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Influence of chemical reaction on MHD flow of a visco-elastic fluid through porous medium with constant suction

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ABSTRACT

The present work analyzes the influence of chemical reaction on free convective flow of a visco-elastic incompressible and electrically conducting fluid past an infinite vertical porous plate through porous medium with constant suction in presence of uniform transverse magnetic field. The necessity of the present analysis arises as the most industrial fluids exhibit the visco-elastic behavior. The coupled nonlinear partial differential equation are turned to ordinary by super imposing a solution with steady and time dependent transient part. Finally, the set of ordinary differential equations is solved with a perturbation scheme to meet the inadequacy of boundary condition. The solutions for the velocity, temperature and concentration are obtained and displayed graphically for pertinent parameters such as magnetic parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number, Eckert number and permeability parameter. It is observed that the magnetic parameter and the Schmidt number retard the velocity of the flow field while the Grashof number for heat and mass transfer, the porosity parameter have accelerating effect on the velocity of the flow field at all points. Further, the Prandtl number reduces the temperature and Schmidt number and chemical reaction parameter diminishes the concentration distribution of the flow field at all points.

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Introduction

During recent years, the flow of viscous fluid through a porous media has been a subject of intensive studies because of its natural occurrence and importance in many industrial and engineering problems. The production of petroleum and natural gases, well drilling and lodging require many predictions based on results of fluid flow through a porous medium. The flow of blood through lungs and arteries are also examples of flow through porous media. The motion of the fluid is affected by so many factors. The boundaries of the fluid affect the flow to have stationery boundaries, fluctuating boundaries, moving boundaries, oscillatory boundaries and so on. The fluid motion in ducts, parallel plate channels, rectangular channels, parabolic boundaries, circular boundaries have been studied due to their importance in engineering and technology. At present, considerable attention has been given to the study of hydro magnetic convective flow of viscous fluids in connection with theories of fluid motion, two-phase flows, stratified flows, flow of immiscible fluids, flow through porous media, flow with suction/injection, flow in presence of heat source/heat flux, flow past a porous/hot/accelerated vertical plate, flow through channels of different shape with varied restriction and so on. New ideas have been added to the literature to possible applications in geophysics, engineering problems, geothermal energy, stem stimulation of oil field, food drying and heat pipes. A comprehensive review on this area has been made by many researchers some of them are Nield and Bejan [1] and Ingham and Pop [3, 4].

The analyses of the flow properties of non-Newtonian fluids are very important in the fields of fluid dynamics because of their technological application. Mechanics of non-Newtonian fluids present challenges to engineers, physicists and mathematicians. In addition, the effects of magnetic field on the non-Newtonian fluid also have great importance in engineering applications; for instance, MHD generators, plasma studies, geothermal energy excitations and in the field of aerodynamics for boundary layer control, etc. Moreover MHD flows in porous media have received wide coverage on the development of noval energy generation systems and interest in astrophysical and geophysical fluid dynamics. Due to the non-linearity of the Navier-Stokes equations and the



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inapplicability of the superposition principle for nonlinear partial differential equations, exact solutions are difficult to obtain and are few in number under certain conditions.

The most common type of body force, which acts on a fluid, is due to gravity so that the body force can be defined as in magnitude and direction by the acceleration due to gravity. Sometimes, electromagnetic effects are important. The electric and magnetic fields themselves must obey a set of physical laws, which are expressed by Maxwell's equations. The solution of such problems requires the simultaneous solution of the equations of fluid mechanics and of electromagnetism. One special case of this type of coupling is the field known as magnetohydrodynamic (MHD).

Many natural phenomena and engineering applications are susceptible to magneto-hydrodynamic (MHD) analysis. From the technological point of view, magneto-hydrodynamic flow finds application in the fields of stellar and planetary magneto-spheres, aeronautics, meteorology, solar physics, cosmic fluid dynamics, chemical engineering, electronics, induction flowmetry, MHD generators, MHD accelerators, construction of turbine and other centrifugal machines. Very often, along with the free convection currents caused by the temperature difference, the flow is also affected by the difference in concentration of material constituents. In engineering application, the concentration differences are created either by injecting foreign gases or by coating a substrate with a material, and subsequently heating it, so that the material evaporates.

Sivaraj et al. [4] have examined the MHD mixed convective flow of viscoelastic and viscous fluid in a vertical porous channel. Yih [5] discussed the viscous and Joule heating effects on Non-Darcy MHD natural convection flow over a permeable sphere in porous media with internal heat generation. Sonth et al. [6] have studied the heat and mass transfer in a visco-elastic fluid over an accelerating surface with heat source/sink and viscous dissipation. Devika et al. [7] have investigated the MHD oscillatory flow of a visco-elastic fluid in a porous channel with chemical reaction. Mishra et al. [8] have discussed the mass and heat transfer effect on MHD flow of a visco-elastic fluid through porous medium with oscillatory suction and heat source. Das et al. [9] have examined the mass transfer effect on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Das et al. [10] have studied the effects of mass transfer on flow past an impulsive started infinite vertical plate with constant heat flux and chemical reaction.

Abdussattar [11] studied the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Anand Rao and Prabhakar Reddy [12] have analyzed the numerical solution of mass transfer in MHD free convective flow of a viscous fluid through a vertical channel. Chamkha [13] studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Hossain and Mandal [14] have discussed the effects of mass transfer and free convection on the unsteady MHD flow past a vertical porous plate with constant suction. Sahoo et al. [15] have examined the magneto-hydrodynamic unsteady free convective flow past an infinite vertical plate with constant suction and heat sink.

Vijaya Sekhar and Viswanadh Reddy [16] have studied the chemical reaction effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Gireesh Kumar and Satyanarayana [17] have discussed the mass transfer effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Ambethkar [18] investigated the numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction and heat source or sink. Alam [19] analyzed the MHD free convective MHD flow through porous media of a rotating visco-elastic fluid past an infinite vertical porous plate with heat and mass transfer in the presence of a chemical reaction. Venkata Srinivasa Rao et al. [21] have discussed the chemical reaction effects on unsteady MHD free convective flow past an exponentially accelerated vertical porous plate. Shivaiah and Ananda Rao [22] have examined the chemical reaction effect on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Mansour et al. [23] have analyzed the effect of chemical reaction and viscous dissipation on MHD natural convection flows saturated in porous media with suction or injection.

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Jaiswal and Soundalgekar [24] have studied the oscillating plate temperature effects on a flow past an infinite vertical porous plate with constant suction and embedded in a porous medium. Raptis and Kafousias [25] have analyzed the magnetohydrodynamic free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Noushima Humera et al. [26] have discussed the unsteady MHD free convective viscoelastic flow and mass transfer through porous medium. Choudhury and Dey [27] have examined the free convective visco-elastic flow with heat and mass transfer through a porous medium with periodic permeability. Bejan and Khair [28] have looked the heat and mass transfer by natural convection in a porous medium. Sharma [29] discussed the free convection effects on the flow past a porous medium bounded by a vertical infinite surface with constant suction and constant heat flux. Sarangi and Jose [30] have studied the unsteady MHD free convective flow and mass transfer through a porous medium with constant suction and constant heat flux. Das and Jana [30] have analyzed the heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [31] discussed the MHD effects on free convection and mass transform flow through a porous medium with variable temperature.

Convection problems associated with heat sources within fluid saturated porous media are of great practical significance, for that there are a number of practical applications in geophysics and energy-related problems, such as recovery of petroleum resources, geophysical flows, cooling of underground electric cables, storage of nuclear waste materials ground water pollution, fiber and granular insulations, solidification of costing, chemical catalytic reactors and environmental impact of buried heat generating waste.

Rajput and Sahu [32] have examined the combined effects of chemical reactions and heat generation/absorption on unsteady transient free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion. Alam [33] have discussed the viscous dissipation effects with MHD natural convection flow on a sphere in presence of heat generation. Rajesh [35] studied the heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium.

The mixture of polymethyl mehacrylate and pyridine at 25^oC containing 30.5g of polymer per liter behaves very nearly as the Walter's liquid model B [36, 37, 38]

In view of all such studies, the unsteady MHD free convective visco-elastic (memory) flow of incompressible and electrically conducting fluid with chemical reaction gain importance and attention in recent years. In view of this, the main object of the present investigation is to study the effect of chemical reaction on the unsteady MHD free convective visco-elastic fluid flow past an infinite vertical porous plate through porous medium with constant suction. In the course of analysis it is assumed that the magnetic field of uniform strength is applied and induced magnetic field is neglected and also we observe that how various parameters affect the flow past an infinite vertical plate.

Formulation of the problem:

The unsteady free convective flow of a chemically reacting visco-elastic (Walters B) fluid past an infinite vertical porous plate through a porous medium with a constant suction in presence of a transverse magnetic field is considered. Let \mathbf{x}' - axis be ablong the plate in the direction of the flow and \mathbf{y}' - axis normal to it. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. Boussineq's approximation, the governing equation and boundary conditions are given by



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$$\frac{\partial v'}{\partial y'} = 0$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T'_{\infty}) + g\beta^*(C' - C'_{\infty}) + v' \frac{\partial^2 u'}{\partial {y'}^2} - B_1\left(\frac{\partial^3 u'}{\partial t' \partial {y'}^2} + v' \frac{\partial^3 u'}{\partial {y'}^3}\right) - v' \frac{\partial^2 u'}{\partial y'} = g\beta(T' - T'_{\infty}) + g\beta^*(C' - C'_{\infty}) + v' \frac{\partial^2 u'}{\partial {y'}^2} - B_1\left(\frac{\partial^3 u'}{\partial t' \partial {y'}^2} + v' \frac{\partial^3 u'}{\partial {y'}^3}\right) - v' \frac{\partial^2 u'}{\partial t'} = g\beta(T' - T'_{\infty}) + g\beta^*(C' - C'_{\infty}) + v' \frac{\partial^2 u'}{\partial {y'}^2} - B_1\left(\frac{\partial^3 u'}{\partial t' \partial {y'}^2} + v' \frac{\partial^3 u'}{\partial {y'}^3}\right) - v' \frac{\partial^2 u'}{\partial t'} = g\beta(T' - T'_{\infty}) + g\beta^*(C' - C'_{\infty}) + v' \frac{\partial^2 u'}{\partial {y'}^2} - B_1\left(\frac{\partial^3 u'}{\partial t' \partial {y'}^2} + v' \frac{\partial^3 u'}{\partial {y'}^3}\right) - v' \frac{\partial^2 u'}{\partial t'} = g\beta(T' - T'_{\infty}) + g\beta^*(C' - C'_{\infty}) + v' \frac{\partial^2 u'}{\partial {y'}^2} - B_1\left(\frac{\partial^3 u'}{\partial t' \partial {y'}^2} + v' \frac{\partial^3 u'}{\partial {y'}^3}\right) - v' \frac{\partial^2 u'}{\partial t'} + v' \frac{\partial^2 u'}{\partial {y'}^2} + v' \frac{\partial^3 u'}{\partial {y'}^3} + v' \frac{\partial^3 u'}{\partial {y'}^3}$$

$$\sigma B_0^2 \frac{u}{\rho} - \frac{v \, u}{k'} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + S' (T' - T'_{\infty}) + \frac{v'}{c_p} \left(\frac{\partial u'}{\partial y'}\right)^2$$
(3)

$$\frac{\partial c'}{\partial t'} + v' \frac{\partial c'}{\partial y'} = D \frac{\partial^2 c'}{\partial {y'}^2} - K'_r (C' - C'_\infty)$$
(4)

On disregarding the Joulean heat dissipation, the boundary conditions of the problem are:

$$u' = 0, T' = T'_{\infty} + \varepsilon (T'_{w} - T'_{\infty}) e^{iw't'},$$

$$C' = C'_{\infty} + \varepsilon (C'_{w} - C'_{\infty}) e^{iw't'} at y = 0$$

$$u' \to 0, T' \to T'_{\infty}, C' \to C'_{\infty} as y' \to \infty$$
(5)

From (1) we have

$$\boldsymbol{v}' = -\boldsymbol{v}_0$$
 (6)

Introducing the non-dimensional quantities and parameters: $y'v_0 = t'v_0^2 = 4v\omega'$

$$y = \frac{y \cdot v_{0}}{v}, \ t = \frac{t \cdot v_{0}}{4v}, \ \omega = \frac{4v\omega}{v_{0}^{2}}, v = \frac{\eta_{0}}{\rho}, \ Pr = \frac{v}{\kappa}, \ \kappa = \frac{K_{0}}{\rho C_{p}}, \ S = \frac{S' \cdot v_{0}^{2}}{4v} T = \frac{(T' - T'_{\infty})}{(T'_{w} - T'_{\infty})}, \ C = \frac{(C' - C'_{\infty})}{(C'_{w} - C'_{\infty})}, \ Kr = \frac{K'_{r} v}{v_{0}^{2}}, Sc = \frac{v}{p}, \ Gr = \frac{vg\beta(T'_{w} - T'_{\infty})}{v_{0}^{3}}, \ Gc = \frac{vg\beta^{*}(C'_{w} - C'_{\infty})}{v_{0}^{3}}, Ec = \frac{v_{0}^{2}}{c_{p}(T'_{w} - T'_{\infty})}, \ k = \frac{k' v_{0}^{2}}{\rho}$$

$$(7)$$

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GrT + GcC + \frac{\partial^2 u}{\partial y^2} - R_m \left[\frac{1}{4} \frac{\partial^3 u}{\partial t \partial y^2} - \frac{\partial^3 u}{\partial y^3} \right] - \left(M + \frac{1}{k} \right) u \tag{8}$$

$$\frac{Pr}{4}\frac{\partial T}{\partial t} - Pr\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{PrST}{4} + PrEc\left(\frac{\partial u}{\partial y}\right)^2$$
(9)

$$\frac{Sc}{4}\frac{\partial C}{\partial t} - Sc\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - KrScC$$
(10)

The corresponding boundary conditions are:

$$u = 0, T = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} at y = 0$$

$$u \to 0, \quad T \to 0, \quad C \to 0 \quad as \ y \to \infty$$
(11)

Method of Solution

In order to reduce the system of partial differential equations (8) – (10) to a system of ordinary differential equations in the nondimensional form, we assume the following for velocity, temperature and concentration distribution of the flow field as the amplitude ε (\ll 1) of the permeability variation is very small.

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$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y) e^{i\omega t} \\ T(y,t) &= T_0(y) + \varepsilon T_1(y) e^{i\omega t} \\ C(y,t) &= C_0(y) + \varepsilon C_1(y) e^{i\omega t} \end{aligned}$$
(12)

where u, T and C are velocity, temperature and concentration. Substituting (12) in equations (8), (9) and (10), equating harmonic (coefficient of ε^{0}) and non-harmonic terms (coefficient of ε), we get Zero order of ε

$$R_m u_0^{\prime\prime\prime} + u_0^{\prime} + u_0^{\prime} - \left(M + \frac{1}{k}\right) u_0 = -[GrT_0 + GcC_0]$$
⁽¹³⁾

$$T_0'' + PrT_0' + \frac{PrST_0}{4} = -PrEc(u_0')^2$$
(14)

$$C_0'' + ScC_0' - KrScC_0 = 0$$
(15)

First order of $\boldsymbol{\varepsilon}$

$$R_m u_1'' + u_1' + u_1' - \left(M + \frac{1}{k} + \frac{i\omega}{4}\right) u_1 = -[GrT_1 + GcC_1]$$
(16)

$$T_{1}^{\prime\prime} + PrT_{1}^{\prime} - \frac{\Pr(S-i\,\omega)T_{1}}{4} = -2PrEcu_{0}^{\prime}u_{1}^{1}$$
(17)

$$C_{1}^{\prime\prime} + ScC_{1}^{\prime} - \left(Kr + \frac{i\,\omega}{4}\right)ScC_{1} = 0$$
⁽¹⁸⁾

The boundary conditions now reduce to

$$\begin{array}{l} u_0 = u_1 = 0, T_0 = T_1 = 1, C_0 = C_1 = 1 \text{ at } y = 0 \\ u_0 \to u_1 \to 0, T_0 \to T_1 \to 0, C_0 \to C_1 \to 0 \text{ at } y \to \infty \end{array}$$
 (19)

The equations (13) and (16) are third order but two boundary conditions are available. Therefore the perturbation method has been applied using R_m ($R_m \ll 1$), the elastic parameter as the perturbation parameter,

$$u_{0}(y) = u_{00}(y) + R_{m}u_{01}(y) + O(R_{m}^{2})$$

$$u_{1}(y) = u_{10}(y) + R_{m}u_{11}(y) + O(R_{m}^{2})$$
(20)

Inserting Eq. (20) into (13) and (16) and equating the coefficients of R_m^0 and R_m to zero we have the following sets of ordinary differential equations

Zero order of R_m

$$u_{00}^{\prime\prime} + u_{00}^{\prime} - \left(M + \frac{1}{k}\right)u_{00} = -[GrT_0 + GcC_0]$$
⁽²¹⁾

$$u_{10}^{\prime\prime} + u_{10}^{\prime} - \left(M + \frac{1}{k} + \frac{i\omega}{4}\right)u_{10} = -[GrT_1 + GcC_1]$$
⁽²²⁾

First order of R_m

$$u_{01}^{\prime\prime} + u_{01}^{\prime} - \left(M + \frac{1}{k}\right)u_{01} = -u_{00}^{\prime\prime\prime}$$
(23)

$$u_{11}'' + u_{11}' - \left(M + \frac{1}{k} + \frac{i\omega}{4}\right)u_{11} = -u_{10}'''$$
(24)

The corresponding boundary conditions are:

$$\begin{array}{c} u_{00} = u_{01} = u_{10} = u_{11} = 0 \text{ at } y = 0 \\ u_{00} \to u_{01} \to u_{10} \to u_{11} \to 0 \text{ as } y \to \infty \end{array}$$
 (25)

Using the multi parameter perturbation technique and assuming $Ec \ll 1$, we write

$$T_{0} = T_{00} + EcT_{01}$$

$$T_{1} = T_{10} + EcT_{11}$$
(27)

Using equations (26) and (27) in the equations (14), (15), (17), (18), (21), (22), (23) and (24) and equating the coefficient of Ec^{0} and Ec^{1} , we get the following set of differential equations:

Zero order of Ec

-

1

$$u_{000}'' + u_{000}' - \left(M + \frac{1}{k}\right)u_{000} = -\left[GrT_{00} + GcC_{0}\right]$$
⁽²⁸⁾

$$u_{011}^{\prime\prime} + u_{011}^{\prime} - \left(M + \frac{1}{k}\right)u_{011} = -u_{000}^{\prime\prime\prime}$$
⁽²⁹⁾

$$u_{100}'' + u_{100}' - \left(M + \frac{1}{k} + \frac{i\omega}{4}\right)u_{100} = -[GrT_{10} + GcC_1]$$
(30)

$$u_{111}'' + u_{111}' - \left(M + \frac{1}{k} + \frac{1\omega}{4}\right)u_{111} = -u_{100}'''$$
(31)

$$T_{00}^{\prime\prime} + PrT_{00}^{\prime} + \frac{PrS}{4}T_{00} = 0$$
(32)

$$T_{10}^{\prime\prime} + PrT_{10}^{\prime} - \frac{Pr(S-i\,\omega)}{4}T_{10} = 0$$
(33)

First order of Ec

$$u_{001}'' + u_{001}' - \left(M + \frac{1}{k}\right)u_{001} = -[GrT_{01}]$$
(34)

$$u_{012}'' + u_{012}' - \left(M + \frac{1}{k}\right)u_{012} = -u_{001}''$$

$$u_{012}'' + u_{012}' - \left(M + \frac{1}{k} + \frac{i\omega}{\omega}\right)u_{012} = -\left[CrT + CrC\right]$$
(35)

$$u_{101}'' + u_{101}' - \left(M + \frac{1}{k} + \frac{1}{4}\right)u_{101} = -[GrT_{11} + GcC_{11}]$$
(36)

$$u_{112}'' + u_{112}' - \left(M + \frac{1}{k} + \frac{i\omega}{4}\right)u_{112} = -u_{101}''$$
(37)

$$T_{01}^{\prime\prime} + PrT_{01}^{\prime} + \frac{PrS}{4}T_{01} = -Pr(u_{000}^{1})^{2}$$
(38)

$$T_{11}^{\prime\prime} + PrT_{11}^{\prime} - \frac{Pr(S-i\,\omega)}{4}T_{11} = -2Pru_{000}^{\prime}u_{100}^{\prime}$$
(39)

The corresponding boundary conditions are:

$$\begin{array}{c} u_{000} = u_{001} = u_{011} = u_{012} = 0 \\ u_{100} = u_{111} = u_{101} = u_{112} = 0 \\ T_{00} = 1, T_{01} = 0, T_{10} = 1, T_{11} = 0 \text{ at } y = 0 \\ u_{000} \rightarrow u_{001} \rightarrow u_{011} \rightarrow u_{012} \rightarrow 0 \\ u_{100} \rightarrow u_{111} \rightarrow u_{101} \rightarrow u_{112} \rightarrow 0 \\ T_{00} \rightarrow 0, T_{01} \rightarrow 0, T_{10} \rightarrow 0, T_{11} \rightarrow 0 \text{ as } y \rightarrow \infty \end{array}$$

$$(40)$$

SOLUTION OF THE PROBLEM:

Solving these differential equations from (28) - (39) and (15), (18), using boundary conditions (40), and then making use of equations (26), (27), finally with the help of equations (12) we obtain the velocity, temperature and concentration as follows:

$$\begin{split} & u(y,t) = \left\{ \alpha_5 e^{\alpha_2 y} + \alpha_3 e^{\beta_2 y} + \alpha_4 e^{\gamma_2 y} + Ec \left(\alpha_{13} e^{\alpha_2 y} + \alpha_6 e^{\beta_2 y} + \alpha_7 e^{2\alpha_2 y} + \alpha_8 e^{2\beta_2 y} + \alpha_9 e^{2\gamma_2 y} + \alpha_{10} e^{(\alpha_2 + \beta_2) y} + \alpha_{11} e^{(\beta_2 + \gamma_2) y} + \alpha_{12} e^{(\alpha_2 + \gamma_2) y} \right) \right\} + R_m \left\{ \alpha_{17} e^{\alpha_2 y} + \alpha_{14} e^{\alpha_2 y} + \alpha_{15} e^{\beta_2 y} + \alpha_{16} e^{\gamma_2 y} + Ec \left(\alpha_{26} e^{\alpha_2 y} + \alpha_{18} e^{\alpha_2 y} + \alpha_{19} e^{\beta_2 y} + \alpha_{20} e^{2\alpha_2 y} + \alpha_{21} e^{2\beta_2 y} + \alpha_{22} e^{2\gamma_2 y} + \alpha_{23} e^{(\alpha_2 + \beta_2) y} + \alpha_{24} e^{(\beta_2 + \gamma_2) y} + \alpha_{25} e^{(\alpha_2 + \gamma_2) y} \right) \right\} + \varepsilon e^{i \,\omega t} \left\{ \left\{ \alpha_{31} e^{\alpha_{28} y} + \alpha_{29} e^{\beta_{11} y} + \alpha_{30} e^{\gamma_4 y} + Ec \left(\alpha_{46} e^{\alpha_{28} y} + \alpha_{36} e^{\beta_{11} y} + \alpha_{37} e^{(\alpha_2 + \alpha_{28}) y} + \alpha_{38} e^{(\alpha_2 + \beta_{11}) y} + \alpha_{39} e^{(\alpha_2 + \gamma_4) y} + \alpha_{40} e^{(\beta_2 + \alpha_{28}) y} + \alpha_{41} e^{(\beta_2 + \beta_{11}) y} + \alpha_{42} e^{(\beta_2 + \gamma_4) y} + \alpha_{33} e^{\beta_{11} y} + \alpha_{39} e^{\beta_{11} y} + \alpha_{40} e^{(\gamma_4 + \beta_{11}) y} \right\} + R_m \left\{ \alpha_{35} e^{\alpha_{28} y} + \alpha_{32} e^{\alpha_{28} y} + \alpha_{30} e^{\beta_{11} y} + \alpha_{30} e^{\beta_{11} y} + \alpha_{45} e^{(\gamma_4 + \gamma_4) y} \right\} + R_m \left\{ \alpha_{52} e^{\alpha_2 + \alpha_{28}) y} + \alpha_{50} e^{(\alpha_2 + \beta_{11}) y} + \alpha_{51} e^{(\alpha_2 + \gamma_4) y} + \alpha_{51} e^{(\alpha_2 + \alpha_{28}) y} + \alpha_{50} e^{(\alpha_2 + \beta_{21}) y} + \alpha_{50} e^{(\alpha_2 + \beta_{22}) y} + \alpha_{50} e^{(\alpha_2 + \beta_{21}) y} + \alpha_{50} e^{(\alpha_2 + \beta_{21}) y} + \alpha_{50} e^{(\alpha_2 + \beta_{22}) y} + \alpha_{50}$$

$$T(y,t) = \left\{ e^{\beta_2 y} + Ec \left\{ \beta_{11} e^{\beta_2 y} + \beta_5 e^{2\alpha_2 y} + \beta_6 e^{2\beta_2 y} + \beta_7 e^{2\gamma_2 y} + \beta_8 e^{(\alpha_2 + \beta_2)y} + \beta_9 e^{(\beta_2 + \gamma_2)y} + \beta_{10} e^{(\alpha_2 + \gamma_2)y} \right\} + \varepsilon e^{i\omega t} \left\{ e^{\beta_4 y} + Ec \left\{ \beta_{21} e^{\beta_{11} y} + \beta_{12} e^{(\alpha_2 + \alpha_{30})y} + \beta_{13} e^{(\alpha_2 + \beta_4)y} + \beta_{15} e^{(\beta_2 + \alpha_{30})y} + \beta_{16} e^{(\beta_2 + \beta_4)y} + \beta_{17} e^{(\beta_2 + \gamma_4)y} + \beta_{18} e^{(\gamma_2 + \alpha_{30})y} + \beta_{19} e^{(\gamma_2 + \beta_4)y} + \beta_{20} e^{(\gamma_2 + \gamma_4)y} \right\} \right\}$$

$$(42)$$

(41)

$$C(y,t) = e^{\gamma_2 y} + \varepsilon e^{i\omega t} e^{\gamma_4 y}$$
(43)

Results and discussion

The problem of mass transfer on unsteady hydromagnetic free convective visco-elastic (memory) flow of incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and chemical reaction through porous medium has been formulated, analyzed and solved by using multi-parameter perturbation technique. Approximate solutions have been derived for the velocity, temperature and concentration. The effects of the flow parameters such as magnetic parameter (M), Grashof number for heat and mass transfer (Gr, Gc), Schmidt number (Sc), Chemical reaction parameter (Kr), Prandtl number (Pr) Eckert number (Ec) and permeability parameter (k) on the velocity, temperature and concentration profiles of the flow field are presented with help of graphs.

The influences of chemical reaction parameter Kr on the velocity and concentration across the boundary layer are presented in Fig. 1(a) and 1(b). It is seen that the velocity as well as concentration across the boundary layer decreases with an increase in the chemical reaction parameter Kr.



Fig: 1(a) - Velocity profiles for different values of chemical reaction parameter Kr when S = -0.5, Gr = 5, Gc = 5, Sc = 0.6, Ec = 0.001, M = 10.0, k = 1.0





The variations of velocity and concentration distributions of flow field with the diffusion of foreign mass such as hydrogen (Sc = 0.22), helium (Sc = 0.30) and water vapor (Sc = 0.60) are shown in Fig. 2(a) and 2(b). The velocity and concentration distributions are decreases at all points of the flow field with the increase of the Schmidt number. This shows that the heavier the diffusing species have a greater retarding effect on the velocity and concentration distributions of the flow field.



Fig: 2(a) - Velocity profiles for different values of Schmidt number Sc when Kr = 0.2, Gr = 5.0, Gc = 5.0, S = -0.5, Ec = 0.001, M = 10.0 k = 1.0





Fig. 3(a) and 3(b) exhibits the velocity and temperature distribution for pertinent parameter, the Prandtl number (Pr). The Prandtl number (Pr) defines the ratios of momentum diffusivity to thermal diffusivity. It is observed than increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that increasing values of Prandtl number equivalent to increase the thermal conductivities and therefore heat is able to diffuse away from the heated plate more rapidly. Thus, it is concluded that in case of smaller Prandtl number as the thermal boundary layer is thicker, the rate of heat transfer is reduced in the presence of elastic elements.



Fig: 3(a) - Velocity profiles for different values of Prandtl number Pr when Kr = 0.2, Gr = 5.0, Gc = 5.0, S = -0.5, Ec = 0.001, Sc = 0.6, M = 10.0, k = 1.0



Fig: 3(b) - Temperature profiles for different values of Prandtl number Pr when Kr = 0.2, Gr = 5.0, Gc = 5.0, S = -0.5, Ec = 0.001, Sc = 0.6, M = 10.0, k = 1.0

The effect of the viscous dissipation parameter i.e., the Eckert number Ec on the velocity and temperature are shown in Fig. 4(a) and 4(b) respectively. The Eckert number Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert umber implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Fig. 4(a) and 4(b).



Fig: 4(a) - Velocity profiles for different values of Eckert number Ec when Kr = 0.2, Gr = 5.0, Gc = 5.0, S = -0.5, Sc = 0.6,

M = 10.0, k = 1.0.



Fig: 4(b) - Temperature profiles for different values of Eckert number Ec when Kr = 0.2, Gr = 5.0, Gc = 5.0, S = -0.5, Pr = 1.0, Sc = 0.6, M = 10.0, k = 1.0

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Fig. 5(a) and 5(b) demonstrate that an increase in the thermal Grashof number (Gr) and the solutal Grashof number (Gc) leads to rise the fluid velocity due to enhancement in buoyancy force sharply and attains distinctive maximum value near to the wall of the porous plate and then decays to the free steam value in the presence of elastic elements. Physically Gr > 0 means heating of the fluid or cooling or the boundary surface, Gr < 0 means cooling of the fluid or heating of the boundary surface and Gr = 0 corresponds to the absence of the free convection current.



Fig: 5(a) - Velocity profiles for different values of thermal Grashof number Gr when Kr = 0.2, Gc = 5.0, S = -0.5, Pr = 1.0, Sc = 0.6, M = 10.0, Ec = 0.001, k = 1.0



Fig: 5(b) - Velocity profiles for different values of solutal Grashof number Gc when Kr = 0.2, Gr = 5.0, S = -0.5, Pr = 1.0,

Sc = 0.6, M = 10.0, Ec = 0.001, k = 1.0



Fig: 6 - Velocity profiles for different values of magnetic parameter M when Kr = 0.2, Gr = 5.0, S = -0.5, Sc = 0.6,

Gc = 5.0, Ec = 0.001, k =1.0



Fig: 7. Velocity profiles for different values of permeability parameter k when Kr = 0.2, Gr = 5.0, S = -0.5, Sc = 0.6, Gc = 5.0, Ec = 0.001, M = 10.0

For various values of the magnetic parameter M, the velocity distribution is plotted in Fig. 6. It can be seen that as M increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convection flow.

Fig. 7 shows the effect of the porosity parameter on the dimensionless velocity distribution. It is observed that the velocity increases as the porosity increases.

Conclusions:

1. The magnetic parameter M retards the velocity of the flow field at all points due to the magnetic pull of the Lorentz force acting on the flow field.

2. The Grashof numbers of heat transfer Gr and mass transfer Gc accelerate the velocity of the flow field.

3. The Schmidt number Sc has a retarding effect on the velocity of the flow field. Heavier the diffusing species, the more is the retarding effect on the fluid velocity.

4. A growing chemical reaction parameter Kr leads to decrease in velocity of the flow field.

5. The effect of porosity parameter k is to enhance the velocity of flow field at all points.

6. The Prandtl number Pr reduces the temperature of the flow field at all points. Higher the Prandtl number the sharper is the reduction in temperature of the flow field.

7. The concentration distribution of the flow field decreases at all points as the Schmidt number Sc increases. This means the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field.

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