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Electromagnetic Transitions and Structures of even–even $^{76-90}Kr$ Isotopes within Interacting Boson Model

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ABSTRACT

The Interacting Boson Model is applied to the even $^{76-90}Kr$ isotopes . Excitation energies, electromagnetic transition strengths, quadrupole and magnetic dipole moments, and δ(E2/M1) multipole mixing ratios, monopole transitions and mixed symmetry states have been described systematically. It is seen that the properties of low-lying levels in these isotopes, for which the comparison between experiment and IBM-2 calculations is possible, can be satisfactorily characterized by the Interacting Boson Model-2.

Introduction

The even–even $^{76-90}Kr$ isotopes are the members of the chain existing around the mass region $A \cong 80$ and they are settled away from both the proton closed-shell number at 28 and neutron closed shell at 50. The interacting boson model-2 (IBM-2) has recently been applied to many light isotopes of *Kr* with emphasizing on the energy levels and on the electromagnetic transition probabilities rates.

Several some theoretical and experimental studies of even-even *Kr* isotopes have been carried out: Kaup and Gelberg [1] , have performed systematic analysis of Kr isotopes in IBM-2 , reproduced energy levels. Helleister and Lieb [2] study the energy levels and Electric transition probability and compare with experimental data. Meyer *et al.*, [3] investigated the nuclear structure of the ^{82}Kr isotope , using in-beam spectroscopy studies and compare the experimental data with the results of IBM-2.

Glannatiempo *et al.*, [4] studied the life-time of the 0_2^+ level in the ^{80}Kr isotope and compare with the calculated value of IBM-2. Deibaksh *et al.*, [5] have performed the IBM-2 calculation on *Kr* isotopes , using two-different approaches. The first approach based on the energy of bosons that $\varepsilon_\pi = \varepsilon_\nu$, and the second approach based on the difference the energy of proton boson and energy of neutron boson $\varepsilon_\pi \neq \varepsilon_\nu$. The results of IBM-2 in a good agreement with experimental data accept for the state 2_3^+ . Giannatiempo *et al.*, [6] have studied the symmetry property of the bands in $^{74-82}Kr$ isotopes by calculating *F*-spin and the n_d component of the wave function of the states of these bands.

Shi Zhu-Yii *et al.*,[7] have studied by using a microscopic sd IBM-2+2q.p. approach, the levels of the ground-band, γ -band and partial two-quasiparticle bands for $^{72-84}Kr$ isotopes are calculated. The data obtained are in good agreement with the experimental results, and successfully reproduce the nuclear shape phase transition of $^{72-84}Kr$ isotopes at zero temperature. The ground-state band is described successfully up to $J^\pi = 18^+$ and E_x = 10 MeV. Based on this model, the aligned requisite minimum energy has been deduced. The theoretical calculations indicate that no distinct change of nuclear states is caused by the abruptly broken pair of a boson, and predict that the first backbending of *Kr* isotopes may be the result of aligning of two quasi-neutrons in orbit $g_{9/2}$, which gains the new experimental support of the measurements of *g* factors in the $^{78-86}Kr$ isotopes.

Al-Khudair and Gui-Lu [8] studied the level structure of $^{76-82}Kr$ isotopes within framework of IBM-2 , and performed that the $J^\pi = 2^+$ (one-phonon mixed symmetry state) and $J^\pi = 1^+, 2^+, 3^+$ (two-phonons mixed symmetry states), and have been identified by analyzing the wavefunction of *M1* transition.

Turkan *et al.*, in 2006 [9] have determined the most appropriate Hamiltonian that is needed for present calculations of nuclei about the $A \cong 80$ region by the view of Interacting Boson Model-2 (IBM-2). After obtaining the best Hamiltonian parameters, level energies and B(E2) probabilities of some transitions in $^{88-90}Kr$ nuclei were estimated. Results are compared with previous experimental and theoretical data and it is observed that they are in good agreement.

Turkan *et al.*, in 2009 [10] studied The quadrupole moments of $^{76,78,80,82,84,88}Kr$ and $^{74,76,78,80,82}Se$ isotopes are investigated in terms of the interacting boson model (IBM), and it was found that a good description of them can also be concluded in this model. Before

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the quadrupole moments were calculated, the positive-parity states and electromagnetic-transition rates ($B(E2)$) of even-mass *Kr* nuclei have also been obtained within the framework of IBM. It was seen that there is a good agreement between the presented results and the previous experimental data. The quadrupole moments of the neighboring *Se* isotopes were also obtained and it was seen that the results are satisfactorily agree well with the previous experimental data.

The aim of this work is to calculate the energy levels and electromagnetic transitions probabilities $B(E2)$ and $B(M1)$, multipole mixing ratios and monopole matrix elements in *Kr* isotopes, using the IBM-2, mixed symmetry states for these isotopes have been studied in this research, and to compare the results with the experimental data.

The Model

In the IBM-2 the structure of the collective states in even-even nuclei is calculated by considering a system of interacting neutron (ν) and proton (π) boson s($l=0$) and d($l=2$). The boson Hamiltonian can be written as [11]:

$$H = E_d(n_{d\nu} + n_{d\pi}) + \kappa Q_\nu^{(2)}.Q_\pi^{(2)} + V_{\pi\pi} + V_{\nu\nu} + M_{\nu\pi} \quad \text{.....(1)}$$

where

$$Q_\rho = (s_\rho^{+} d_\rho + d_\rho^{+} s_\rho)^{(2)} + \chi_\rho (d_\rho^{+} d_\rho)^{(2)} \qquad \rho = \nu, \pi \quad \text{.....(2)}$$

κ is the quadrupole-quadrupole strength and $V_{\rho\rho}$ is the boson-boson interaction, which is given by the equation:

$$V_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_\rho^L \left(\left[d_\rho^{+} d_\rho^{+} \right]^{(L)} . \left[\tilde{d_\rho} \tilde{d_\rho} \right]^{(L)} \right)$$

and $$M_{\nu\pi} = L_2 (s_\nu^{+} d_\pi^{+} - d_\nu^{+} s_\pi^{+})^{(2)} (s_\nu d_\pi - d_\nu s_\pi)^{(2)} - 2\sum_{k=1,2} L_k (d_\nu^{+} d_\pi^{+})^k (d_\nu d_\pi)^{(2)} \quad \text{.....(3)}$$

The Majorana term $M_{\pi\nu}$ shifts the states with mixed proton-neutron symmetry with respect to the totally symmetric ones. Since little experimental information is known about such states with mixed symmetry, we did not attempt to fit the parameters appearing in eq. (3) , but rather took constant values for all *Kr* isotopes. :

$$T(E2) = e_\pi Q_\pi + e_\nu Q_\nu \quad \text{.....(4)}$$

The quadrupole moment Q_ρ is in the form of equation (2), for simplicity, the χ_ρ has the same value as in the Hamiltonian. This is also suggested by the single j-shell microscopy, e_π and e_υ are proton and neutron boson effective charges respectively. In general, the E2 transition results are not sensitive to the choice of e_ν and e_π, whether $e_\nu = e_\pi$ or not.

The reduced electric quadrupole transition probability B (E2) is given by:

$$B(E2; I_i \to I_j) = \frac{1}{2I_i + 1}\left(\left| < I_f \| T(E2) \| I_i > \right|^2 \right) \quad \text{.....(5)}$$

The M1transition operator is given :

$$T(M1) = \sqrt{3/\pi}\,(g_\pi L_\pi + g_\upsilon L_\upsilon) \qquad (6)$$

where $L_\upsilon (L_\pi)$ is the neutron and (proton) angular momentum operator

$$L_p^{(1)} = \sqrt{10}\,(d^{+} d)^{(1)}$$

where g_π and g_υ are the effective boson (proton, neutron) gyromagnetic –factors.

The reduced magnetic dipole transition probability B(M1) is given by:

$$B(M1; I_i \to I_j) = \frac{1}{2I_i + 1}\left(\left| < I_f \| T(M1) \| I_i > \right|^2 \right) \quad \text{.....(7)}$$

In the IBM-2 , the monopole transition (E0) operator is given by [18]

$$T(E0) = \beta_{0\rho} (d^{+} \times d)_\rho^{(0)} + \gamma_{o\rho} (s^{+} \times s)_\rho^{(0)} \qquad (8)$$

which is related to the transition matrix $\rho(E0)$ by the expression [12]:

$$\rho_{if}(E0) = \frac{Z}{R_o^2} \sum \beta_{0\rho}^{-} < f \left| d_\rho^{+} \times d_\rho \right| i > \quad \text{.....(9)}$$

where R_0 is the nuclear radius constant ($R_0 = 1.25 * 10^{-15}$m).

The Monopole transition probability is defined by :

$$B(E0; I_i \to I_f) = e^2 R_0^2 \rho^2 (E0) \quad \text{.....(10)}$$

The two parameters $\beta_{0\pi}$ and $\beta_{0\nu}$ in eq.9, may be estimated by fitting the isotopic shift, which is different in the mean square radius $\Delta < r^2 >$ between neighboring isotopes in their ground state [2]:

$$\Delta < r^2 >=< 0_1 \left| r^2 \right| 0_1 >_A - < 0_1 \left| r^2 \right| 0_1 >_{A+1}$$

$$\Delta < r^2 >= \beta_{0\pi}^{\sim}[< 0_1 \left| d_\pi^+ d_\pi^{\sim} \right| 0_1 >_{N_\nu} - < 0_1 \left| d_\pi^+ d_\pi^{\sim} \right| 0_1 >_{N_\nu}] + \beta_{0\nu}^{\sim}[< 0_1 \left| d_\nu^+ d_\nu^{\sim} \right| 0_1 >_{N_\nu}$$

$$- < 0_1 \left| d_\nu^+ d_\nu^{\sim} \right| 0_1 >_{N_\nu}] - \gamma_{0\nu} \text{.....(11)}$$

Results and Discussion

Hamiltonian Interaction Parameters

Since the Hamiltonian contain many parameters it is unpractical and not very meaningful to vary all parameters freely. Instead it is convenient to use the behavior of the parameters predicted by a microscopic point of view as a zeroth-order approximation. In a simple shell-model picture based upon degenerate single nucleon levels [13] the expected dependence of $\varepsilon, \kappa, \chi_\nu$ and χ_π on neutron (N_ν) and proton (N_π) boson numbers can be expressed as:

ε = constant, $\kappa = \kappa_\pi \kappa_\nu$, $\kappa_\rho = \sqrt{\frac{\Omega_\rho - N_\rho}{\Omega_\rho - 1}} \kappa_\rho^{(0)}$, $\chi_\rho = \frac{\Omega_\rho - 2N_\rho}{\sqrt{\Omega_\rho - N_\rho}} \chi_\rho^{(0)} \quad \rho = \pi, \nu$ (12)

Here $\kappa_\rho^{(0)}$ and $\chi_\rho^{(0)}$ are constants, and Ω_ρ is the pair degeneracy of the shell. We see that while κ_ρ has always the same sign, χ_ρ changes sign in the middle of the shell.

In realistic cases the estimates of eq.1 are expected to be valid only approximately. In our approach we have imposed somewhat weaker constrains on the parameters: (**i**) it is assumed that within a series of isotones (isotones) $\chi_\nu (\chi_\pi)$ does not vary at all; (**ii**) the parameters ε, κ and χ_ν are assumed to be smooth functions of (N_ν).

Concerning the sign of χ_ν and χ_π a complication arises. From very simple microscopic consideration it follows that the $\chi's$ (which also determine to a large extent the sign of the quadrupole moment of the first excited state 2_1^+ are negative in the region where the valence shell is less than half filled (particle-boson) and positive in the region where the valance shell is more than half filled (hole-boson). Quantitatively, such a behavior was confirmed in other phenomenological calculations with IBM-2. For example in a study of the Ba isotopes with $72 < N < 80$ good fit to the energy levels was obtained with $\chi_\nu \approx 0.90$ [14,15]. Since in the naïve shell-model picture in the Kr region both neutrons and protons are hole-like and therefore both $\chi's$ would be positive, there would be no way to obtain an *SU*(5) type spectrum, which requires opposite signs of χ_ν and χ_π. This indicates that the situation is not so simple and that more complicated effects play a role , such as a possible nonclosedness of the Z= 50 or the *N* = 82 core. Although the Hamiltonian invariant under simultaneous change in sign of both χ_ν and χ_π and thus equally good fits to energy spectra can be obtained for both combinations $\chi_\nu > 0$ and $\chi_\pi < 0$. Namely, only with this choice the observed sign of the mass quadrupole moment of the 2_1^+ state in *Kr* can be reproduced .

The remaining parameters play a less important role and are used mainly to improve the fit with experiment . In this work only $C_{0\nu}$ and $C_{2\nu}$ representing part of the d-boson conserving interaction between neutron bosons, were used as free parameters independent of (N_π). Finally, the values of ξ_2 and ξ_1 were vary from isotope to another, ξ_3 kept constant. The parameters used for the various isotopes are shown in table 1 .

It seen that parameters are constant or vary smoothly: within a series of isotopes χ_π does not vary, the variation in ε is very small and there is a slight decrease of the value of κ for the lighter Kr isotopes. The change in character of the spectra through a series of isotopes is essentially due to two effects: (**i**) the decrease of the value of χ_ν, and (**ii**) the decrease of the number of neutron bosons N_ν. We note that the behaviors of $\varepsilon, \kappa, \chi_\nu$ and χ_π is I qualitative agreement with microscopic considerations. It was found that both $C_{0\nu}$ and $C_{2\nu}$ vary for the isotopes. Such a behavior agree with the trend found in other regions [16]. The positive value of ξ_2 guarantees that no low-lying anti-symmetric multiplets occur for which there is no experimental evidence.

Table 1: IBM-2 Hamiltonian parameters for *Kr* isotopes, all parameters in MeV units except χ_π and χ_ν are dimensionless

ξ_3	ξ_2	ξ_1	$C_{4\pi}$	$C_{2\pi}$	$C_{0\pi}$	$C_{4\nu}$	$C_{2\nu}$	$C_{0\nu}$	χ_π	χ_ν	κ	ε	N	N_ν	N_π	**Isotopes**
0.11	0.032	0.051	0.30	0.30	-1.2	0.11	0.20	-1.20	-0.60	0.41	-0.080	0.701	9	5	4	$^{76}_{36}Kr_{40}$
0.11	0.130	0.102	0.30	0.31	-1.2	0.11	0.27	-0.66	-0.60	0.52	-0.090	0.722	8	4	4	$^{78}_{36}Kr_{42}$
0.11	0.050	0.242	0.05	-0.21	-0.48	0.11	0.11	-0.22	-0.60	0.60	-0.081	0.890	7	3	4	$^{80}_{36}Kr_{44}$
0.11	0.050	0.182	0.07	-0.18	1.43	-0.37	-0.82	0.11	-0.60	0.70	-0.081	0.960	6	2	4	$^{82}_{36}Kr_{46}$
0.11	0.440	0.601	0.38	0.25	0.14	0.0	0.0	0.0	-0.60	0.81	-0.080	0.949	5	1	4	$^{84}_{36}Kr_{48}$
0.11	0.450	0.601	0.0	-0.18	-0.30	0.12	0.12	0.12	-0.60	0.62	-0.087	0.920	5	1	4	$^{88}_{36}Kr_{52}$
0.11	0.510	0.632	0.17	0.17	0.17	0.11	0.11	0.11	-0.60	059	-0.992	0.862	6	2	4	$^{90}_{36}Kr_{54}$

Energy Spectra

The calculated excitation energies of positive parity levels to $^{76\text{-}90}Kr$ isotopes are given in table 1 and displayed in Figs.(1-7). The agreement between the calculated and experimental values is satisfactory.

Using the parameters in table 1, the estimated energy levels are shown in the figures, along with experimental energy levels. As can be seen, the agreement between experiment and theory is quite good and the general features are reproduced well. We observe the discrepancy between theory and experiment for high spin states. But one must be careful in comparing theory with experiment, since all calculated states have a collective nature, whereas some of the experimental states may have a particle-like structure. Behavior of the ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ of the energies of the first 4_1^+ and 2_1^+ states are good criteria for the shape transition [17]. The value of $R_{4/2}$ ratio has the limiting value 2.0 for a quadrupole vibrator, 2.5 for a non-axial gamma-soft rotor and 3.33 for an ideally symmetric rotor. $R_{4/2}$ remain nearly constant at increase with neutron number. The estimated values change from isotope to another (see table 2), this meaning that their structure seems to be varying from axial gamma soft to quadrupole vibrator $U(5) \to O(6)$. Since *Kr* nucleus has a rather vibrational-like character, taking into account of the dynamic symmetry location of the even-even *Kr* nuclei at the *IBM* phase Casten triangle where their parameter sets are at the $U(5) \to O(6)$ transition region and closer to *U*(*5*) character and we used the multiple expansion form of the Hamiltonian for our approximation.

The shape transition predicted by this study is consistent with the spectroscopic data for these nuclei. *K*r are typical examples of isotopes that exhibit a smooth phase transition from vibrational nuclei *SU*(*5*) to soft triaxial rotors *O*(6).**[9]**

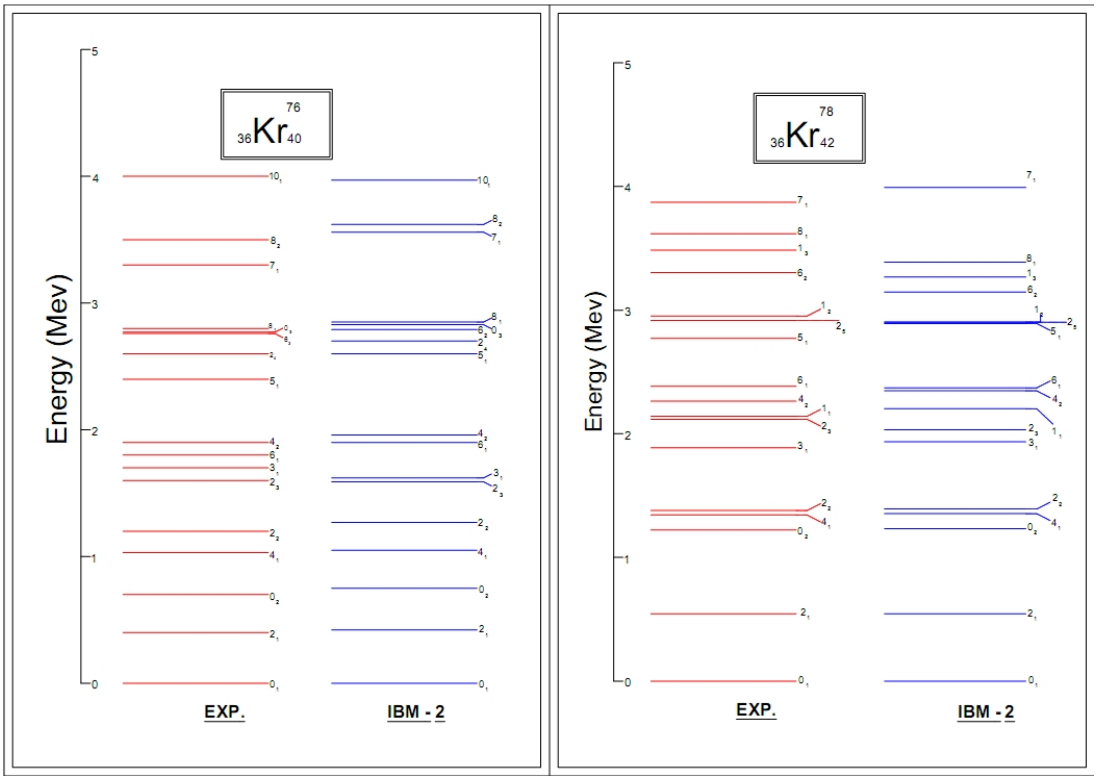
Table 2: Energy ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ for *Kr* isotopes

$E(4_1^+)/E(2_1^+)$	$^{76}_{36}Kr_{40}$	$^{78}_{36}Kr_{42}$	$^{80}_{36}Kr_{44}$	$^{82}_{36}Kr_{46}$	$^{84}_{36}Kr_{48}$	$^{88}_{36}Kr_{52}$	$^{90}_{36}Kr_{54}$
Exp. [17]	2.44	2.459	2.327	2.345	2.375	2.121	3.043
IBM-2	2.49	2.467	2.312	2.354	2.329	2.116	3.043

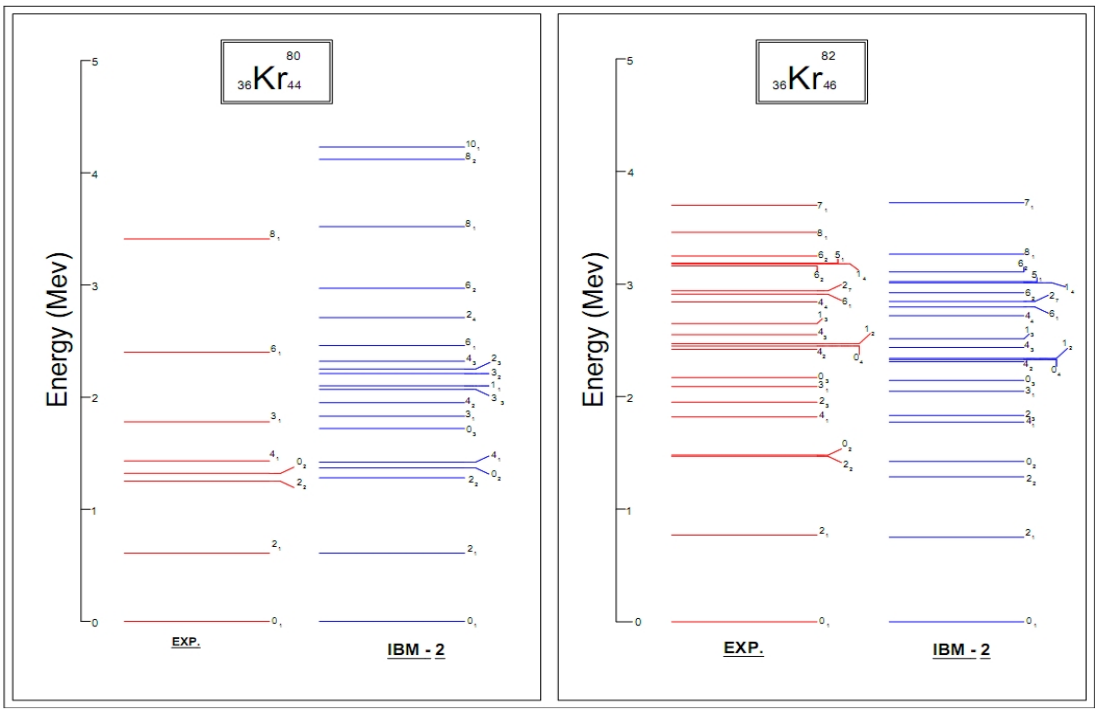
In the Figures we show the results of our calculations for the energies of the ground state band ($2_1^+, 4_1^+, 6_1^+, 8_1^+$ and 10_1^+) in the $^{76\text{–}90}Kr$ isotopes . We observe the discrepancy between theory and experiment for $J^\pi = 6^+, 8^+$ in *Kr* isotopes with neutron bosons ($N = 42,44,46,48$), However, one must careful in comparing theory with experiment, since all calculated low-lying states have a collective nature.

The order of the 0_2^+ and 3_1^+ is correctly predicted in $^{76\text{–}90}Kr$ isotopes and we remark that the energy of the 3_1^+ state is predicted systematically too high. This is a consequence of the presence of a Majarona term $M_{\pi\nu}$ in the Hamiltonian (eq. 3). We have chosen the parameters of the Majarona force in such a way that it pushes up states which are not completely symmetric with respect to proton and neutron bosons, since there is no experimental evidence for such states. However, experimental information becomes available about these states with mixed symmetry, this situation could possibly be improved. In the present case it would have been possible to further higher its energy by constant the value of ξ_3 .

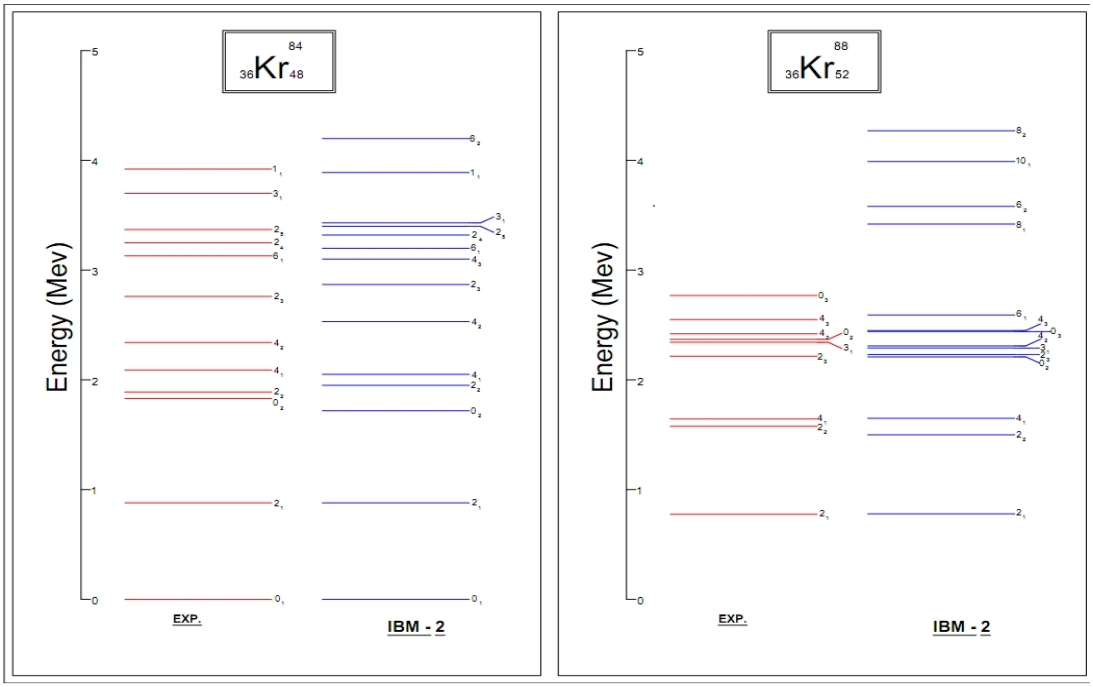
The position of the 2_3^+ state relative to the 0_2^+ state especially in $^{78}Kr, ^{82}Kr$ and ^{88}Kr isotopes. The moment of inertia of the ground state band increase, the quasi γ-band is pushed up, and also 0_2^+ state becomes a member of a $K = 0$ β-band.



Figures 1& 2 : A comparison between the experimental energy levels from IBM-2 calculations for ^{76}Kr and ^{78}Kr [18]



Figures 3& 4 : A comparison between the experimental energy levels from IBM-2 calculations for ^{80}Kr and ^{82}Kr [18]



Figures 5 & 6 : A comparison between the experimental energy levels from IBM-2 calculations for ^{84}Kr and ^{88}Kr [18]

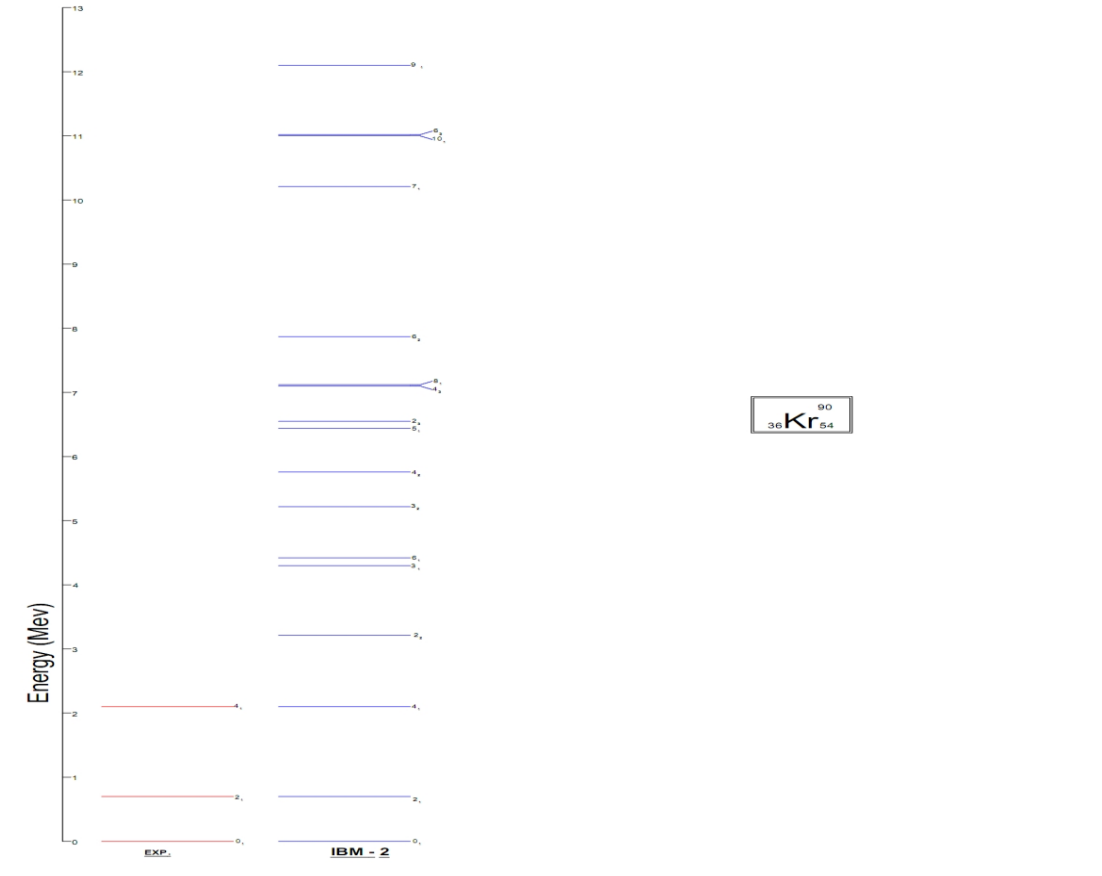


Figure 7: A comparison between the experimental energy levels from IBM-2 calculations for ^{90}Kr [18]

Electric Transition Probability

The effective boson charges e_π and e_ν were calculated by plotting $\boldsymbol{M}$ against N_ν / N_π where[19]:

for the $U(5)$ limit……(13)

$$M_1 = (1/N_\pi)\left[NB(E2;2_1^+ \to 0_1^+\right]^{1/2} e.b = e_\pi + e_\nu N_\nu / N_\pi$$

for the $O(6)$ limit……(14)

$$M_2 = (1/N_\pi)\left(\frac{5N}{N+4}B(E2;2_1^+ \to 0_1^+\right)^{1/2} e.b = e_\pi + e_\nu N_\nu / N_\pi$$

where $B(E2)$ is the reduced transition probability, N_π and N_ν are the boson numbers of proton and neutron respectively, $N = N_\pi + N_\nu$ is the total boson number. The difference between the effective charge and the charge of the single nucleon is referred to as the polarization charge. The value of effective charge may depend somewhat on the orbit of the nucleon. In particular, the polarization effect decreases when the binding energy of the nucleon becomes small.

Figure: 8 represent the relation between $\left[NB(E2;2_1^+ \to 0_1^+ / N_\pi^2\right]^{1/2}$ and $e_\pi + e_\nu N_\nu / N_\pi$ for the SU(5) limit, $\left(\frac{5N}{N+4}B(E2;2_1^+ \to 0_1^+ / N_\pi^2\right)^{1/2}$ and $e_\pi + e_\nu N_\nu / N_\pi$ for *O*(6) limit. The linearity is indeed present giving $e_\pi = 0.0288\ eb$ and $e_\nu = 0.209\ eb$ in the SU(5) limit and $e_\pi = 0.0575\ eb$, $e_\nu = 0.1047\ eb$ in the O(6) limit .The best fit to $^{76-90}Kr$ isotopes was obtained $e_\pi = 0.04315\ eb$ and $e_\nu = 0.15685\ eb$ .This result gives a clear indication that the rotational contribution in nuclear motion in this region is very high.

We use used these results of effective charges to calculate the electric transition probabilities using the NPBEM code. The results are presented in table 3.

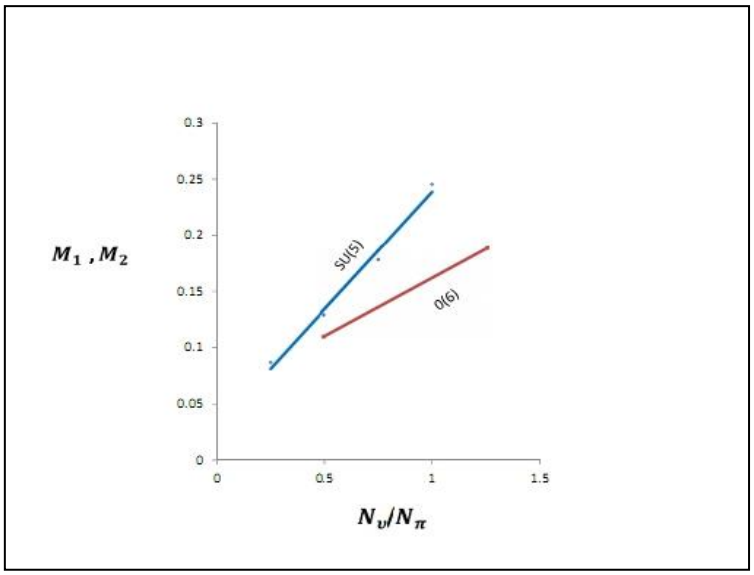


Figure 8: The plot of the quantity M_1 and M_2 versus N_ν / N_π for several even-even nuclei.

Table 3: Electric Transition Probability for *Kr* isotopes in e^2b^2 units

IBM-2	Exp.[8,18]	$I_i^+ \to I_f^+$	Isotopes
0.165	0.164(57)	$2_1 \to 0_1$	
0.00521	0.0090	$2_2 \to 0_1$	
0.0087	0.0038	$2_2 \to 2_1$	
0.00071	-	$2_3 \to 2_1$	
0.1322	-	$0_2 \to 2_1$	
0.0065	0.0019	$3_1 \to 2_1$	^{76}Kr
0.1871	-	$3_1 \to 2_2$	
0.0004	-	$3_2 \to 2_1$	
0.1413	0.198(190)	$3_2 \to 2_2$	
0.0098	0.0011(4)	$4_1 \to 2_1$	
0.1320	0.0858(28)	$4_2 \to 2_1$	
-	-	$4_2 \to 2_2$	
0.0731	0.0209(76)	$4_2 \to 4_1$	
0.0007	-	$1_1 \to 2_1$	
0.0021	-	$1_1 \to 2_2$	
0.1861	-	$1_1 \to 2_3$	
-0.501	-	**$Q(2_1)$**	
0.1202	0.1206(79)	$2_1 \to 0_1$	

0.0029	0.003(4)	$2_2 \rightarrow 0_1$	^{78}Kr
0.0825	0.0118(39)	$2_2 \rightarrow 2_1$	
0.00025	-	$2_3 \rightarrow 2_1$	
0.1861	-	$0_2 \rightarrow 2_1$	
0.1462	-	$3_1 \rightarrow 2_1$	
0.0060	-	$3_1 \rightarrow 2_2$	
0.00019	-	$3_2 \rightarrow 2_1$	
-	-	$3_2 \rightarrow 2_2$	
0.1876	0.1740(138)	$4_1 \rightarrow 2_1$	
0.1092	0.1147(158)	$4_2 \rightarrow 2_1$	
-	-	$4_2 \rightarrow 2_2$	
0.0622	0.0474(118)	$4_2 \rightarrow 4_1$	
0.0396	-	$1_1 \rightarrow 2_1$	
0.1261	-	$1_1 \rightarrow 2_2$	
0.0071	-	$1_1 \rightarrow 2_3$	
-0.481	-	**$Q(2_1)$**	
0.0756	0.0727(43)	$2_1 \rightarrow 0_1$	^{80}Kr
0.0026	0.0038	$2_2 \rightarrow 0_1$	
0.0772	0.0511(102)	$2_2 \rightarrow 2_1$	
0.0002	-	$2_3 \rightarrow 2_1$	
0.0903	-	$0_2 \rightarrow 2_1$	
0.0019	0.011(3)	$3_1 \rightarrow 2_1$	
0.0782	0.0695(102)	$3_1 \rightarrow 2_2$	
0.0002	-	$3_2 \rightarrow 2_1$	
-	-	$3_2 \rightarrow 2_2$	
0.0971	0.899(122)	$4_1 \rightarrow 2_1$	
0.0072	0.0005(3)	$4_2 \rightarrow 2_1$	
0.117	0.1021(613)	$4_2 \rightarrow 2_2$	
0.041	-	$4_2 \rightarrow 4_1$	
0.0287	-	$1_1 \rightarrow 2_1$	
0.1291	-	$1_1 \rightarrow 2_2$	
0.0431	-	$1_1 \rightarrow 2_3$	
-0.361	-	**$Q(2_1)$**	
0.0469	0.0450(14)	$2_1 \rightarrow 0_1$	^{82}Kr
0.0006	0.0002	$2_2 \rightarrow 0_1$	
0.0072	0.0053	$2_2 \rightarrow 2_1$	
-	-	$2_3 \rightarrow 2_1$	
0.0428	0.0317(105)	$0_2 \rightarrow 2_1$	
0.0008	-	$3_1 \rightarrow 2_1$	
0.0631	-	$3_1 \rightarrow 2_2$	
0.0002	-	$3_2 \rightarrow 2_1$	
-	-	$3_2 \rightarrow 2_2$	
0.0620	0.0676(253)	$4_1 \rightarrow 2_1$	
0.0081	0.0024(4)	$4_2 \rightarrow 2_1$	
0.0202	0.0195(4)	$4_2 \rightarrow 2_2$	
0.0320	0.0812(13)	$4_2 \rightarrow 4_1$	
0.0005	-	$1_1 \rightarrow 2_1$	
0.00021	-	$1_1 \rightarrow 2_2$	
0.0627	-	$1_1 \rightarrow 2_3$	
-0.29	-	**$Q(2_1)$**	
0.0239	0.0244(11)	$2_1 \rightarrow 0_1$	^{84}Kr
0.0002	0.0052(13)	$2_2 \rightarrow 0_1$	
0.0301	0.0239(87)	$2_2 \rightarrow 2_1$	
0.0005	-	$2_3 \rightarrow 2_1$	
0.0297	-	$0_2 \rightarrow 2_1$	
0.0002	-	$3_1 \rightarrow 2_1$	
0.00019	-	$3_1 \rightarrow 2_2$	
0.0422	-	$3_2 \rightarrow 2_1$	
-	-	$3_2 \rightarrow 2_2$	
0.0402	0.0479(65)	$4_1 \rightarrow 2_1$	
0.00019	0.0003	$4_2 \rightarrow 2_1$	
0.00420	0.0035(5)	$4_2 \rightarrow 2_2$	
0.0221	-	$4_2 \rightarrow 4_1$	
0.0001	-	$1_1 \rightarrow 2_1$	
0.0003	-	$1_1 \rightarrow 2_2$	
0.0272	-	$1_1 \rightarrow 2_3$	
-0.23	-	**$Q(2_1)$**	

Continued B(E2) for Kr isotopes

IBM-2	Exp.	$I_i^+ \rightarrow I_f^+$	Isotopes
0.0322	-	$2_1 \rightarrow 0_1$	^{88}Kr
0.003	-	$2_2 \rightarrow 0_1$	
0.622	-	$2_2 \rightarrow 2_1$	
0.007	-	$2_3 \rightarrow 2_1$	
0.622	-	$0_2 \rightarrow 2_1$	
0.003	-	$3_1 \rightarrow 2_1$	
0.025	-	$3_1 \rightarrow 2_2$	
0.001	-	$3_2 \rightarrow 2_1$	
-	-	$3_2 \rightarrow 2_2$	
0.527	-	$4_1 \rightarrow 2_1$	
0.0781	-	$4_2 \rightarrow 2_1$	
0.321	-	$4_2 \rightarrow 2_2$	
0.737	-	$4_2 \rightarrow 4_1$	
0.003	-	$1_1 \rightarrow 2_1$	
0.0009	-	$1_1 \rightarrow 2_2$	
0.632	-	$1_1 \rightarrow 2_3$	
-0.22	-	**$Q(2_1)$**	
0.652	-	$2_1 \rightarrow 0_1$	^{90}Kr
0.137	-	$2_2 \rightarrow 0_1$	
0.521	-	$2_2 \rightarrow 2_1$	
0.001	-	$2_3 \rightarrow 2_1$	
0.346	-	$0_2 \rightarrow 2_1$	
0.187	-	$3_1 \rightarrow 2_1$	
0.276	-	$3_1 \rightarrow 2_2$	
0.0015	-	$3_2 \rightarrow 2_1$	
-	-	$3_2 \rightarrow 2_2$	
0.972	-	$4_1 \rightarrow 2_1$	
0.622	-	$4_2 \rightarrow 2_1$	
0.432	-	$4_2 \rightarrow 2_2$	
0.821	-	$4_2 \rightarrow 4_1$	
0.002	-	$1_1 \rightarrow 2_1$	
0.001	-	$1_1 \rightarrow 2_2$	
0.197	-	$1_1 \rightarrow 2_3$	
-	-	**$Q(2_1)$**	

0.20			
- 0.361	-	**$Q(2_1)$**	

Experimental data are taken from [8,18]

It is well known that absolute gamma ray transition probabilities offer the possibility of a very sensitive test of nuclear models and the majority of the information on the nature of the ground state has come from studies of the energy level spacing. The transition probability values of the exited state in the ground state band constitute another source of nuclear information. Yrast levels of even-even nuclei (I_i= 2,4,6,.....) usually decay by *E*2 transition to the lower lying yrast level with $I_f = I_i - 2$.

In table 3, we show the $B(E2;2_1^+ \to 0_1^+)$ and $B(E2;4_1^+ \to 2_1^+)$ values , which are of the same order of magnitude and display a typical decrease towards the middle of the shell.

As a consequence of possible *M*1 admixture the $B(E2;2_2^+ \to 2_1^+)$ quantity is rather difficult to measure. For Kr isotopes, we give the different, conflicting experimental results and we see that no general feature n be derived from them, from these values seems to increase for $^{76\text{–}78\text{–}80}Kr$ and decrease for $^{82\text{–}84}Kr$.

In the table we show $B(E2;2_2^+ \to 0_1^+)$ values. Experimentally the results are radically different for the Kr isotopes , the value. In the some Kr isotopes the value seems to increased towards the middle of the shell, whereas in another Kr isotopes is decreased. Our calculations could not reproduce these contradictory features simultaneously.

The quantity $B(E2;0_2^+ \to 2_1^+)$, which is shown in table 3, provides a second clue for identifying intrude 0^+ states. If the experimental $B(E2;0_2^+ \to 2_1^+)$ value largely deviates from the results of our calculation, it is very likely the observed 0_2^+ states does not correspond to the collective state, but it is rather an intruder state. In ^{82}Kr isotope, there is a good agreement between experimental and calculated $B(E2;0_2^+ \to 2_1^+)$ value. This confirm our earlier statement about the nature of the lowest 0^+ state in this isotope.

The electric transition probabilities from the mixed-symmetry state $I = 1^+$ to the symmetric states $(2_1^+, 2_2^+)$ is weak collective *E*2 transition. The E2 transition between the 1^+ and the 2_1^+ ground state is small, whereas E2 transitions are large between fully-symmetric states and between mixed-symmetry states.

To conclude this section on the *E*2 properties, we give the results for the quadrupole moments $Q(2_1^+)$ of the first excited 2^+ state in table 3. We show complication of theoretical results. The general features of these results is clear, namely an increased in the negative quadrupole moment with increasing neutron number.

Magnetic Transition Probability

The *B*(*M*1) reduced transition probabilities were calculated using eq.6, and the boson gyromagnetic factors g_π, g_ν were estimated using the fact that $g = Z/A$ and the relation [20]

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \text{.....(15)}$$

and one of the experimental $B(M1;2_2^+ \to 2_1^+)$= 0.0429 μ_N^2 [8] for ^{76}Kr isotope, were used to produce a suitable estimation for the boson gyromagnetic factors. These values are $g_\pi = 0.782\ \mu_N$ and $g_\nu = 0.328\ \mu_N$. They are different from those of the rare–earth nuclei, $(g_\pi - g_\nu = 0.65\mu_N)$, suggested by Van Isacker *et al* [21] also used $g_\pi = 1$ and $g_\nu = 1$ to reduce the number of the model parameters in their calculation of *M*1 properties in deformed nuclei. The results of our calculation are listed in table 4. A good agreement between the theory and the available experimental data is achieved. As can be seen from the table yields to a simple prediction that M1 matrix elements values for gamma to ground and transitions should be equal for the same initial and final spin. Also the size of gamma to ground matrix elements seems to decrease as the mass number increases.

The results shows that the transitions between low-lying collective states are relatively weak. This is because of the increase of the anti-symmetric component in the wave functions introduced by *F-spin* breaking in the Hamiltonian. The magnitude of M1 values increases with increasing spin for $\gamma \to g$ and $\gamma \to \gamma$ transitions and we see:

1-By fitting B(M1) from 2_γ to 2_g we always get small value for $g_\pi - g_\nu$ compared with the value basis on the microscopic calculations $g_\pi - g_\nu = 1$.

2-There are evidences that M1 small mode exists in all spectra.

3-one cannot make decisive conclusions related to the agreement between theoretical and experimental data from the above table due to the lack of experimental data. However both experiments and theory predicts small M1 component which is due to symmetry and forbiddances of band crossing gamma transitions.

4-The $\gamma \to \gamma$ M1 matrix elements are larger than the $\gamma \to g$ M1 matrix elements by a factor of 2 to 3. Again, this agree qualitatively with the perturbation expressions derived in [22] .

5-The size of the $\gamma \to g$ M1 matrix elements seems to decrease with increasing mass. Specially, a change in $\gamma \to g$ M1 strengths occurs when the gamma band crosses the beta band.

The *M1* properties of collective nuclei are certainly very sensitive to various, even small, components in the wave functions either of collective or non-collective character. In the $^{88-90}Kr$ isotopes it was shown that the inclusion of excitations across the major shell and two quasi-particle states is important. One excepts that also for $^{88-90}Kr$ isotopes (which are near to closed shell for neutron) similar effects come into play. As above analysis suggests they can manifest in considerable renormalization of IBM-2 boson g-factors from their slandered values. The magnetic dipole moment for first excited state is given by

$$\mu(2_1^+) = g_\pi L_\pi + g_\nu L_\nu \quad ....(16)$$

where $g_\pi(g_\nu)$ is the *g*-factor for the correlated proton (neutron) boson and $L_\pi(L_\nu)$ is the corresponding angular momentum operator. According to the microscopic foundation of the model, $g_\pi(g_\nu)$ is expected to depend, in first approximation on proton (neutron) number $N_\pi(N_\nu)$ only, $g_\pi = g_\pi(N_\pi)$ and $g_\nu = g_\nu(N_\nu)$. The IBM-2 calculations for $\mu(2_1^+)$ are listed in table 4, we see a good agreement with experimental data.

It is clear that the two effects contribute to the dependence of the magnetic moments on proton and neutron number: the dependence of g_π and g_ν on proton and neutron number and the variation of the matrix elements of the operator $L_\pi(L_\nu)$ with N_π and N_ν . As will be better shown below , the former effect is the related to the shell structure of the orbits, while the latter is

Table 4: Reduced transitions probability B(M1) in Kr Isotopes in μ_N^2 units

$^{90}_{36}Kr_{54}$		$^{88}_{36}Kr_{52}$		$^{84}_{36}Kr_{48}$		$^{82}_{36}Kr_{46}$		$^{80}_{36}Kr_{44}$		$^{78}_{36}Kr_{42}$		$^{76}_{36}Kr_{40}$		$I_i^+ \to I_f^+$
IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	
0.0310	-	0.0271	-	0.0291	0.0256(53)	0.00291	0.0010	0.00065	0.0004(1)	0.0236	0.0157(21)	0.0031	0.0429	$2_2 \to 2_1$
0.00035	-	0.0732	-	0.0671	-	0.00775	-	0.0132	-	0.0097	0.0001	0.0883	-	$2_3 \to 2_1$
0.0078	-	0.00033	-	0.0003	-	0.0008	-	0.00099	0.0007(2)	0.0026	-	0.0192	0.0154	$3_1 \to 2_1$
0.00083	-	0.0072	-	0.0064	-	0.00078	-	0.00201	0.0015(4)	0.00087	-	0.0030	-	$3_1 \to 2_2$
0.00052	-	0.0008	-	0.00071	-	0.0079	-	0.0062	-	0.0167	-	0.0301	-	$3_2 \to 2_1$
0.0022	-	0.00052	-	0.00041	-	0.0432	0.1414(208)	0.0062	0.0054(35)	0.0052	0.0046	0.0159	0.0172(62)	$4_2 \to 4_1$
0.0342	-	0.032	-	0.0022	-	0.0002	-	0.0172	-	0.00281	-	0.00322	-	$1_1 \to 2_1$
0.0031	-	0.0056	-	0.00231	-	0.004	-	0.00145	-	0.001431	-	0.0013	-	$1_1 \to 2_2$
0.379	-	0.032	-	0.019	-	0.0221	-	0.021	-	0.0172	-	0.0131	-	$1_1 \to 2_3$
0.0001	-	0.002	- 0.330(3)	2.510	2.24(28)	0.077	0.80(4)	0.081	-	0.084	0.86(2)	0.0891	-	$\mu(2_1^+)$

Experimental data are taken from [8 ,18]

Mixing Ratio $\delta(E2/M1)$

We evaluate the mixing ratio $\delta(E2/M1)$ for *Kr* isotopes, depends on the equation [23].

$$\delta(E2/M1) = 0.835E_\gamma \frac{< I_i \| T(E2) \| I_f >}{< I_i \| T(M1 \| I_f >} \quad .....(17)$$

These are compared with experimental and theoretical results in table 5, where one can see good agreement with estimated and experimental values. The variations in sign of the *E2/M1* mixing ratios from one nucleus to another for the same class of transitions, and within a given nucleus for transitions from different spin states, suggest that a microscopic approach is needed to explain the data theoretically. For such reason, the sign of the mixing ratio is not taken into consideration. Sign convention of mixing ratios has been explained in detail by Lange *et al*. [23]

These results exhibit disagreement in some cases, with one case showing disagreement in sign. However, it is a ratio between very small quantities and any change in the dominator that will have a great influence on the ratio. The large calculated value for

$2_2^+ \to 2_1^+$ is not due to a dominate E2 transition, but may be under the effect of very small *M*1 component in the transition. Moreover, the large predicted value for transition $2_2^+ \to 2_1^+$ in ^{80}Kr compared with experimental value may be related to high predicted energy level value of the IBM-2; $E(2_2^+)$ =1.287 MeV, while the experimental value is 1.256 MeV. We are unable to bring the energy value of this state close to experimental value simply by changing the Majorana parameters.

Table 5: Mixing Ratios $\delta(E2/M1)$ in Kr Isotopes in MeV eb/μ_N units

$^{90}_{36}Kr_{54}$		$^{88}_{36}Kr_{52}$		$^{84}_{36}Kr_{48}$		$^{82}_{36}Kr_{46}$		$^{80}_{36}Kr_{44}$		$^{78}_{36}Kr_{42}$		$^{76}_{36}Kr_{40}$		$I_i^+ \to I_f^+$
IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	
-6.327	-	0.9107	-	0.914	$10^{+\infty}_{-8}$	0.717	$0.8 \le \delta \le 6.4$	5.480	$16^{+\infty}_{-8}$	1.0865	-	1.189	-	$2_2 \to 2_1$
1.1176	-	-0.835	-	0.1435	-	-0.549	-	0.5317	-	0.165	-	0.087	-	$2_3 \to 2_1$
4.805	-	0.912	-	0.647	-	1.114	4.2(2)	0.4447	-	7.182	-	1.9935	-	$3_1 \to 2_1$
-10.255	-	0.3915	-	4.150	-	5.858	2..13(7)	2.833	-	0.0989	-	0.4289	-	$3_1 \to 2_2$
0.7333	-	3.792	-	7.372	-	0.0274	-	0.2710	-	0.1729	-	2.565	-	$3_2 \to 2_1$
18.4123	-	10.121	-	2.979	-	0.3909	-	0.6478	-	2.376	-	-1.630	-	$4_2 \to 4_1$
4.942	-	22.701	-	0.5674	-	2.1323	-	1.605	-	1.3559	-	0.0887	-	$1_1 \to 2_1$
3.617	-	21.432	-	0.0614	-	0.6455	-	2.104	-	1.7973	-	0.0492	-	$1_1 \to 2_2$
0.527	-	0.0327	-	3.560	-	0.7104	-	0.855	-	0.0466	-	2.227	-	$1_1 \to 2_3$

Experimental data are taken from [8, 23]

Electric Monopole Matrix Element $\rho(E0)$

The *E*0 transition occurs between two states of the same spin and parity by transferring the energy and zero units of angular momentum, and it has no competing gamma ray. The *E*0 transition is present when there is a change in the surface of the nucleus. For example, in nuclear models where the surface is assumed fixed, *E*0 transitions are strictly forbidden, such as in shell and IBM models. Electric monopole transitions are completely under the penetration effect of atomic electrons on the nucleus, and can occur not only in $0^+ \to 0^+$ transition but also, in competition with gamma multipole transition, and depending on transition selection rules that may compete in any $\Delta J = 0$ decay such as a $2^+ \to 2^+$ or any $I_i = I_f$ states in the scheme. When the transition energy greater than $2moc^2$, monopole pair production is also possible.

The *E*0 reduced transition probability is given in eq. 9. The parameters in equation 9, can be predicted from the isotope shift [20] (see table 7), since such data are not available for *Kr* isotopes, we calculate these parameters by fitting procedure into two experimental values of isotopic shifts (equation (2-59)). The parameters which were subsequently used to evaluate the $\rho(E0)$ -values were; $\beta_{0\pi} = 0.062\ eb$, $\beta_{0\nu} = -0.021\ eb$ and $\gamma_{0v}=0.032$ fm^2 . From the table 6, in general there is no experimental data to compare with the IBM-2 calculations**.**

The monopole matrix element is important for nuclear structure and the model predictions due to their sensitivity for the nuclear shape. We conclude that more experimental work is needed to clarify the band structure and investigate an acceptable degree of agreement between the predictions of the models and the experimental data.

We also find good agreement between the calculated and experimental values for isotope shifts for all mercury isotopes (table 7) but the isomer shift result for *Kr* isotopes is in poor agreement with the experimental value.

Table 6: Monopole matrix element for *Kr* isotopes

^{90}Kr	^{88}Kr	0.0082	^{82}Kr	^{80}Kr	^{78}Kr	^{76}Kr	$I_i^+ \to I_f^+$
0.075	0.0094	0.0082	0.0750	0.070	0.0551	0.0431	$0_2 \to 0_1$
0.0009	0.00092	0.00089	0.00083	0.00080	0.00073	0.0067	$0_3 \to 0_1$
0.0102	0.0099	0.00094	0.00089	0.00034	0.0045	3.18*10-3	$0_3 \to 0_2$
0.013	0.0099	0.0096	0.0076	0.0056	0.0034	0.0002	$2_2 \to 2_1$
0.00070	0.00067	0.00065	0.00052	0.00046	0.0041	0.00034	$2_3 \to 2_1$
0.00872	0.00531	0.00519	0.00395	0.00351	0.0027	0.0021	$2_3 \to 2_2$
0.000742	0.00072	0.000611	0.00059	0.00057	0.00041	0.00035	$3_2 \to 2_1$

Table 7: **Isotopic Shifts for *Kr* Isotopes**

Nucleus	$\Delta < r^2 > \ fm^2$	
	Exp.[17]	***IBM-2***
$^{76}_{36}Kr_{40} - ^{78}_{36}Kr_{42}$	-	-0.009
$^{78}_{36}Kr_{42} - ^{80}_{36}Kr_{44}$	-	-0.0148
$^{80}_{36}Kr_{44} - ^{82}_{36}Kr_{46}$	-0.028(5)	-0.0298
$^{82}_{36}Kr_{46} - ^{84}_{36}Kr_{48}$	-0.040(4)	-0.055
$^{84}_{36}Kr_{48} - ^{86}_{36}Kr_{46}$	0.071(3)	0.0810
$^{86}_{36}Kr_{50} - ^{88}_{36}Kr_{52}$	0.37997)	0.431
$^{88}_{36}Kr_{52} - ^{90}_{36}Kr_{54}$	0.751	0.655

Mixed Symmetry States

One of the advantage of the IBM-2 is ability of reproducing the mixed symmetry states. These states are created by a mixture of the wave function of protons and neutrons that are observed in most even-even-even nuclei. This mixed symmetry states (*MSS*) has been observed in many nuclei. In more vibrational and $\gamma - soft$ nuclei This mixed symmetry states (*MSS*) has been observed in many nuclei. In more vibrational and gamma soft nuclei. We expect the lowest MSS with $I = 2^+$ state, while in rotational nuclei observed as the $I^\pi = 1^+$ state. In *Kr* isotopes we see that when the states

$I^\pi = = 2_2^+, 2_4^+$ and 3_1^+ are strongly dominated by the $F=F_{max}$, the strongest contribution to the $I^\pi = 2_3^+, 3_2^+$ states is the one with $F=F_{max}$-1. We can see the $I^\pi = 2_3^+, 3_2^+$ states as a mixed symmetry states in *Kr* isotopes.

In this work, we proposed that the 2_3^+ state decays to the first excited state with an energy 1.598 MeV in ^{76}Kr with a mixing ratio $\delta(E2/M1)$ = 1.189 which means it is dominated by the M1 transition, with $B(M1)$ equal to 0.0031 μ_N^2 . In ^{78}Kr isotope, for the third I^π = 2^+ state at energy 1.685 MeV excitation is close to the experimental data for 1.755 MeV. The energy is well reproduced by the calculation, where the choice of the Majarona parameters plays a crucial role. This state is quite pure F_{max}-1 with $R = < J \left| F^2 \right| I > / F_{\max}(F_{\max} + 1) = 50\%$. The excitation energy of 3_2^+ state is 2.399 MeV with mixing ratio $\delta(E2/M1; 3_2^+ \to 2_1^+) = 2.565$, $B(M1; 3_2^+ \to 2_1^+) = 0.00301 \mu_N^2$. In the ^{80}Kr, the calculation predicted the 2_3^+ state at 2.251 MeV with $R = 83\%$.

In other $^{82\text{–}84\text{–}88\text{–}90}Kr$ isotopes the states 2_3^+ and 3_2^+ are mixed symmetry states their excitation energies are closed with available experimental data and the values of $R = 73\%, 75\%, 72\%$ and 80% respectively .

In all Kr isotopes that the second 3^+ states to be the lowest $I^\pi = 3^+$ mixed symmetry states with two phonon excitation. The low-lying levels with angular momentum greater than 3^+ with a large mixed symmetry states component are predicted in this work.

The energy fit to several levels is very sensitive to the parameters in the Majorana term which also strongly influence the magnitude and sign of the multipole mixing ratios of many transitions. In particular we find that the calculated energies of a number of states are affected in a very similar way and these might be considered to have a mixed-symmetry origin, or contain substantial mixed-symmetry components. Those with a mixed-symmetry origin have no counterpart in IBM-1**.** The energy dependence of the 2_2^+ and 2_4^+ levels is consistent with the mixed-symmetry character of the 2_3^+ level being shared with neighboring states[24,25].

The influence of the parameters on these states is shown in table 1. The ξ_2 term strongly affects the energies of all of the levels considered to have a mixed-symmetry character or to contain mixed-symmetry components . In obtaining this plot the ξ_1 and ξ_3 terms were maintained at their best-fit values. The mixing ratio data, discussed in the above section have a strong dependence on ξ_2 and show that ξ_2 cannot be zero in our fit.

The 1^+ level is strongly affected by changing ξ_1, while the 3_1^+ level energy depends on the ξ_3 value. The 2_3^+ mixed-symmetry state and the predominantly symmetric 2_2^+ and 2_4^+ levels are largely unaffected by changing ξ_1, or ξ_3 in contrast to their dependence on ξ_2.

Most experimentally observed low-spin levels, apart from 1^+ states below.5 MeV; have their counterpart in the IBM-2 level spectrum although the energy match is not good in every case. It also appears that we may identify the members of the family of mixed-symmetry states corresponding to the [*N*-1,1] representation [12]. The small *E2/M1* mixing ratios are consistent with this interpretation but level lifetimes are required for a firmer identification.

In *Kr* isotopes, all hitherto discovered MSS have been reviewed in [9]. It has been shown that the lowest lying MSS is the one quadrupole phonon *MSS* labeled as $2_{1,ms}^+$, $3_{1,ms}^+$ and characterized by a weakly-collective *E*2 transition probability to the ground state and a large *M*1 transition to the 2_1^+ state.

Table 6, contains the calculated $\rho(E0)$ values. In general there is no experimental data to compare the IBM-2 results. It must also be remarked that the comparatively large $\rho(E0)$ values for transitions from the 2_3^+ mixed-symmetry state and from the 2_1^+ and 2_2^+ states indicate that substantial E0 components occur in these decays from mixed-symmetry states. The *E*0 matrix element describing such decay is proportional to $\beta_{0\pi}$ and $\beta_{0\nu}$, although the β values are small, their sign difference results in the E0 matrix being greatest.

Conclusion

We have presented results of calculations of the properties of the $^{76\text{-}90}Kr$ isotopes found in many cases good agreement between our calculations and experiment. However, we have also found that $^{76\text{-}90}Kr$ isotopes, that there are several 0^+ states whose properties cannot be reproduced by our calculations. These intruder states are presumably associated with other with the other low-lying degrees of freedom. Since they appears to be present also in other mass regions, it is clear that they must be explicitly included in the calculations if one wants to describe all observed low-lying states.

The shape transition predicted by this study is consistent with the spectroscopic data for $^{76\text{-}90}$Kr are typical examples of isotopes that exhibit a smooth phase transition from vibrational nuclei to soft triaxial rotors U(5)→O(6). The comparison of some B(E2), B(M1), mixing ratios for these isotopes with the experimental data show that these isotopes exist along the U(5)-O(6) side of the IBM triangle.

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