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A study of the internal heat transfer by natural convection through a cubic enclosure

Andima N. Robert[1,*], Johana K. Sigey[1], Jaconiah A. Okello[1] and James Okwoyo[2]

[1]Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, Nairobi, Kenya.

[2]School of Mathematics, University of Nairobi, Nairobi, Kenya.

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ABSTRACT

In this study, we consider natural convection in a cubic enclosure with the heater and window placed on opposite ends of adjacent vertical walls. For analysis a complete set of non-dimensional equations governing a Newtonian fluid and boundary conditions are recast into velocity or vector potential to eliminate the need for solving the continuity equation. The governing equation with the boundary equations are then descritized using three point central and forward difference approximation for non-uniform mesh. The resulting finite difference equations are then solved using Matlab computer software. The solutions shall be presented at the Reynolds number 2,000 with Prandtl number 0.71. The results are discussed and presented in graphical forms. The room is divided into two regions with those near the heater having high temperatures as those near the window have low temperatures. Mixing of hot and cold fluid and turbulence makes the other areas warm.

Nomenclature

Roman Symbols

C_p Specific heat capacity at constant pressure

g Acceleration due to gravity in n/m^2

h Mesh interval in the case of a uniform mesh with the same interval in the x, y and z directions. Or static enthalpy as used in equation

i, j, k Integer variables

M The number of mesh points in the x-direction or molecular mass of air

p Thermodynamic pressure in N/m^2

s Specific entropy

t Time in seconds

T Thermodynamic temperature in Kelvins, k

u, u, U Instantaneous velocity component in the x-direction, fluctuation velocity and mean velocity in the x-direction respectively in m/s

x, y, z Co-ordinate direction in the i, j and k direction

Fr Frounde Number = $\frac{U_*}{\sqrt{gL_R}}$

Re Reynolds number = $\frac{\rho_R U_* L_R}{\mu_R}$

Eu Euler Number = $\frac{P_R}{\rho_R U_*^2}$

Pr Prandtl Number = $\frac{\mu_R c_{pR}}{\lambda_R}$

Greek symbols

β Coefficient of volumetric expansion in m^3

δ_{ij} Kronecker delta

λ Thermal conductivity

Tele:

E-mail addresses: robandi80@gmail.com

Φ Dissipation function or scalar transport unknown
μ First coefficient of viscosity
ρ Density

Subscripts

i, j, k Denotes the i^{th}, j^{th} and k^{th} mesh points in the x, y and z directions respectively

Abbreviations

Fig. Figure

Introduction

Natural convection is observed as a result of the motion of the fluid due to density changes arising from heating process. The movement of the fluid in natural convection results from the buoyancy forces imposed on the fluid when its density of the heat transfer surface is described as a result of thermal expansion of the fluid in anon-uniform temperature distribution. Convection heat transfer is dependent on the movement of the fluid and the development of the flow of the fluid is also affected by the shape of the heat transfer surfaces. Both numerical and experimental methods have been used to obtain the solution of heat transfer and fluid flow problems. In most cases, the experimental methods are costly and time consuming due to necessary of expensive prototypes and instrumentations. On the other hand, numerical methods can offer considerable savings in design time and cost Saha (2007).

Convective heat transfer at the interior surfaces of buildings accounts for a great proportion of the overall thermal load Strachan *et al* (1999). Modeling programs rely on convective heat transfer correlations to quantify the magnitude of convection at each building surface. The correlation values vary widely among different programs and often presume natural convection to be the dominant mode of convective heat transfer. This can seriously underestimate the convective heat loss in mechanically ventilated buildings Awbi and Hatton (2000). An incorrect assumption in the modeling of convection, such as inaccurate convection correlations, can translate into a large error in the overall estimation of total heat transfer within the building. In a study examining the significance of varied convective heat transfer coefficients, Lomas (1996) found that the cooling load could vary as much as 27%. Considering the building energy consumption prediction, the degree of sensitivity of energy simulation to convection calculation models is approximately Lomas *et al* (1996). Convection plays an important role in the accuracy of the overall heat transfer calculation. A good overall heat transfer model will link the accuracy of radiation and conduction models to the accuracy of the convection model. In energy modeling tools, indoor radiative heat transfer is usually described by precise radiation equations.

The inaccuracy in radiation models comes from the neglect of participating objects within the room such as people, computers, and lights. Conduction is limited by the accuracy of one-dimensional homogeneous models used to represent multidimensional heat transfer occurring in nonhomogenous materials or building corners. Convection on the other hand is limited by the accuracy of the analytical or semi-empirical correlation used for a characteristic space geometry and airflow distribution. This is the accuracy the new convection correlations seek to address.

There are significant differences in convective heat transfer not only between mechanically and naturally ventilated rooms, but also among different types of air distribution and heating and cooling systems such as: displacement ventilation, mixing ventilation with various ceiling and side wall diffusers, floor registers, radiant floor heating, and cooled ceiling panels Novoselac et al. (2006). Among these systems are ceiling slot diffusers, one of the most common airflow distribution systems in modern commercial buildings with large glass facades. With these systems, cold (or hot) air is supplied in the perimeter zones of buildings along the window surfaces to dissipate heat (or cover heat losses) at the source locations Awbi (2003).

Ceiling slot diffusers in perimeter building zones are unique in their proximity to the window. This causes the vertical air supply jet to attach itself to adjacent surfaces. This phenomenon is termed the Coanda effect, and it transports the jet momentum along the window and wall surface.

In addition to this forced convection, large temperature differences between supply air and window surfaces cause buoyancy. Depending on the direction of the surface heat flux (positive for summer or negative for winter), the surface-air temperature difference can generate a force that acts in the same or the opposite direction to the ceiling supply jet. Consequently, heat transfer at the window surfaces is potentially affected by both forced and natural convection. While at high volume flow rates one would expect the correlations to be dominated by forced convection, at lower flow rates, the large air temperature stratification could cause the effects of buoyancy to dominate. As mentioned before, the most commonly used convection correlations at window surfaces are based on the assumption that natural convection is the dominant mode of convective heat transfer. However, without knowledge of which effect dominates, it is difficult to know whether currently used models underestimate or overestimate convective heat transfer in commercial buildings with ceiling slot diffusers.

While specific correlations have been developed and utilized in energy analysis software for decades, there is currently a gap in the literature concerning forced air cooling and heating systems, including ceiling slot diffusers located near the window Beausoleil-Morrison and Strachan (1999). As mentioned above, this is particularly important because this supply air configuration represents a large proportion of office ventilation systems. Furthermore, while numerous studies have been conducted to investigate heat transfer correlations in mixed convective regimes, Chen *et al*. Novoselac (2005), none of these studies focused on ceiling slot diffusers near the window surface, nor on the development of correlations specifically pertaining to the window.

Typical office buildings utilize not only ceiling slot diffusers close to the window, but also blinds which control the day lighting within the room and reduce the penetration of solar heat through the window. While the primary purpose of the blinds is to reduce heat gains from shortwave (solar) radiation, they also influence convective heat transfer as well as heat transfer in a room by long wave radiation.

Mathematical Formulation

Natural convection in an enclosure arising from localized heating and cooling is encountered in a number of practical occurrences, such as the use of convectional heaters in rooms. Besides forced convection, natural convection plays a major role in determination of the indoor climate. The temperature and velocity fields in a room depend mainly on temperature of any heat source and window as well as air ventilation flow rate.

This is a numerical study of turbulent flow in a cubic enclosure. The flow of heat is one form of Newtonian motion. In this project, we will consider natural convection in a three dimensional square enclosure in the form of a room with the heater and window placed on opposite walls.

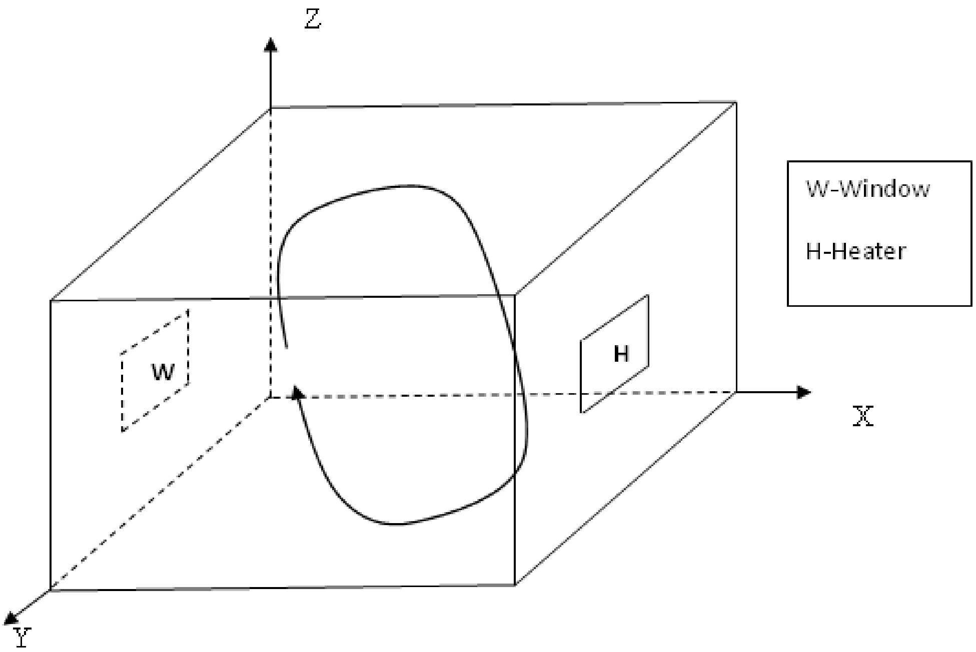

Figure 1

Fig. 1 above is a model of a room with a heater placed on one wall and a window on the opposite wall (Y-Z planes). The other walls, the floor and the top planes are taken to be insulated (X-Y planes and X-Z planes).

Governing Equations

We consider the equations governing the behavior of Newtonian fluids experiencing heat or mass transfer. These fundamental equations of fluid dynamics are based on the following universal laws of conservation; conservation of mass (continuity), momentum and energy. These equations are presented in tensor form as well as in Cartesian form useful for computer programming.

Consider a fluid in which the density ρ is a function of position x_j (j=1,2,3), let u_j (j=1,2,3) denote the components of the velocity. Hence in writing the various equations, use of the notation of Cartesian tensors with the usual summation convection is applied.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \tag{1}$$

$$\frac{\partial}{\partial x_t}(\rho u_i) + \frac{\partial}{\partial x_j}(\partial u_j u_i) = -\frac{\partial \rho}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \mu_s \delta_{ij}\frac{\partial u_k}{\partial x_k}\right] \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho c_p T\right) + \frac{\partial}{\partial x_j}\left(\rho c_p u_j T\right) = \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) + \beta T\left(\frac{\partial p}{\partial t} + \frac{\partial u_j p}{\partial x_j}\right) + \Phi \tag{3}$$

Equations (2) for momentum and (3) for energy are descritized using three point central difference approximation as shown below;

Method of solution

Equation (2) can be written in a non-dimensional form as;

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - v\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) - \frac{Eu}{\rho_R}\frac{\partial \rho}{\partial y} - \frac{\Theta g_i}{(Fr)^2} \tag{4}$$

Using Taylor's central difference approximation;

$$u_{i,j}^{n+1} = \left(1 - \frac{4k}{h^2\,\mathrm{Re}}\right)u_{i,j}^n + \left(\frac{k}{h^2\,\mathrm{Re}} - \frac{vk}{2h}\right)u_{i+1,j}^n + \left(\frac{k}{h^2\,\mathrm{Re}} + \frac{vk}{2h}\right)u_{i-1,j}^n + \left(\frac{k}{h^2\,\mathrm{Re}} - \frac{vk}{2h}\right)u_{i,j+1}^n + \left(\frac{k}{h^2\,\mathrm{Re}} + \frac{vk}{2h}\right)u_{i,j-1}^n - \left(\frac{Eu}{\rho_R} + \frac{\Theta g_i}{(Fr)^2}\right) \tag{5}$$

Equation (3) can also be written as;

$$\frac{\partial \Theta}{\partial t} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) - 2v\left(\frac{\partial \Theta}{\partial x} + \frac{\partial \Theta}{\partial y}\right) \tag{6}$$

The central difference approximation equation for the equation is given by;

$$\frac{\Theta_{i,j}^{n+1} - \Theta_{i,j}^n}{k} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}\,h^2}\left\{\left(\Theta_{i+1,j}^n - 2\Theta_{i,j}^n + \Theta_{i-1,j}^n\right) + \left(\Theta_{i,j+1}^n - 2\Theta_{i,j}^n + \Theta_{i,j-1}^n\right)\right\} - \frac{v}{h}\left\{\left(\Theta_{i+1,j}^n - \Theta_{i-1,j}^n\right) + \left(\Theta_{i,j+1}^n - \Theta_{i,j-1}^n\right)\right\} \tag{7}$$

Equations (5) and (7) together with boundary conditions are used in a code with Matlab software to simulate the expected results.

Results and discussions

Temperature distribution

Fig. 2(a), and Fig. 4(a) shows the vertical temperature fields in the planes y = 0.1 and y = 0.9 respectively. Being equidistant from the heater and the window there are lower temperatures than in the middle plane. The temperature decreases as you move towards the wall containing the window. The lowest temperatures are recorded on the wall containing the window. It is warmest at the wall containing the heater.

Fig. 3(a) shows the isotherms at the vertical plane y = 0.5. It is the region coinciding with the window and the heater. At this plane, the temperatures are higher. This is because of the distance from the heater but there are higher temperatures to the left wall of the enclosure due to the effect of the heater. The right wall here recorded the lowest temperatures due to the cooling from the window.

The horizontal plane z = 0.5 is shown in Fig. 5. Coinciding with the window and the heater, the plane records mixed temperatures with high temperatures on the region close to the heater and low temperatures close to the window.

The room is divided into a number of regions with those near the heater having high temperatures as those near the window have low temperatures. Mixing of hot and cold fluid and turbulence makes the other areas warm. Natural convection also plays a key role in the variation of temperature distribution in the room.

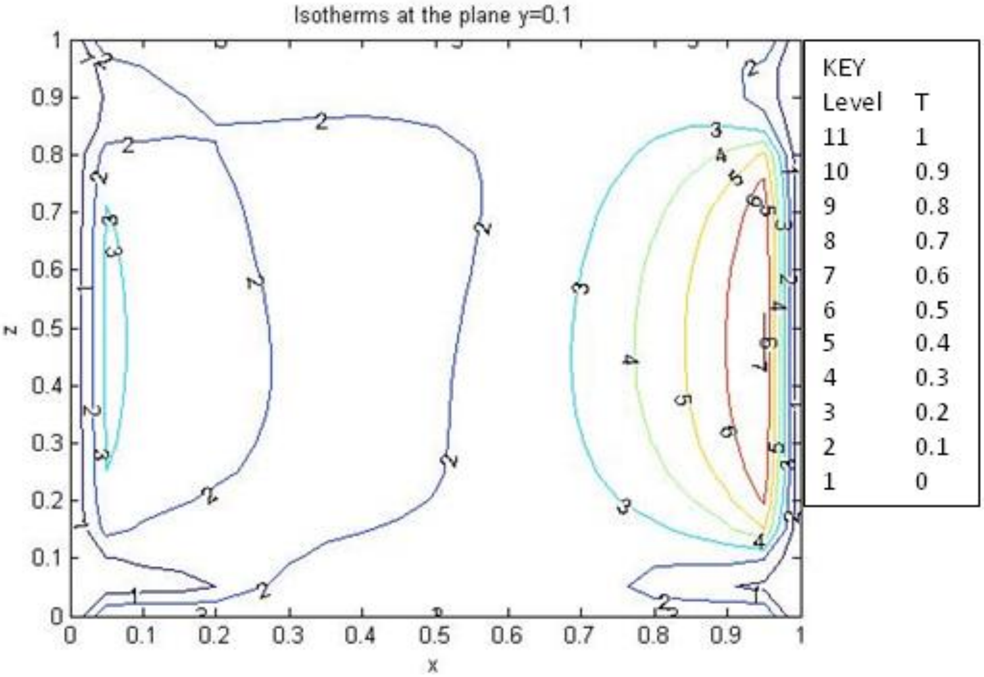

Figure 2(a): Isotherms at the plane y = 0.1 for Re = 2000

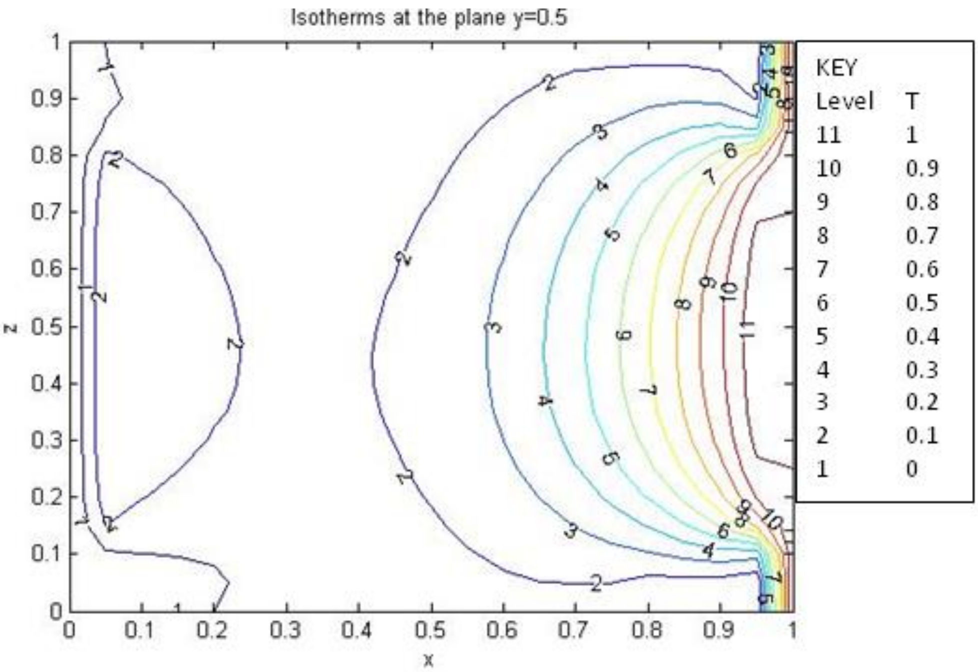

Figure 3(a): Isotherms at the plane y = 0.5 for Re = 2000

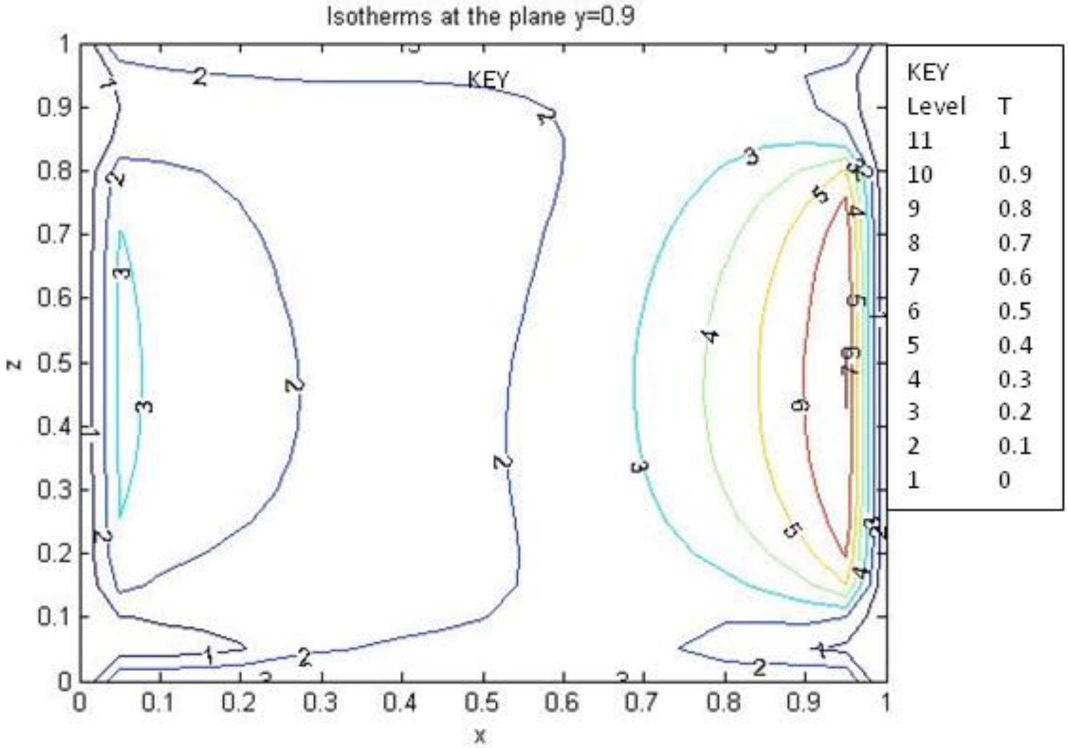

Figure 4(a): Isotherms at the plane y = 0.9 for Re = 2000

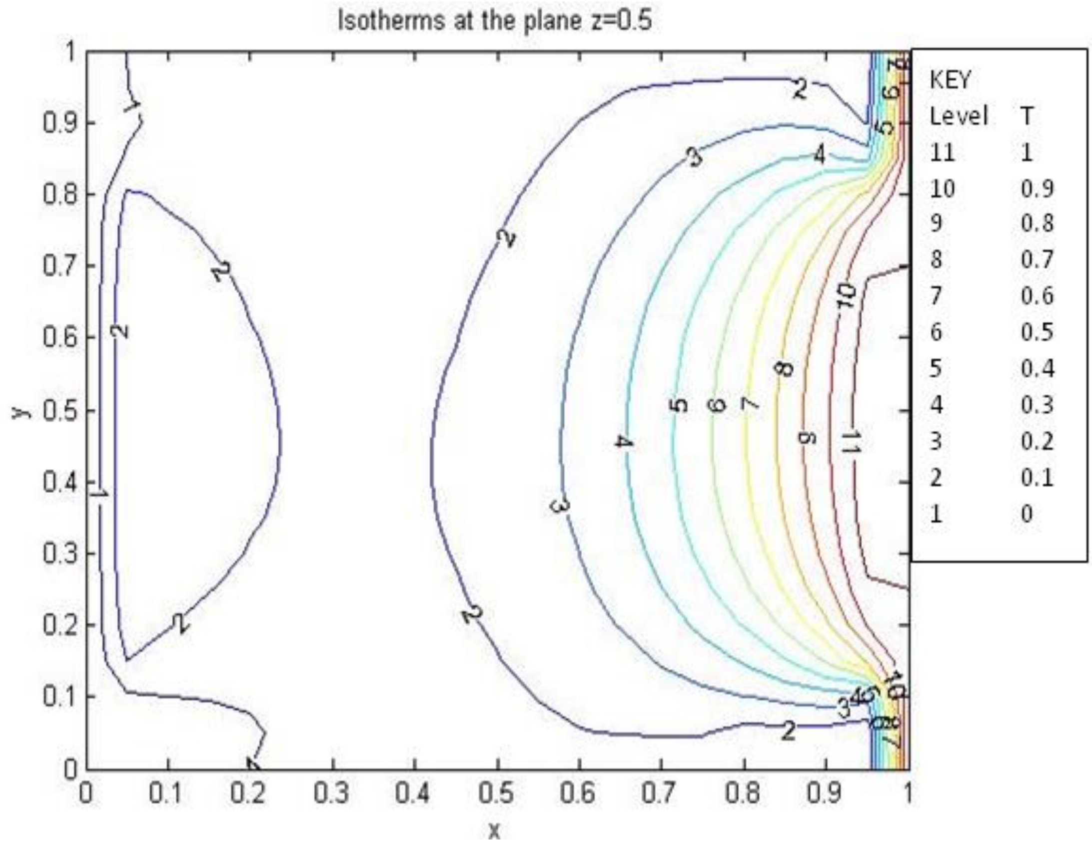

Figure 5: Isotherms at the plane z = 0.5 for Re = 2000

Velocity flow fields

The structure of the flow can easily be seen in the vector plots which have been selected from the X-Y plane. The vector plots are in the planes z = 0.1, z = 0.5 and z = 0.9 (see fig. 2(b), fig. 3(b) and fig. 4(b)).

In fig. 3(b) where the plane is at x = 0.5, the structure of the flow shows one circular motion in different directions; one in the clockwise direction (to the right) while the other (to the left) is anti-clockwise. This is due to the strong convective motion developed by the heating and high Reynolds number.

The warm fluid gains energy becomes less dense and gains in velocity resulting in an upward motion at the centre where the effect of the heaters is felt, while on the sides the cold fluid descends. This is due to buoyancy effects. The velocity of the descending fluid is strongest near the windows while the rising velocity is strongest near the heaters (at the bottom).

Fig. 2(b) and Fig. 4(b) shows the flow patterns in the planes y = 0.1 and y = 0.9 respectively. They have similar patterns since they have the same temperature effects with more strength in upward motion since they are close to the source of heat. On the sides there is mixing up of warm and cold fluid resulting into slow movement of the fluid particles.

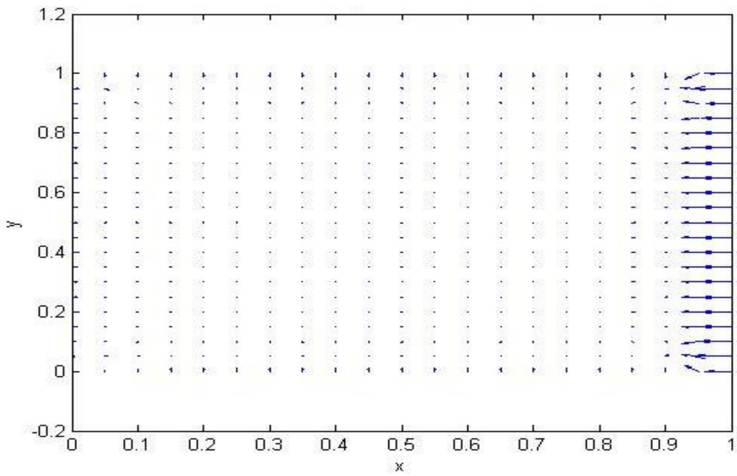

Figure 2(b): Velocity vector plot at the plane y = 0.1 for Re = 2000

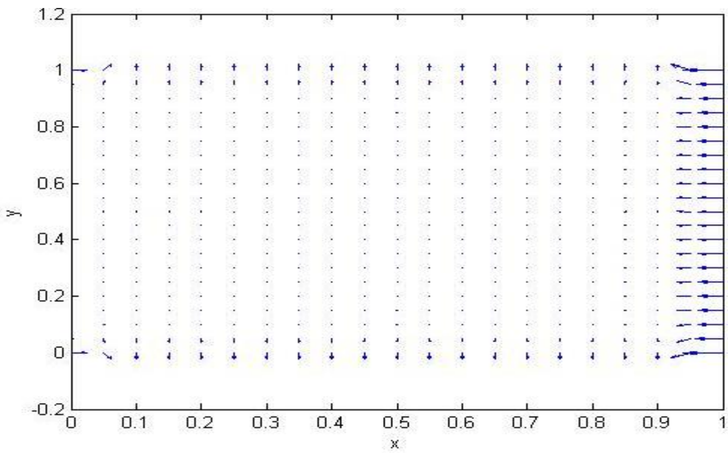

Figure 3(b): Velocity vector plot at the plane y = 0.5 for Re = 2000

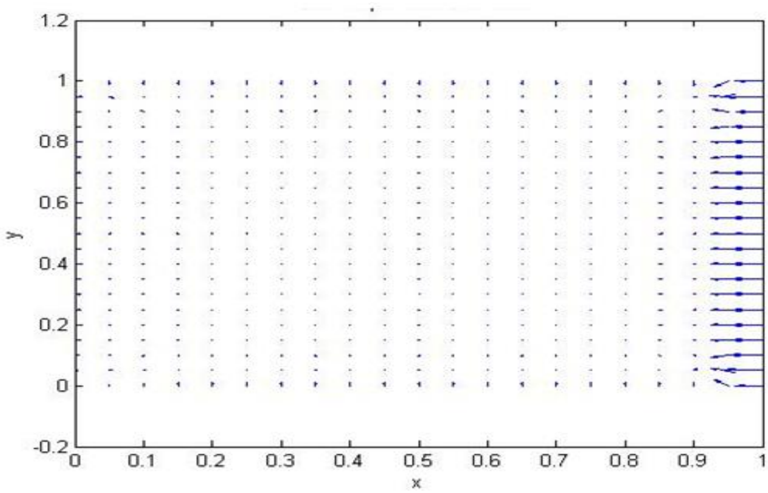

Figure 4(b): Velocity vector plot at the plane y = 0.9 for Re = 2000

Conclusion

In this we investigate temperature and velocity profiles in a cubic enclosure brought about by heating and cooling. A three dimensional cubic enclosure in the form of a room with a heater and a window placed on opposite walls was considered in this study. The flow was considered laminar hence the solutions were obtained for Reynolds number 2000 and Prandtl number 0.71.

To analyze the flow and heat transfer rates, a complete set of equations governing Newtonian fluid and boundary conditions are recast into vector potential to eliminate the need for solving the continuity equations. The governing equations with the boundary conditions are descritized using three point central difference approximations for a non-uniform mesh. The resulting finite difference equations are then solved using Matlab simulation software.

The results show that natural convection plays an important role in temperature distribution in an enclosure. The room is divided into a number of regions with those near the heater having high temperatures as those near the window have low temperatures. This helps in keeping some of the items at stated temperatures. Convective currents caused by buoyancy forces due to temperature differences between room air and air in contact with hot and cold surfaces also play a major role in determining the velocity profiles. The study has a wide application in engineering. A good example is in an appliance combining a fridge and an oven.

References

1. Aydin O. and Yang J. [2000]: Natural convection in enclosures with localized heating in below and symmetrically cooling from sides *Int. .1. Number. Methods: Meat Fluid Row* ; **K**. pp518 -529.

2. Ampofo F. and Karayiannis T. G. [2003]. "Experimental benchmark data for turbulent natural convection in an air filled square cavity". *International Journal of Heat and Vass Transfer* **46**. Pp3551 -3572 [Links].

3. Batchelor G. K. (1954) Heat Trasfer by Free Convection Across a Closed Cavity Between Vertical Boundaries at Different Temperature, *Appl. Mathematics*, **1**, pg 209 -233.

4. Boousinesq, J. (1903)Theorie Analytique deha chanteur, Gauthier-villars, **2**.

5. Deng Q. H. Tang G.f and Li. Y. (2002). A combined temperature scale for analyzing natural convection in rectangular enclosures with discrete wall heat sources. *International journal of Heat and Mass transfer*. **45**. pp 3437-3446 (Links).

6. Eckert, E.R. and Carson, W. D. (1961*) Natural convection in Air layer Enclosed between two vertical plates with Different temperature*s. Int, J Heat Mass transfer, **1,** Pg 106-120.

7. Francesco Corvaro and Massimo Paroncini (2008). Natural Convective Heat Transfer in a partially divided enclosure. *A Study on the enfluence of the source position journal.*

8. Martorell I. Herrero J. and Grau F.X. (2003). " Natural Convection form narrow horizontal plates at moderate Rayleigh numbers" *Internal journal of Heat and mass Transfer,* **46**. pp 2389-2402 (Links).

9. Sigey J. K. (2004) Three dimensional Buoyancy driven Natural convection in an Enclosure *PHD thesis* JKUAT Kenya.