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# Analysis of capacity for multiuser mimo downlink systems

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ARTFCLE INFO <u>Min  $\sum w_i x_i$ </u> ABSTRACT Atricte history (n+w)×1 Received in revised form: 10 December 2013; Accepted: 18 December 2013;

# Keywords

Channel state information (CSI), Dirty Paper Coding (DPC), Multi-User MIMO (MU-MIMO), Tomilson Harashima Precoding (THP), Bit error rate (BER).

MIMO techniques increases the spectral efficiency of the transmission and increasing Received: 19 October 2013;  $t = A_{n\times n} X_n$  capacity by using Spatial Multiplexing and Precoding. MIMO Spatial Multiplexing is the simultaneous use of the same frequencies to transmit independent data streams. Precoding (mixing) of the two streams is used to optimize the transmission into the channel so that the receiver has the best chance of recovering the original data streams. Several nonlinear precoding methods including dirty paper coding is used to achieve the capacity region for MIMO. In this paper capacity of the MIMO system and sum rate comparison is analyzed by suggesting a modified precoding technique and a round robin user scheduling algorithm.

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# Introduction

The capacity region of multiple-input multiple-output broadcast channel (mimo-bc), also known as downlink multiuser mimo (dl mu-mimo) channel, is achievable by dirty paper coding (dpc) [1]. however, dpc is too complex for implementation. In this paper, the standard dirty paper coding algorithm is modified such that it accepts arbitrary precoder and proposed modified dirty paper coding algorithm with arbitrary user power allocation and optimize user power allocation such that the weighted sum rate of users is maximized. One of the most promising approaches is block diagonalization (bd), which supports techniques for the broadcast channel including bd and dpc have only considered multi-user interference from different users in the same cell, without taking into account other-cell interference (oci). multiple stream transmission and achieves rate close to the sum capacity [7]-[10]. unfortunately, most precoding in cellular systems, however oci that comes from adjacent cells has been shown to significantly degrade the overall system performance. Moreover, in the multiuser mimo (mu-mimo) downlink channel, the MIMO system throughput can be increased by exploiting the multiuser diversity gain which is a form of selection diversity among users, arises from the separate independent fading channels between the base station (bs) and the multiple users.



Figure 1. System model

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The considered system is a downlink multi-user MIMO system with M transmit antennas at the base station, N receive antennas are located at the mobile station as figure 1 shows. Each MS will feedback the vector index from the codebook I and its corresponding CQI to the BS. The BS will schedule users and select precoding matrix from codebook I and indicates the index of precoding matrix in codebook I to each scheduled users . There are totally X users to be scheduled at the base station and the scheduler will determine K users to be transmitted according to the feedback channel information and the scheduling criterion. Then the scheduled users will be precoded by the precoding matrix before transmission.

The use of codebook is shared by the transmitter and receiver. The codebook is a set of code words, which are the quantized vectors to represent the states of channel condition. In this approach, channel gains are estimated at the receiver side. Then, the index of the appropriate codeword is selected to represent a state of estimated channel gain. Rather than the full CSI, only the corresponding index is fed back to the transmitter side. For a given channel condition H, the codebook method can be represented as use the codebook that is shared by the transmitter and receiver. The codebook is a set of code words, which are the quantized vectors to represent the states of channel condition.

# **Signal Detection Methods**

Let H denote a channel matrix with it  $\Box$  *j*, *i* $\Box$  entry hij for the channel gain between the ith transmit antenna and the jth receive antenna,  $j = 1, 2, \ldots, N_R$  and  $i = 1, 2, \ldots$ ,

N<sub>T</sub>.The spatially-multiplexed userdata and the

corresponding received signalsare representedby x = [x 1, x 2, ..., xN]T and y = [y 1, y 2, ..., yN]Trespectively, where x i and y j denote the transmit signal from the i th transmit antenna and the received signal at the jth receive antenna, respectively. Let z j denote the white Gaussian noise with a variance of  $\sigma$  z at the jth receive antenna, and h I denote the ith column vector of the channel matrix H. Now, the N R x N T MIMO system is represented as

where  $z = [z \ 1, z \ 2, \dots, z \ N]$ R

Therefore, interference signals from other transmit antennas are minimized or nullified in the course of detecting the desired signal from the target transmit antenna. To facilitate the detection of desired signals from each antenna, the effect of the channel is inverted by a weight matrix W such that







The linear detection methods include the zero-forcing (ZF) technique and the minimum mean square error (MMSE) technique and maximum likelihood technique (ML).

a) Zero forcing ordered successive interference cancellation method The zero-forcing (ZF) technique nullifies the interference by the following weight matrix:

W  $_{ZF} = (H^H H)^{-1} H^H$  (3) where (.)  $^H$  denotes the Hermitian transpose operation it inverts the effect of channel as

xZF = x + z ZF (4) b) Minimum mean square error based ordered successive

interference cancellation method In order to maximize the post-detection signal-to

interference plus noise ratio (SINR), the MMSE weight matrix is given as

Ω MMΣE =(H H H+ σ ζ2) –1 H H (5)

the MMSE receiver requires the statistical information of noise  $\sigma z$ . Using the MMSE weight the relation obtained is

xMMSE =x+ z MMSE (6) c) Maximum likelihood method

Maximum likelihood (ML) detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. The ML detection determines the estimate of the transmitted signal vector x as

$$xML$$
 =argmin  $Y \Box HX$  (7)  
where  $Y \Box HX$  corresponds to the maximum likelihood  
metric. The Maximum likelihood method achieves the optimal  
performance as the maximum a posteriori (MAP) detection  
when all the transmitted vectors are equally likely.

#### **User Scheduling Method**

In multiuser wireless networks, an opportunistic scheduling may offer a promising solution for improving the overall system throughput by always selecting the user with the current higher capacity channel. While opportunistic schedulers typically select a single best user for transmission, MIMO systems support transmissions to multiple users simultaneously and hence improves the efficiency of the resources use. It increases the system capacity through the effective use of multiple antennas in achieving a diversity effect from multiple users.

In this scheme one user among K active users is selected at each time slot in a round robin fashion, and all the N<sub>T</sub> spatial channels or transmit antennas are assigned to the selected user for the time slot. This scheduling scheme provides equal channel access chance to users, since every user uses spatial channels every K time slots. When the kth user is selected at the time slot , the system capacity is expressed as

$$N_T$$

$$C_{RRS}(t) = \sum \log_2 \left( 1 \qquad \Box \ \gamma_{k,n}(t) \right) \tag{8}$$

 $\gamma_{k,n}(t)$  denotes the SNR of channel corresponding to the nth transmit antenna and the kth user.

# **Precoding Methods**

v=1

Precoding (mixing) of the two streams is used to optimize the transmission into the channel so that the receiver has the best chance of recovering the original data streams.

The channel state information (CSI) can be completely or partially known on the transmitter side. Exploitation of such channel information allows for increasing the channel capacity and improving error performance. CSI must be estimated at the receiver side and then, fed back to the transmitter side. The various precoding methods are discussed below.

# TH precoding

y=Hx+n

TH precoding pre-subtracts previously precoded symbols intended for other receivers, thus performing spatial equalization, rather than temporal equalization. A fundamental assumption of TH precoding is the availability of perfect Channel State Information (CSI) at the transmitter. Perfect CSI enables the transmitter to precisely pre-subtract the terms that would interfere at the receiver. interference pre-subtraction and channel spatial equalization are performed at the transmitter using a feedback precoding matrix Q and a feed forward precoding matrix P .The vector s contains the data symbol destined for each user. The effect of the modulo operation is equivalent to the addition of the complex quantity. the constellation of the modified data symbols in the vector u = s + i is simply the periodic extension of the original constellation S along the real and imaginary axes.

From this model, it is ovious that v is linearly related to the modified data vector.

$$v = (I + Q)^{\Box I}$$
(9)

the vector v is then linearly precoded to produce the vector of transmitted signals x=Pv

The signals received at each user, y  $_{k}$  , can be written in the vector form

(11)

At the receivers, the modulo operation is necessary to eliminate the effect of the periodic extension of the constellation. Following the modulo operation, the receivers use a quantizer of the actual constellation to detect the received symbols.

(17)

### **Dirty paper precoding (DPC)**

Interference-free transmission can be realized by subtracting the potential interferences before transmission. It is well known that dirty paper coding (DPC) achieves the capacity region of the multiple antenna BC. Dirty paper coding (DPC) is a method of precoding the data such that the effect of the interference can be canceled subject to some interference that is known to the transmitter. However, implementation of DPC requires significant additional complexity at both transmitter and receiver. One of the most interesting results is that weighted sum rate is maximized at asymptotically high SNR by allocating power directly proportional to user weights. The DPC sum rate, which achieves the sum capacity of the MIMO broadcasting channel, can be obtained as  $N_{T}$ 

$$C = \max \sum_{n=1}^{N} \log_2 \left( I + \sum H_k^H Q_k H_k \right)$$
(12)

# Modified dirty paper precoding

On this proposed modified zero-forcing dirty paper precoding algorithm with arbitrary user power allocation and optimize user power allocation such that the weighted sum rate of users is maximized. We obtain closed-form power allocation formula in the case that high-SNR approximation of Shannon capacity is used as user rates. Let B and F be the Dirtypaper feedback and feedforward filters respectively, where B is a unit triangular matrix. The transmitted signal is given by

$$\mathbf{x} = \mathbf{F}\mathbf{B}^{-1}\mathbf{u} \tag{13}$$

Suppose we want the effective linear precoder to be P, where P is an arbitrary N×K matrix for which a pseudo inverse exists. Decompose P as  $P = FB^{-1}$  where B is unit triangular matrix. Then apply QR decomposition on

$$P^{-H} = p(p^{H} P)^{-1} \text{ as follows}$$

$$P^{-H} = Q R \qquad (14)$$

 $\mathbf{B} = \mathbf{R}_{1}^{H} (\operatorname{diag}(\mathbf{R}))^{-1}$ (15)

$$F=Q_{1}(\operatorname{diag}(R_{1}))^{-1}$$
(16)

By taking pseudo inverse Hermitian the result obtained is

 $P=O R_{1-H}$ To optimize power allocation to maximize weighted sum ( $HH^{H}$  $)^{-1}$  where D is diagonal matrix. The weighted sum-rate of users is given by

$$\mathbf{r}(\mathbf{k}) = \sum_{k=1}^{N} W_k \quad \log(1+\rho_k)$$
(18)

where  $W_k$  is the weight associated to user. The user weights could come from MU-MIMO scheduler in a wireless network. The effective linear precoder with the above optimal power allocation can then be written as

 $P = H^{H} (HH^{H})^{-1} diag(R)W$ (19)where W is a diagonal matrix **Results And Discussions** 

The scheduler selects the user with the maximum Shannon capacity. For the simulation number of transmitters and receivers used are 2 and 2. From the codebook the channel quality indicator is found out and it is fed back to the scheduler and the scheduler will schedule particular user with high SNR. Table 1 will give the simulation parameters details.

# **Table 1. Simulation parameters** Simulation parameters Specifications No. of transmitters

No. of receivers No. of users considered4

By using various signal detection methods like zero forcing method, minimum mean square method and maximum likelihood method the signals are recovered. The maximum likelihood method is giving better performance than other methods. In figure 3 bit error rate and signal to noise ratio of various signal detection methods were compared.



Figure 3. Signal detection methods

It is found out that as the number of users increased the total capacity of the system also increased by implementing the round robin scheduling algorithm.

In figure 4 by selecting the user with the higher channel capacity the total capacity of the system with scheduling is analyzed. The results indicate that with the increasing of the codebook size and the user number, the performance will increase.



Figure 4. Round robin scheduling

By implementing modified dirty paper coding it is giving better performance than the existing TH precoding and dirty paper coding methods. In figure 5 by utilizing the approximation to high SNR, the modified dirty paper coding is giving better performance.

Here bit error rate and signal to noise ratio is compared, it is found out that modified dirty paper coding is giving minimum error rate performance than the dirty paper precoding and TH precoding methods.



Figure 5. Precoding Techniques

In figure 6 the sum rate of the modified dirty paper precoding and existing dirty paper coding were compared. It is found out that the modified dirty paper coding maximizes the weighted sum-rate of users.



Figure 6. CDF of sum rate comparison

In figure 7 the capacity of the modified dirty paper precoding is compared for various combination of antennas. It is found out that the capacity of the modified dirty paper precoding is increased with the signal to noise ratio.



# Conclusion

In this paper various precoding techniques were compared. Also the capacity achieved by modified dirty paper coding (DPC) by utilizing the approximation to high SNR is investigated. It is found out that the modified dirty paper coding with four antennas maximizes the weighted sum-rate of users. The system throughput is increased by selecting the user with the high SNR. It provides significant improvement to the system capacity and reduction in error performance.

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