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Interference suppression using smart antenna in CDMA using training sequence based adaptive algorithms

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ABSTRACT

Rapid growth of mobile communication over the past years has tremendously increased the number of users which reduces the capacity of the system due to the increase in the number of interferers. Space Division Multiple Access (SDMA) is one of the promising technologies for future mobile communication. Smart antennas, usually employed at the base station radiate narrow beams to serve different users by using spatial processing. As long as the users are well separated spatially, the same frequency can be reused even when the users are in the same cell. The paper focuses the use of adaptive antenna arrays in CDMA networks to suppress cochannel interference. The training sequence based adaptive algorithms like the LMS, SMI and RLS algorithms are discussed in detail and the effect of number of antenna elements, inter element spacing, effect of number of interferers and the variation in SNR on the training sequence based adaptive algorithms is studied. The beam formation, null steering, maximum side lobe levels and convergence of these algorithms is studied in detail and compared.

Introduction

Smart antenna systems continually monitor their coverage areas and the system adapts to the user's motion providing an antenna pattern that tracks the user, achieving the maximum gain in the user's direction. For this purpose smart antenna base station combines an antenna array with a control unit that optimize reception and radiation patterns dynamically in response to the signal environment, i.e. mobile moving around the coverage area [1-10]. The smart antenna systems can generally be classified as either switched beam or adaptive array systems. In a switched beam system multiple fixed beams in predetermined directions are used to serve the users. Adaptive beam forming uses antenna arrays backed by strong signal processing capability to automatically change the beam pattern in accordance with the changing signal environment [6-8]. Number of researchers has tried for interference suppression using smart antennas using different adaptive algorithms and results are reported [10-19]. In the present paper attempts are made to study performance of these algorithms is studied with the effect of number of antenna elements, inter element spacing, and two additional parameters such as effect of number of interferers and the variation in SNR. In the present paper section 1 deals with the introduction part whereas sections 2, 3 and 4 deals with the theoretical aspects of LMS, RLS and SMI algorithms respectively. Section 5 deals with simulation results where as section 6 deal with the conclusion.

Least Mean Square (LMS) Algorithm

LMS algorithm is an adaptive algorithm which uses a gradient based method of steepest decent, that is, it uses the estimates of the gradient vector from the available data [4], [11], [20-24]. The LMS algorithm was introduced by Widrow and Hoff in 1960 [25] and it uses continues adaptation. From the method of steepest decent, the weight vector equation is given by [2], [11], [21], [22], [26], [27],

$$w(n+1) = w(n) + \frac{1}{2}\mu[-\nabla(E\{e^2(n)\})] \qquad (1)$$

where $w(n+1)$ denotes the weights to be computed at iteration $\boldsymbol{n+1}$. μ is the step-size parameter and controls the convergence characteristics of the LMS algorithm [4], [11], [21], [22], [28], [29]; $\boldsymbol{e^2(n)}$ is the mean square error which is given by

$$e^2(n) = [d^*(n) - y(n)]^2 \qquad (2)$$

where $y(n) = w^h x(n)$, w^h denotes the complex conjugate transpose of the weight vector w.

The gradient vector is computed as

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$$\nabla_w(E\{e^2(n)\}) = -2r + 2Rw(n) \qquad (3)$$

The LMS algorithm uses the instantaneous values of the cross correlation matrix and the covariance matrix instead of their actual values [11], [22] i.e.

$R(n) = x(n)x^h(n)$ and $r(n) = d^*(n)x(n)$

Therefore the weight update can be given by the following equations,

$$w(n+1) = w(n) + \mu x(n)[d^*(n) - x^h(n)w(n)]$$
$$= w(n) + \mu x(n)e^*(n) \qquad (4)$$

The LMS algorithm is initiated with an arbitrary value $\boldsymbol{w(0)}$ for the weight vector at $\boldsymbol{n = 0}$. The successive correction of the weight vector eventually leads to the minimum value of the MSE [11], [20], [22], [24].

The LMS algorithm is seen to converge and stay stable for

$$0 < \mu < \frac{1}{\lambda_{max}} \qquad (5)$$

where $\boldsymbol{\lambda_{max}}$ is the largest eigen value of the correlation matrix $\boldsymbol{R}$. As the correlation matrix is positive definite, all eigen values are positive [30]. If all the interfering signals are noise and there is only one SOI, we can approximate the condition in Eq. (5) as

$$0 \le \mu \le \frac{1}{2trace(R)} \qquad (6)$$

Recursive Least Square (RLS) Algorithm

The RLS algorithm is training signal based adaptive beam forming algorithm that minimizes the MSE and produces an optimum set of weights [31]. The LMS algorithm depends on eigen value spread for convergence. When the array correlation matrix has a large eigen value spread the algorithm converges with slow speed. The RLS algorithm resolves this problem by replacing the gradient step size μ with a gain matrix $\boldsymbol{\hat{R}^{-1}(n)}$ at the n^{th} iteration [21], [33].

The weight update equation for RLS algorithm is computed as follows [14], [32], [33].

$$w(n) = w(n-1) - \hat{R}^{-1}(n)x(n)\mathcal{E}^*\big(w(n-1)\big) \qquad (7)$$

where $\boldsymbol{\hat{R}^{-1}(n)}$ is the gain matrix and it is given as

$$\hat{R}(n) = \alpha\hat{R}^{-1}(n-1) + x^*(n)x^H(n)$$
$$= \sum_{k=0}^{n} \alpha^{n-k} x^*(k)x^H(k) \qquad (8)$$

where $\boldsymbol{\alpha}$ is called the forgetting factor because the update equation tends to de-emphasize the old samples and its value is less than but close to 1. $\boldsymbol{\mathcal{E}(n)}$ denotes the error signal and is given by [33]

$$\mathcal{E}(n) = d(n) - y(n) \qquad (9)$$

where $\boldsymbol{d(n)}$ represents the desired signal and $\boldsymbol{y(n)}$ denotes the output of the beam former. The RLS algorithm updates the required inverse of $\boldsymbol{R(n)}$ using the previous inverse and the present sample as [20], [32], [33]:

$$\hat{R}^{-1}(n) = \frac{1}{\alpha}\left[\hat{R}^{-1}(n-1) - \frac{\hat{R}^{-1}(n-1)x(n)x^H(n)\hat{R}^{-1}(n-1)}{\alpha + x^H(n)\hat{R}^{-1}(n-1)x(n)}\right] \qquad (10)$$

The gain matrix is initialized as

$$\hat{R}^{-1}(0) = \frac{1}{\mathcal{E}_0}I, \mathcal{E}_0 > 0 \qquad (11)$$

The RLS algorithm minimizes the cumulative square error as

$$J(n) = \sum_{k=0}^{n} \alpha^{n-k} |\mathcal{E}(k)|^2 \qquad (12)$$

Sample Matrix Inversion (SMI) Algorithm

The SMI algorithm collects and processes a block of samples of the filter input and the desired output to obtain a block of output samples. However, it requires that the number of interferers and their positions remain constant during the duration of the block acquisition [11], [34]. The SMI algorithm was developed by Reed, Mallett and Brennen in 1974 [35], [36] and is based on the estimate of the correlation matrix. Due to the direct inversion of the auto correlation matrix, R, the SMI algorithm has a faster convergence rate. The auto correlation matrix, R, and the cross correlation matrix, r, are given as [11], [21].

$R = E[x(n)x^H(n)]$ and $r = E[d(n)x(n)]$

The estimates of the matrices over a block size of K [21], [31] can be written as

$$\hat{R} = \frac{1}{K}\sum_{k=1}^{K} x(k)x^H(k) \qquad (13)$$

$$\hat{r} = \frac{1}{K}\sum_{k=1}^{K} d^*(k)x^H(k) \qquad (14)$$

where k is the block number and K is the block length [30]. When new sample arrive the estimate of R is updated using the following equation:

$$R(n+1) = \frac{nR(n) + x(n+1)x^H(n+1)}{n+1} \qquad (15)$$

The weight vector can now be estimated by the following equation [11], [30]:

$$\hat{w} = \hat{R}^{-1}\hat{r} \qquad (16)$$

Based on Eq. (16) the weights will be updated for each incoming block. Due to the estimation of the correlation matrix there is always a residual error in the SMI algorithm. The error due to estimates can be computed by the following equation [11]:

$$e = \hat{R}w_{opt} - \hat{r} \qquad (17)$$

The stability of the SMI algorithm depends on the ability to invert the large covariance matrix [11].

Simulation Results of LMS, RLS And SMI Algorithms

The performance of these algorithms is compared by varying the number of antenna elements; inter element spacing, number of interferers and SNR. Mainly the beam formation, null steering capabilities, maximum side lobe levels and convergence of these algorithms is compared and discussed in detail.

General Comparison

The performance of the algorithms is compared by fixing the number of antenna elements and inters element spacing.

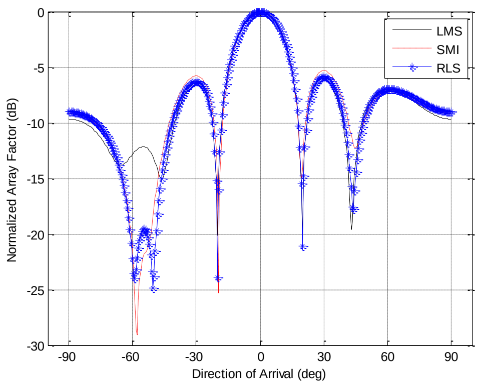

Figure 1: Comparison of LMS, SMI and RLS algorithms based on the Normalized Array Factor in dB.

It is considered that the desired user is arriving at an angle of 0 degree and an interferer at an angle of -60 degree. The spacing between the individual elements is considered to be half wavelength, the number of antenna elements is six and the SNR is 20 dB. Figure 1 shows the maximum side lobe level and null depth for the three algorithms.

From Figure 1 it is seen that the SMI algorithm has the highest level of side lobes. Thus, LMS and RLS algorithms have better performance than the SMI algorithm, because they create side lobes with low power. Considering the null depth, the RLS algorithm has superior performance than the other algorithms. That is, the null depth for LMS algorithm is 12.8 dB below the maximum, the null depth for SMI algorithm is 21.8 dB below the maximum and the null depth for RLS algorithm is 22.7 dB below the maximum. The LMS algorithm has a very weak performance in placing nulls in the direction of the interferers. The three algorithms have the same beam steering capacity.

Due to the estimation of the correlation matrix, there will always be a residual error in the SMI algorithm. Thus, the SMI algorithm has higher amount of residual error than the other algorithms. The RLS algorithm has lower MSE level than the LMS algorithm. The convergence speed of the RLS algorithm is better than the LMS algorithm. For this particular case the RLS algorithm converges around 100 iterations and the LMS algorithm converges around 180 iterations. Generally, the SMI algorithm has fast convergence than the other algorithms.

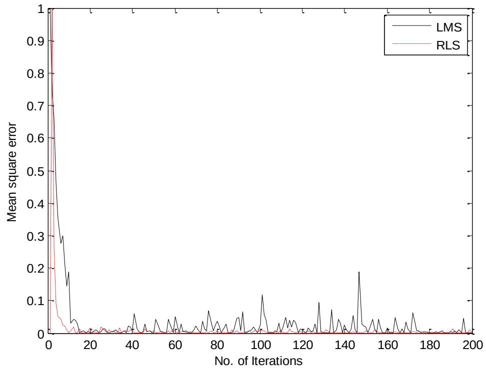

Figure 2: Comparison of LMS and RLS algorithms based on the MSE.

Effect of Number of Antenna Elements on Array Factor

Here it is assumed that the desired user is arriving at an angle of 0 degree and an interferer at an angle of -60 degree. The spacing between the individual elements is considered to be half wavelength and the SNR is 20 dB. Figures 3 and 4 show that the effect of increasing the number of antenna elements on the beam forming and null steering capability of the algorithms.

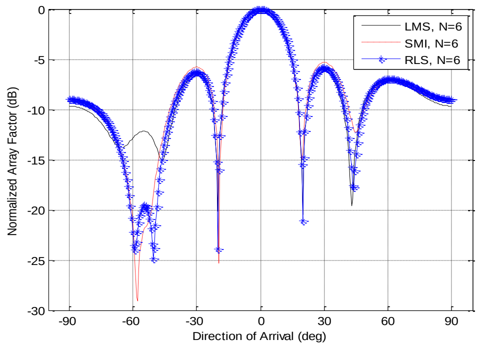

Figure 3: Normalized array factor in dB for LMS, SMI and RLS algorithms when N=6.

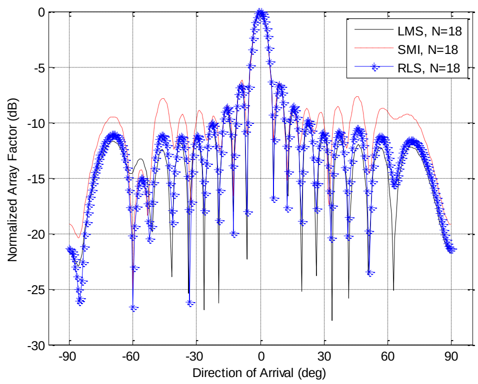

Figure 4: Normalized array factor in dB for LMS, SMI and RLS algorithms when N=18.

When the number of antenna elements increases all the algorithms are able to create sharper beams and deeper nulls but create an increase in the number of side lobes. From Figure 4 it can be seen that when 18 antenna elements are used the beam steering capacity of all the algorithms is the same. Hence the antenna array becomes more directive. Comparing the side lobe level of the three algorithms, SMI algorithm create the highest level of side lobe. The null steering capacity of the RLS algorithm is slightly higher than the other algorithms.

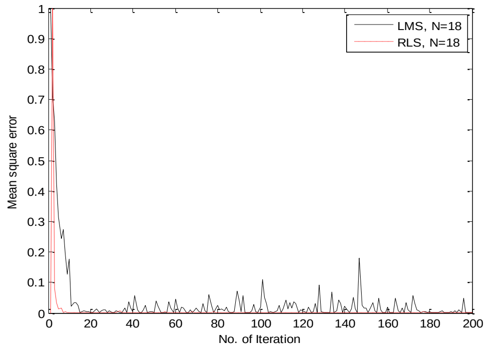

Figure 5: Mean square error analysis of LMS and RLS algorithms using N=18

From Figure 5 it can be concluded that the RLS algorithm has a very small amount of MSE for large number of antenna elements.

Effect of Inter Element Spacing on Array Factor

It is considered that the desired user is arriving at an angle of 0 degree and an interferer at an angle of -60 degree. The number of antenna elements is kept at N=18 and SNR is 20 dB. For this case, the performance of the LMS, SMI and RLS algorithms is compared for d=0.25, d=0.5 and d=1 (the inter element separation distance is given in terms of wavelength).

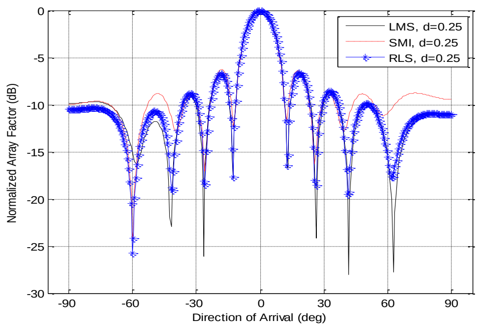

Figure 6: Normalized array factor in dB for LMS, SMI and RLS algorithms when d=0.25.

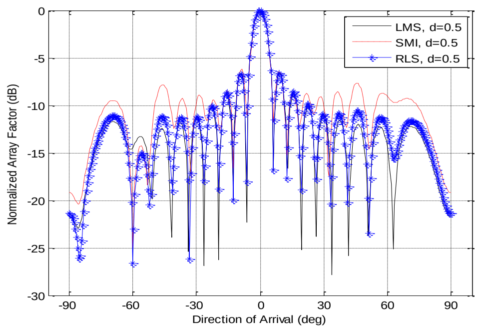

Figure 7: Normalized array factor in dB for LMS, SMI and RLS algorithms when d=0.5.

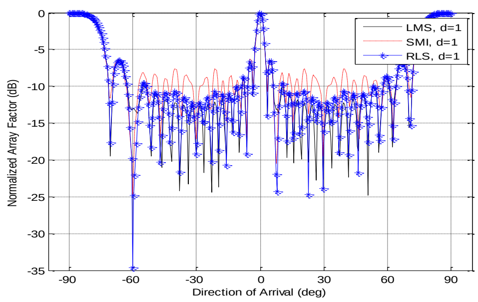

Figure 8: Normalized array factor in dB for LMS, SMI and RLS algorithms when d=1.

It is seen that as the inter element spacing increases the beam width decreases and there will be increased directivity in all of the algorithms at the expense of increased number of side lobes. In the case of the RLS and SMI algorithms the interference rejection capability of the algorithms increases as the inter element spacing increases from half wave length to one wavelength. But, in LMS algorithm the null depth slightly decreases as the inter element spacing increases. The RLS algorithm is able to place deeper nulls than the other algorithms when the inter element spacing is equal to one wavelength. Inter element spacing equal to the wavelength creates grating lobes in all of the algorithms.. The creation of grating lobes highly degrades the performance of the system. From Figure 6 it can be concluded that the SMI algorithm has the highest level of side lobes. For all the algorithms the optimum value of inter element spacing is half wavelength because it creates sharper beams, deeper nulls and no grating lobes.

Effect of Number of Interferers on Array Factor

Assume the DOA of the desired user is 0 degree and the interferers are coming at 60, 30, -30, -60 degrees. That is, there is one desired user and four interferers. For this case, the performance of the LMS, SMI and RLS algorithms is shown for N=6 and N=12 where the inter element spacing is kept at half wavelength and the SNR is 20 dB.

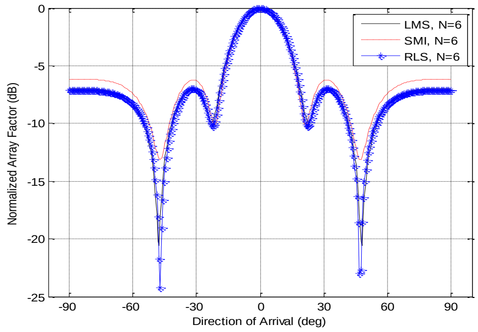

Figure 9: Normalized array factor in dB for LMS, SMI and RLS algorithms when N=6.

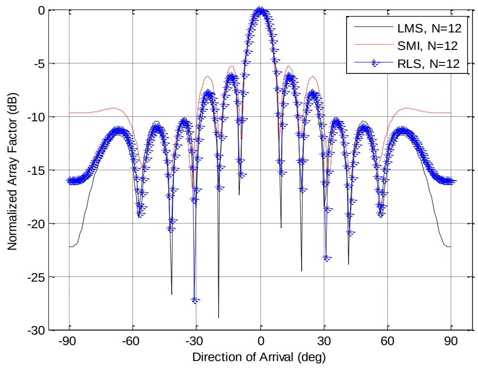

Figure 10: Normalized array factor in dB for LMS, SMI and RLS algorithms when N=12.

From Figures 9 and 10 it is seen that when an array with six elements is used in an environment with four interferers, the RLS algorithm is able to place deeper nulls than the other algorithms. The SMI algorithm creates side lobes with large amount of power than the other algorithms. When the number of antenna elements is increased to 12, all the algorithms are able to create sharper beams. But the null steering capacity of RLS algorithm is still better than the others.

Effect of SNR on Array Factor and Convergence of the Algorithms

It is assumed that that the DOA of the desired user is 0 degree and the interferer is coming at -60 degree. The number of antenna elements is six and the inter element spacing is kept at half wavelength. For this case, the performance of the LMS, SMI and RLS algorithms is shown for different value of SNR in Figure 11 and 13.

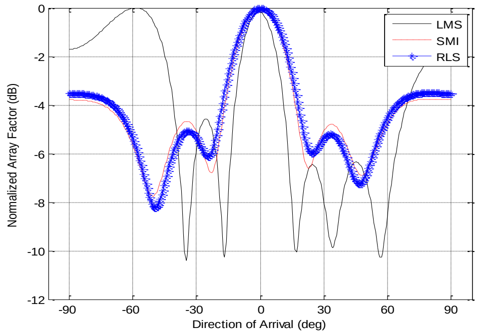

Figure 11: Normalized array factor in dB for LMS, SMI and RLS algorithms when SNR=-20 dB.

From Figure 11 it is seen that when the signal level is lower than the noise, SMI and RLS algorithms are able to locate the desired user and the interferers and algorithms are able to steer the main beam towards the desired user and place nulls in the direction of the interferer. But the LMS algorithm fails to locate the desired user and the interferer. From Figure 12 it can be seen that the MSE of the LMS algorithm increases logarithmically and the algorithm does not converge. Thus, the LMS algorithm fails to operate when the SNR value is less than 0 dB.

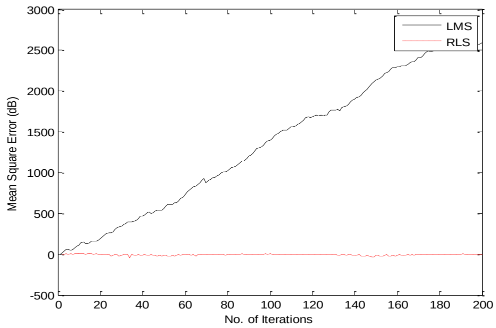

Figure 12: Mean Square Error in dB for LMS and RLS algorithms when SNR=-20dB.

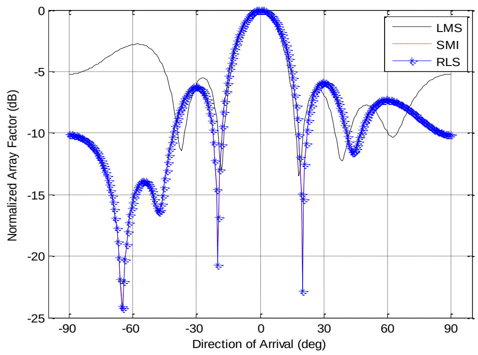

Figure 13: Normalized array factor in dB for LMS, SMI and RLS algorithms when SNR=0 dB.

From Figure 13 it is seen that the SMI and RLS algorithms are able to steer deep null in the direction of the interferer. The LMS algorithm has a weak performance in null steering when the SNR value is 0dB. Comparing Figures 1, 11 and 13 it can be concluded that an increase in SNR enhances the beam formation in the direction of the desired user and null steering capability of the algorithms. When the SNR value increases the side lobe level generated by all the algorithms decrease.

When the SNR value increases, the MSE decreases and this enhances the convergence of both the LMS and RLS algorithms and convergence speed of the RLS algorithm is faster than the LMS algorithm. Thus, the increase in SNR value, increases the convergence of the algorithms and this enhances the null steering capability of the algorithms.

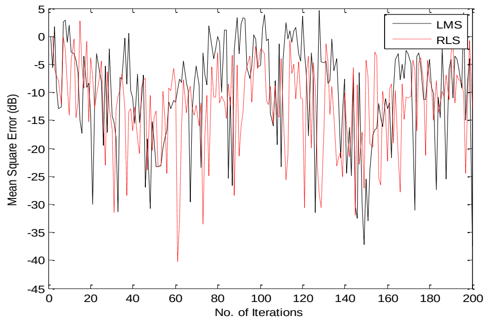

Figure 14: Mean Square Error in dB for LMS and RLS algorithms when SNR=0 dB

Conclusion

Compared to other algorithms, the LMS algorithm is relatively simple because it does not require correlation matrix calculation and matrix inversion. Compared to the LMS algorithm, the RLS algorithm has faster convergence speed and do not exhibit the eigen value spread problem. An increase in SNR value also enhances the beam formation, null steering and the convergence of the algorithms. The forgetting factor affects the performance of the RLS algorithm in terms of convergence rate, misadjustment, tracking and stability. The stability of the SMI algorithm depends on its ability to invert a large correlation matrix. Keeping the number of antenna elements and the inter element spacing constant, the RLS algorithm is able to place deeper nulls than the LMS and SMI algorithms. When the number of antenna elements increase, all the algorithms are able to create sharper beams and deeper nulls. As inter element spacing equal to half wavelength creates sharper beams, deeper nulls and no grating lobes, the optimum value of the inter element spacing is half wavelength. In an environment with many interferers, the RLS algorithm has better null steering capability than the SMI algorithm.

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