A(x + y, q) $\wedge$A(–y, q) $\geq$ A(x + y, q) $\wedge$ A(y, q) }= A(x+y, q). Therefore, A(x+y, q) = A(x, q), for all x and y in R and q in Q. Hence A(x + y, q) = A(x, q) = A(y + x, q), for all x and y in R and q in Q.

Theorem: Let A be a (Q, L)-fuzzy ideal of a ring R. If A(x, q) > A(y, q), for some x and y in R and q in Q, then A(x+y, q)=A(y, q)= A(y+x, q), for all x and y in R and q in Q.

Proof: It is trivial.

Theorem: Let A be a (Q, L)-fuzzy ideal of a ring R such that Im A={α}, where α in L. If A=B$\cup$C, where B and C are (Q, L)-fuzzy ideals of R, then either B $\subseteq$ C or C $\subseteq$ B.

Proof: Let A = B $\cup$ C = { $\langle$ (x , q), A(x, q) $\rangle$ / x in R and q in Q }, B = {$\langle$ (x, q), B(x, q) $\rangle$ / x in R and q in Q } and C = { $\langle$ (x, q), C(x, q) $\rangle$ / x in R and q in Q }. Suppose that neither B $\subseteq$ C nor C $\subseteq$ B. Assume that B(x, q) > C(x, q) and B(y, q) < C(y, q), for some x and y in R and q in Q. Then, α = A(x, q) = (B$\cup$C)(x, q) = B(x, q) $\vee$ C(x, q) = B(x, q) > C(x, q). Therefore, α > C(x, q). And, α = A(y, q) = (B$\cup$C)(y, q) = B(y, q) $\vee$ C(y, q) = C(y, q) > B(y, q). Therefore, α > B(y, q). So that, C(y, q) > C(x, q) and B(x, q) > B(y, q).

Hence B(x+y, q) = B(y, q) and C(x+y, q) = C(x, q), by Theorem 2.11 and 2.12. But then, α=A(x+y, q) =(B$\cup$C)(x+y, q)= B(x+y, q) $\vee$C(x+y, q)}= B(y, q)$\vee$ C(x, q) < α ---------(1). It is a contradiction by (1). Therefore, either B $\subseteq$ C or C $\subseteq$ B is true.

Theorem: If A and B are (Q, L)-fuzzy ideals of the rings R and H, respectively, then A×B is a (Q, L)-fuzzy ideal of R×H.

Proof: Let A and B be (Q, L)-fuzzy ideals of the rings R and H respectively. Let x_1 and x_2 be in R, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in R×H and q in Q. Now, A×B[(x_1, y_1) + (x_2, y_2), q] = A×B((x_1+x_2, y_1+y_2), q) = A(x_1+x_2, q) $\wedge$ B(y_1+y_2, q) $\geq$ {A(x_1, q)$\wedge$A(x_2, q)} $\wedge${B(y_1, q) $\wedge$B(y_2, q)}= {A(x_1, q) $\wedge$B(y_1, q)}$\wedge${A(x_2, q) $\wedge$B(y_2, q)} = A×B((x_1, y_1), q)$\wedge$A×B ((x_2, y_2), q). Therefore, A×B[(x_1, y_1)+(x_2, y_2), q] $\geq$ A×B ((x_1, y_1), q) $\wedge$ A×B((x_2, y_2), q). And A×B [–(x_1, y_1) , q] = A×B((–x_1, –y_1), q) = A(–x_1, q) $\wedge$ B(–y_1, q) $\geq$ A(x_1, q) $\wedge$ B(y_1, q) = A×B((x_1, y_1), q). Therefore, A×B [–(x_1, y_1), q] $\geq$ A×B ((x_1, y_1), q). Now, A×B [(x_1, y_1)(x_2, y_2), q] = A×B((x_1x_2, y_1y_2), q) = A(x_1x_2, q) $\wedge$ B(y_1y_2, q) $\geq$ {A(x_1, q)$\vee$A(x_2, q)}$\wedge${ B(y_1, q) $\vee$ B(y_2, q)}= {A(x_1, q)$\wedge$B(y_1, q)}$\vee${A(x_2, q) $\wedge$ B(y_2, q) } = A×B((x_1, y_1), q) $\vee$ A×B((x_2, y_2), q). Therefore, A×B[(x_1, y_1)(x_2, y_2), q] $\geq$ A×B ((x_1, y_1), q)$\vee$A×B((x_2, y_2), q). Hence A×B is a (Q, L)-fuzzy ideal of R×H.

Theorem: Let A and B be (Q, L)-fuzzy subsets of the rings R and H, respectively. Suppose that e and $e^{|}$are the identity element of R and H, respectively. If A×B is a (Q, L)-fuzzy ideal of R×H, then at least one of the following two statements must hold.

(i) B($e^{|}$, q) $\geq$ A(x, q), for all x in R and q in Q,

(ii) A(e, q) ≥ B(y, q), for all y in H and q in Q.

Proof: Let A×B be a (Q, L)-fuzzy ideal of R×H.

By contra positive, suppose that none of the statements (i) and (ii) holds. Then we can find a in R and b in H such that A(a, q) > B(e^{l}, q) and B(b, q) > A(e, q), q in Q. We have, A×B((a, b), q) = A(a, q) ∧ B(b, q) > A(e, q) ∧ B(e^{l}, q) = A×B((e, e^{l}), q). Thus A×B is not a (Q, L)-fuzzy ideal of R×H. Hence either B(e^{l} , q) ≥ A(x, q), for all x in R and q in Q or A(e, q) ≥ B(y, q), for all y in H and q in Q.

Theorem: Let A and B be (Q, L)-fuzzy subsets of the rings R and H, respectively and A×B is a (Q, L)-fuzzy ideal of R×H. Then the following are true:

(i) if A(x, q) ≤ B(e^{l}, q), then A is a (Q, L)-fuzzy ideal of R.

(ii) if B(x, q) ≤ A(e, q), then B is a (Q, L)-fuzzy ideal of H.

(iii) either A is a (Q, L)-fuzzy ideal of R or B is a (Q, L)-fuzzy ideal of H.

Proof: Let A×B be a (Q, L)-fuzzy ideal of R×H, x and y in R and q in Q. Then (x, e ^{l}) and (y, e ^{l}) are in R×H. Now, using the property A(x, q) ≤ B(e^{l}, q), for all x in R and q in Q, we get, A(x–y, q) = A(x–y, q) ∧ B(e^{l}e ^{l}, q) = A×B(((x–y), (e^{l}e^{l})), q) = A×B [(x, e^{l}) + (–y, e^{l}), q] ≥ A×B((x, e^{l}), q) ∧ A×B((–y, e^{l}), q) = {A(x, q) ∧ B(e^{l} ,q)} ∧{A(–y, q) ∧ B(e^{l}, q)}= A(x, q) ∧A(–y, q) ≥ A(x, q) ∧A(y, q). Therefore, A(x–y, q)≥ A(x, q) ∧ A(y, q), for all x, y in R and q in Q. And, A(xy, q)= A(xy, q)∧B(e^{l}e^{l}, q)}= A×B(((xy), (e^{l}e^{l})), q) = A×B[(x, e^{l})(y, e^{l}), q] ≥A×B((x, e^{l}), q) ∨ A×B((y, e^{l}), q)= {A(x, q)∧B(e^{l} ,q)}∨{A(y, q) ∧ B(e^{l}, q)}= A(x, q)∨ A(y, q). Therefore, A(xy, q)≥ A(x, q)∨ A(y, q), for all x, y in R and q in Q. Hence A is a (Q, L)-fuzzy ideal of R. Thus (i) is proved. Now, using the property B(x, q) ≤ A(e, q), for all x in H and q in Q, we get, B(x–y, q) = B(x–y, q)∧ A(ee, q) =A×B(((ee) ,(x–y)), q)=A×B[(e, x)+(e, –y), q]≥ A×B((e, x), q) ∧ A×B((e, –y), q) = { B(x, q)∧A(e, q)}∧{ B(–y, q) ∧ A(e, q)}= B(x, q) ∧ B(–y, q) ≥ B(x, q) ∧B(y, q). Therefore, B(x–y, q) ≥ B(x, q) ∧ B(y, q), for all x and y in H and q in Q. And, B(xy, q) = B(xy, q) ∧ A(ee, q) = A×B (((ee) ,(xy)), q) = A×B[(e, x)(e, y), q] ≥ A×B((e, x), q) ∨ A×B((e, y), q) ={ B(x, q) ∧ A(e, q) }∨ { B(y, q) ∧ A(e, q)}= B(x, q) ∨ B(y, q). Therefore, B(xy, q) ≥ B(x, q) ∨ B(y, q), for all x and y in H and q in Q. Hence B is a (Q, L)-fuzzy ideal of H. Thus (ii) is proved. (iii) is clear.

Theorem: Let A be a (Q, L)-fuzzy subset of a ring R and V be the strongest (Q, L)-fuzzy relation of R with respect to A. Then A is a (Q, L)-fuzzy ideal of R if and only if V is a (Q, L)-fuzzy ideal of R×R.

Proof: Suppose that A is a (Q, L)-fuzzy ideal of R. Then for any x = (x_1, x_2) and y = (y_1, y_2) are in R×R and q in Q. We have, V(x–y, q) = V[(x_1, x_2)–(y_1, y_2), q] = V((x_1–y_1 , x_2–y_2), q) = A((x_1–y_1), q)∧A((x_2–y_2), q) ≥{A(x_1, q) ∧A(–y_1, q)}∧{A(x_2, q) ∧ A(–y_2, q)}={A(x_1, q) ∧A(x_2, q)}∧{A(–y_1, q)∧A(–y_2, q)}={A(x_1, q)∧ A(x_2, q) }∧ {A(y_1, q) ∧ A(y_2, q)}= V((x_1, x_2), q) ∧ V((y_1, y_2), q) = V(x, q)∧V (y, q). Therefore, V((x–y), q) ≥V(x, q)∧V(y, q), for all x and y in R×R and q in Q. And we have, V(xy, q) = V [(x_1, x_2)(y_1, y_2), q] = V((x_1y_1 , x_2y_2), q) = A((x_1y_1), q) ∧ A((x_2y_2), q) ≥ {A(x_1, q) ∨ A(y_1, q) }∧{A(x_2, q) ∨ A(y_2, q) }= {A(x_1, q) ∧A(x_2, q)}∨ {A(y_1, q) ∧ A(y_2, q)}= V((x_1, x_2), q) ∨ V((y_1, y_2), q) = V(x, q)∨V(y, q). Therefore, V((xy), q) ≥V (x, q)∨ V(y, q), for all x and y in R×R and q in Q. This proves that V is a (Q, L)-fuzzy ideal of R×R. Conversely, assume that V is a (Q, L)-fuzzy ideal of R×R, then for any x = (x_1, x_2) and y= (y_1, y_2) are in R×R, we have A(x_1–y_1, q) ∧ A(x_2–y_2, q) = V((x_1–y_1, x_2–y_2), q) = V[(x_1, x_2) – (y_1, y_2), q] = V(x–y, q) ≥ V(x, q)∧V (y, q) = V((x_1, x_2), q) ∧ V ((y_1, y_2), q) = {A(x_1, q) ∧ A(x_2, q)}∧{A(y_1, q) ∧ A(y_2, q)}. If we put x_2 = y_2 = e, where e is the identity element of R. We get, A((x_1–y_1), q) ≥ A(x_1, q)∧ A(y_1, q), for all x_1 and y_1 in R and q in Q. And A(x_1y_1, q) ∧ A(x_2y_2, q) } = V((x_1y_1, x_2y_2), q) = V[(x_1, x_2)(y_1, y_2), q] = V(xy, q) ≥ V(x, q) ∨ V(y, q) = V((x_1, x_2), q) ∨ V((y_1, y_2), q) = {A(x_1, q) ∧A(x_2, q) }∨ {A(y_1, q) ∧A(y_2, q)}. If we put x_2 = y_2 = e, where e is the identity element of R. We get, A((x_1y_1), q) ≥ A(x_1, q) ∨ A(y_1, q), for all x_1 and y_1 in R and q in Q. Hence A is a (Q, L)-fuzzy ideal of R.

Reference

1. Akram.M and Dar.K.H, On fuzzy d-algebras, Punjab university journal of mathematics, 37(2005), 61-76.

2. Anthony.J.M. and Sherwood.H, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69,124 -130 (1979)

3. Asok Kumer Ray, On product of fuzzy subgroups, Fuzzy sets and systems, 105, 181-183 (1999).

4. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35, 512-517 (1971).

5. Biswas.R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35,121-124 (1990).

6. Davvaz.B and Wieslaw.A.Dudek, Fuzzy n-ary groups as a generalization of rosenfeld fuzzy groups, ARXIV-0710.3884VI(MATH.RA) 20 OCT 2007, 1-16.

7. Goguen.J.A., L-fuzzy Sets, J. Math. Anal. Appl. 18 145-147(1967).

8. Kog.A and Balkanay.E, θ-Euclidean L-fuzzy ideals of rings, Turkish journal of mathematics 26 (2002) 149-158.

9. Kumbhojkar.H.V., and Bapat.M.S., Correspondence theorem for fuzzy ideals, Fuzzy sets and systems, (1991)

10. Mohamed Asaad, Groups and fuzzy subgroups, Fuzzy sets and systems, North-Holland, (1991).

11. Mustafa Akgul, Some properties of fuzzy groups, Journal of mathematical analysis and applications, 133, 93-100 (1988).

12. Palaniappan. N & K. Arjunan, 2007. Operation on fuzzy and anti fuzzy ideals, Antartica J. Math., 4(1): 59-64.

13. Rajesh Kumar, Fuzzy Algebra, Volume 1, University of Delhi Publication Division, July -1993.

14. Salah Abou-Zaid, On generalized characteristic fuzzy subgroups of a finite group, Fuzzy sets and systems, 235-241 (1991).

15. Sidky.F.I and Atif Mishref.M, Fuzzy cosets and cyclic and abelian fuzzy subgroups, Fuzzy sets and systems, 43(1991) 243-250.

16. Solairaju.A and Nagarajan.R, A New Structure and Construction of Q-Fuzzy Groups, Advances in fuzzy mathematics, Volume 4, Number 1 (2009) pp. 23-29.

17. Solairaju.A and Nagarajan.R, Lattice Valued Q-fuzzy left R-submodules of near rings with respect to T-norms, Advances in fuzzy mathematics,Vol 4, Num. 2, 137-145(2009).

18. Solairaju.A and Nagarajan.R, “Q-Fuzzy left R-subgroups of near rings with respect to t-norms”. Antarctica Journal of Mathematics, 5(2008) 1-2, 59-63.

19. Zadeh.L.A, Fuzzy sets, Information and control, Vol.8, 338-353 (1965).