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Optimal Multistage Scheduling of PMU Placement in Electric Power Systems

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$Min \sum_{i=1}^{n} w_i.x_i$

$s.t\ A_{(n+w)\times n} X_{n\times 1} \ge b_{(n+w)\times 1}$

$Min \sum_{i=1}^{n} w_i.x_i$

$s.t\ \ y = A_{n\times n} X_{n\times 1} \ge b_{n\times 1}$

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ABSTRACT

This paper presents an optimization model to calculation of the minimum number of phasor measurement units (PMUs) for complete observability of electrical power networks. Minimum PMU placement problem has multiple solutions. We propose a flow chart and indicators in order to find all solution and rank these multiple solutions, respectively. The proposed model is capable of obtaining the full set of optimal solutions instead of only one or partial solutions. The integer linear programming (ILP) approach is implemented to solve this model. We propose a procedure for multistaging of PMU placement in a given time horizon due to financial and physical constraints using an ILP framework. In addition, network expansion scenarios associated with intermediate stages are taken into account. Numerical studies are conducted on a nine-bus system and the IEEE 57-bus system and the results are analyzed.

Introduction

Phasor Measurement Unit (PMU) based on GPS technique is widely used to monitor the state of a power system. The PMU measurement is received by time sampling based on the same time reference synchronized by the GPS, so it could provide power engineers with immediate and precise measurements. By applying the PMU measurements in different areas in power systems such as state estimation, protection, load shedding, voltage collapses etc., the reliability and stability in power system are expected to be improved. However, due to a high cost of PMUs or nonexistence of communication facilities in certain buses, it is impossible to place a PMU on every bus in the network, either as a stand-alone unit or relay-based function. So an optimal placement of PMUs is required for better power quality.

Various plans have been proposed to find the optimal location of PMU. In [1, 2], the authors use a simulated annealing technique to find the optimal PMU locations. Since this approach needs an initial random guess for a solution, the computational burden of this method is heavy and this method may not provide full optimal solutions over a global range. In [3], a genetic algorithm (GA) is used to find the optimal PMU locations. GA is an advanced global search method which is based on biological phenomena. The advantage of the GA method is that it can produce multiple solutions in order to provide power companies with more options. The drawback of this approach is that it may not result in the minimum number of PMUs making the system observable. And the optimal number of PMUs should be known beforehand in order to implement the GA method. In [4], the condition number of the normalized measurement matrix is proposed as a criterion for selecting candidate solutions. However, the result reported is not the optimal one. PSO (Particle Swarm Optimization) and BPSO (Binary Particle Swarm Optimization) are among the other methods used by the researchers [5, 6].

All the papers mentioned above only archive one or partial solutions in PMU placement and have some limitations. In reality, a power company may run into many difficulties during the procession of PMUs installation on account of multiple reasons such as the nonexistence of communication facility, the limited channels of a PMU and the inconvenience of installation. Also, just one placement scheme may not satisfy the power company's demand. It is necessary to provide power companies with a full set of solutions with the same minimum number of PMUs in order to make a flexible choice. To overcome the above limitations, in this paper the PMU placement problem is staged over a planning horizon because of managing its expenses [7]. Accordingly, the solution obtained for the PMU placement problem is assumed as the horizon year objective and intermediate stages are defined. The multistage problems maximize the average probability of the system observability, subject to a limited number of PMUs and in the context of

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horizon year placement schedule. The paper considers the expansion of power systems along with the multistage PMU placement. The expansion of generation facilities and transmission networks would respond to the load growth. The expansion scenarios would influence the observability as the network topology changes. Here, the observability analysis is carried out at each time stage based on the corresponding network topology.

The remaining sections of this paper are organized as follows. Observability analysis with PMU is presented in Section II. Then PMU placement problem formulation and a model based on Integer Linear Programming (ILP) are presented to obtain the full set of optimal placement solutions. Concept and formulation of the probabilistic observability are presented in Section IV. Section V covers the formulation of multistage PMU placement. Section VI examines performance of the proposed techniques on two test systems. Finally in Section VII conclusion is presented.

Observability analysis with PMU

A PMU installed on a certain bus is able to measure the voltage magnitude and phase angle of the local bus and the branch current phasor of all branches emerging from this bus. The voltage magnitude and phase angle of the neighboring bus can be computed using voltage drop equations. Thus the buses monitored by a PMU are directly observable, the neighboring buses connected to the PMU buses are indirectly observable and the other buses which are not associated with the PMU buses are unobservable. The following graph explains the bus observability in a system[8].

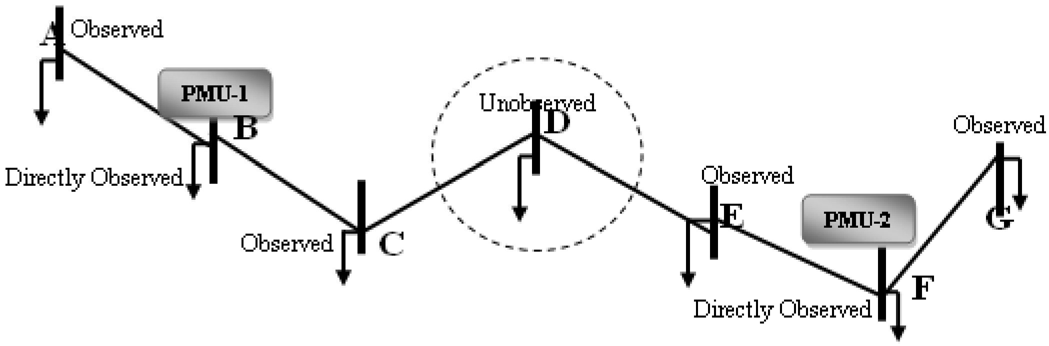

Fig. 1 PMU Observability Analysis

In Fig. 1, the network has 7 buses from A to bus G. Assume two PMUs are located on bus B and bus F, so bus B and F are directly observable. Bus A, C, E and G are all connected to bus B and F so they are indirectly observably. Bus D is not associated with any PMU bus, so bus D is unobservable. So in this 7-bus system example, 6 buses are observable and 1 bus is not observable. Thus, this system is not a completely observed system. A completely observed system means all the buses in this system should be directly or indirectly observed with a proper PMU placement scheme.

PMU placement problem formulation

A PMU is able to measure the voltage phasor of the installed bus and the current phasors of all the lines connecting to this bus. That is to say, a PMU can make the installed bus and its neighboring buses observable. The objective of placing PMUs in power systems is to determine a minimal set of PMUs such that the whole system is observable.

Therefore the placement of PMUs makes a problem that is to find a minimal set of PMUs such that a bus must be reached at least once by the set of the PMUs.

Now the optimal placement of PMUs can be formulated as a problem of Integer Linear Programming (ILP) [9]:

$$Min \sum_{i=1}^{n} w_i . x_i \qquad (1)$$
$$s.t \quad y = A_{n \times n} X_{n \times 1} \geq b_{n \times 1}$$

Where n is total number of buses in the network and w is the cost function for the installed PMUs or the weight matrix for the buses that can vary based on the importance of every bus. w is normally equal to unit $n \times n$ matrix. In this equation, x, A and b are defined as bellow:

$$A_{n\times n}(i,j)=\begin{cases}1 & i=j\\ 1 & \text{if buses } i \text{ and } j \text{ are connected}\\ 0 & \text{otherwise}\end{cases}$$

$$x_{n\times 1}(i)=\begin{cases}1 & \text{if PMU installed in bus } i\\ 0 & \text{otherwise}\end{cases}$$

$$b_{n\times 1}=\begin{bmatrix}1 & 1 & 1 & \dots & 1 & 1\end{bmatrix}^T$$

The inequality in function (1) is used for complete monitoring the system. The *i*th row in *Ax* matrix is the number of times that the *i*th bus is monitored which should be at least one.

A power company probably has many problems during the installation process of PMUs, such as unavailability of communication facilities, limitation of PMU transferring or unsuitable location of PMU installation. Also a unique placement plan does not satisfy the power company requirements. Hence, it seems essential to supply a full set of solutions with minimum number of PMUs for power company to achieve a flexible choice. Moreover, a model is proposed which can obtain the full set of optimal solutions instead of only one or partial solutions. For the same answer repeating avoidance matrix $A_{n\times n}$ and the vector $b_{n\times 1}$ are determined as followed:

$$A_{(n+1)\times n}=\begin{bmatrix} & A_{n\times n} & & \\ -x_{s1,1} & -x_{s1,2} & \dots & -x_{s1,n}\end{bmatrix}_{(n+1)\times n}$$

$$b_{(n+1)\times 1}=\begin{bmatrix}1 & 1 & \dots & 1 & -(m-1)\end{bmatrix}^T_{1\times(n+1)}$$

Assume $X_{s1}=\begin{bmatrix}x_{s1,1} & x_{s1,2} & \dots & x_{s1,n}\end{bmatrix}^T$ is an optimal solution and *m* is the optimal number of PMUs which are needed.

These changes are based on the fact that the inequality $X_{s1}^T X_{s2}\le(m-1)$ should be acceptable if X_{s2} is a different solution from X_{s1}; if X_{s2} is the same solution as X_{s1}, then $X_{s1}^T X_{s2}=m$. Multiplying -1 on both sides of the inequality, it reaches to $(-X_{s1})^T X_{s2}\ge-(m-1)$.

If *w* solutions are already known, the matrix $A_{n\times n}$ and the vector $b_{n\times 1}$ will be:

$$A_{(n+w)\times n}=\begin{bmatrix} & A_{n\times n} & & \\ -x_{s1,1} & -x_{s1,2} & \dots & -x_{s1,n}\\ -x_{s2,1} & -x_{s2,1} & \dots & -x_{s2,n}\\ . & . & . & .\\ . & . & . & .\\ . & . & . & .\\ -x_{sw,1} & -x_{sw,2} & \dots & -x_{sw,n}\end{bmatrix}_{(n+w)\times n}$$

$$b_{(n+w)\times 1}=\begin{bmatrix}1 & 1 & \dots & 1 & -(m-1) & \dots & -(m-1) & -(m-1)\end{bmatrix}^T_{1\times(n+w)}$$

The formulations for the modified model are:

$$\begin{aligned}&Min\ \sum_{i=1}^{n} w_i . x_i\\ &s.t\ \ A_{(n+w)\times n}X_{n\times 1}\ge b_{(n+w)\times 1}\end{aligned} \qquad (2)$$

Matrix *A* and vector *b* will modify after each iteration. If the optimal number of PMUs will exceed *m* once, the iteration stops. The flow chart in following shows the process for finding all solutions.

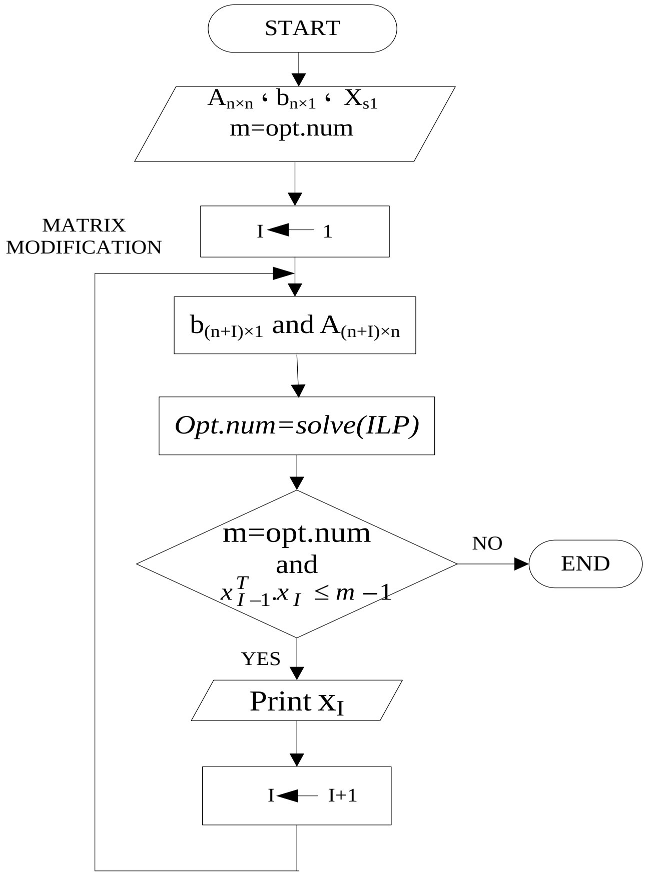

Fig. 2 Flow chart of process to find all solutions

In following the problem of selecting the best optimized answer of all solutions will be discussed. In this section, we introduce *Bus Observability Index* (BOI) for bus-*i* (β_i) as a numbers of PMU$_s$ which make the same bus observable, directly or indirectly. finally, maximum bus observability index is limited to *maximum connectivity* (η_i) of a bus plus one, i.e.

$$\beta_i \le \eta_i + 1$$

Now we define *System Observability Redundancy Index* (SORI) as the sum of BOIs for all busses of a system. Then

$$\gamma = \sum_{i=1}^{n} \beta_i$$

Where (γ) represents SORI. Consider a six-bus system shown in Fig. 3; it seems that a minimum of two PMUs are required to find out the system observability. Assume two optimal solutions which are shown in Fig. 3 [7].

For the PMU placement as given in Fig. 3(a), BOI (β_i) for busses 1 to 6 are 1, 2, 1, 1, 2, and 1, respectively. This makes SORI, $\gamma_a = 8$. Alternatively, for PMU placement in Fig. 3(b), BOI for busses 1 to 6 are unity, making $\gamma_b = 6$. Therefore, the PMU placement with maximum SORI in Fig. 3(a) must be chosen to suitable placement.

There is an advantage with maximizing SORI which a larger area of system is remained observable in case of a PMU outage. Suppose, in Fig. 3(a), one PMU outage will indicate, two busses remaining unobservable, as against for loss of single PMU for system, three busses remaining unobservable in Fig. 3(b).

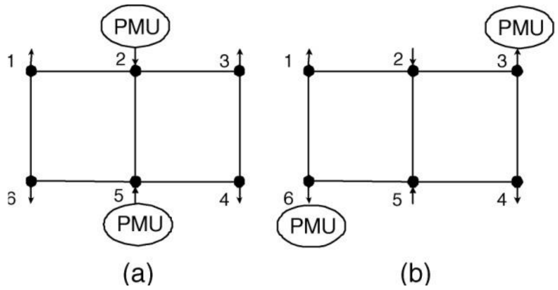

Fig 3. A six-bus system with system observability (a) 6 and (b) 8

Probability of observability

In observability analyses, there is a possibility to determine a separate set of probabilistic indices for specified buses and the whole system. For each bus, the probability of observability is determined and a system index is proposed for average value of probabilities. The system index provides a quantitative insight on the power network observability.

In this section probability of observability of bus i is defined as follows [11]:

$$PO_i = 1 - \prod_{j \in I} (1 - u_j A_{ij}); \quad \forall i \tag{3}$$

Where u_j is a binary variable which invetigate the PMU installation at bus j. A_{ij} is a constant value representing the probability of observability of bus because of the PMU located at bus. This value is mathematically defined as

$$A_{ij} = a_{ij} A_j^{V_m} A_j^{PMU} A_j^{Link} A_{ij}^{C_m} A_{ij}^{Line}; \quad \forall i, \forall j \tag{4}$$

Where $A_j^{V_m}$ and $A_{ij}^{C_m}$ are the availability of PMU voltage measurement at bus j and the availability of PMU current measurement at line ij, respectivelyll . These values could be calculated in terms of the availability of potential transformers (PTs) and current k (CTs). Consider that three-phase measurements are required for each phasor calculation [12]. A_j^{PMU} and A_j^{Link} are the probability of successful operation of the PMU at bus j and its communication link, respectively. Also, A_{ij}^{Line} represents the availability of line ij. In (4), $A_{ii}^{Line} = A_{ii}^{Cm} = 1; \ \forall i$ and a_{ij} is the binary connectivity parameter defined as

$$a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if buses } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases}$$

The formulation of average probability of observability (APO) which is the average value of bus indices is defined as follows:

$$APO = \frac{1}{N_b} \sum_{i \in I} PO_i \tag{5}$$

More discussion on (3) and (5) is available in [13]. The average function is employed to maintain the *APO* between zero and one.

Problem formulation of multistage PMU placement

The implementation of PMU's installation is in multi-stage, Because of the construction lead time and PMU investment cost. The problem formulation for the multistage scheduling of the PMU placement is explained in this section. An optimization model will be applied for a multistage PMU placement. The optimization model maximizes APO of network in each stage. PMU placement scheme is calculated initially In [7], the final year and the multi-stage scheduling is defined in the context of the final solution. The final multi-stage solution should be the same as that of the single stage solution. A similar approach to the staging problem is applied also [11].

In this paper, the staging of PMU placement is determined by observability probabilistic indices. Assume that the PMU placement at the last stage will result in N PMUs in $\tilde{S}^H$ where

$$\tilde{S}^H = \left\{ i \in I^H \,\middle|\, u_i^H = 1 \right\} \tag{6}$$

H is superscript implying the horizon year, $\tilde{S}^t$ is a set of available candidate buses for the PMU installation at stage t, i is index for bus number, I is a set of buses, u_i is binary decision variable that is equal to one if a PMU is installed at bus i and zero otherwise; and n^t is number of PMUs to be installed at stage t.

The solution of PMU stages determines S^t (as a subset of $\tilde{S}^H$) corresponding to stages

$t=1, 2, \ldots, H$ with n^t representing the number of PMUs installed at stage t. Obviously

$$n^1 + n^2 + \cdots + n^H = N \qquad (7)$$

$$S^1 \cup S^2 \cup \cdots \cup S^H = \tilde{S}^H \qquad (8)$$

Each candidate bus should have one PMU; so, S^t; $t=1, 2, \ldots, H$ are nonintersecting. According to the network expansion schedule, a number of new buses should be added to the network at every stage and the PMU placement candidate buses for $t=1, 2, \ldots, H-1$ would be

$$\tilde{S}^t = \left\{ i \in I^t \middle| u_i^H = 1 \right\} \qquad (9)$$

Here, $\tilde{S}^t \subset \tilde{S}^H$ since $I^t \subset I^H$. We assume PMUs placed at earlier stages cannot be relocated. So, the set S^t would not include the earlier buses. Mathematically, S^t would be a subset of $\tilde{S}^t - \left\{ S^1 \cup S^2 \cup \cdots \cup S^{t-1} \right\}$. At the last stage, the remaining candidate buses which are not equipped with PMUs are considered. So, the solution of the last stage is known. The optimization problem for $t=1, 2, \ldots, H-1$ is formulated as [11]:

$$\max \; APO^t = f(\bar{u}^t)$$

Subject to

$$\bar{u}_i^t = \sum_{t'=1}^{t} u_i^{t'}; \quad i \in \tilde{S}^t$$

$$n^t = \sum_i u_i^t \; ; \quad i \in \tilde{S}^t - \left\{ S^1 \cup S^2 \cup \cdots \cup S^{t-1} \right\}$$

The objective function (10) is to maximize the average probability of observability at stage t. Note that the transmission lines added in stage t are considered in the calculation of APO^t. APO^t would incorporate the new PMUs associated with stage t and the PMUs installed at earlier stages. So, APO^t is defined as a function of $\bar{u}^t$ with elements defined by (11). $\bar{u}_i^t$ for $i \in \tilde{S}^t - \left\{ S^1 \cup S^2 \cup \cdots \cup S^{t-1} \right\}$ is equal to the problem variable $\bar{u}_i^t$ (which is either zero or one) and $\bar{u}_i^t$ for $i \in \left\{ S^1 \cup S^2 \cup \cdots \cup S^{t-1} \right\}$ is equal to one. The number of PMUs is constrained by financial and physical limitation (12).

Numerical Study

In this section we investigate two case studies which are nine-bus system and the IEEE 57-bus system to numerically analyze the proposed approach. The cost of PMU installation is negligible and both case studies are simulated by using the suggested formulation in Section III and V. The integer linear programming models in this study are solved using the function *bintprog* in MATLAB software package.

Nine-Bus System

The present system, depicted in Fig. 4 with solid lines, has nine buses, nine transmission lines and the system expansion includes three stages. At the first stage, no new buses or transmission lines are added. Stage 2 includes a new transmission lines between buses 4 and 6, and Stage 3 consists of a new bus with the associated transmission line, i.e., bus 10 and line 9–10. The expansion scenarios are also depicted in Fig. 4 [11].

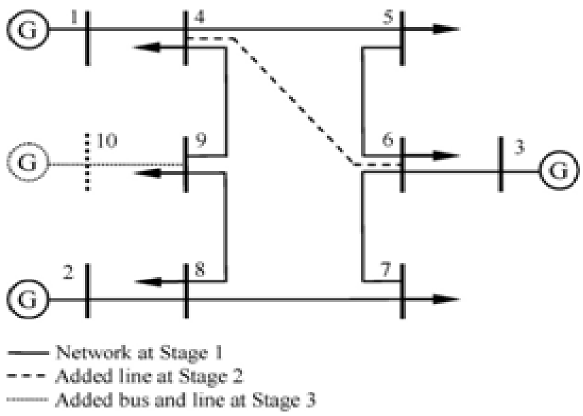

Fig. 4. Nine-bus system with the expansion scenarios [11].

The reliability data are given in the Appendix.

The process of placement for the period of three years is presented in this section. First of all, we need to determine the number of required PMUs to complete observability of network in third year which the network included buses 1 to 10. Using the method presented in Section III, we can determine a full set of solutions for 10 buses system and APO for each set of solutions. The set of solutions are shown in Table I.

Table I. Full set of solutions for the nine-bus system with the expansion scenarios

Test System	No. of PMUs	Number	Locations	SORI	APO
Nine-Bus System	4	1	4, 6, 8, 9	18	0.9930
		2	1, 6, 8, 9	15	0.9911
		3	1, 2, 6, 9	13	0.9892
		4	3, 4, 8, 9	15	0.9905
		5	1, 2, 6, 10	11	0.9878
		6	1, 6, 8, 10	13	0.9908
		7	2, 4, 6, 9	16	0.9921
		8	2, 4, 6, 10	14	0.9914
		9	3, 4, 8, 10	13	0.9893
		10	4, 6, 8, 10	16	0.9928

Consider the values of Table I, firs set of solutions is chosen to install PMUs, because of high values of SORI and APO among the other solutions set. $\tilde{S}^3 = \{4,6,8,9\}$

It is assumed that one PMU in first year, two PMUs in second year and one PMU in third year will install in the network. Means it will be: $n_1=1$, $n_2=2$, $n_3=1$. In each stage (year) placement problem must maximize APO^t; t=1, 2.

Table II. Results of the multistage PMU placement for the nine-bus system

Stage number *(t)*		1	2	3
Number of PMUs at each stage		1	2	1
PMU installation buses at each stage		8	4, 6	9
APO^t		0.43815434	0.99304556	0.99301117
$PO_i^t; i =$	1	0	0.9792888	0.97972888
	2	0.98546465	0.98546465	0.98546465
	3	0	0.98319012	0.98319012
	4	0	0.99983750	0.99999728
	5	0	0.99967616	0.99967616
	6	0	0.99983750	0.99983750
	7	0.98338790	0.99977661	0.99977661
	8	0.99015970	0.99015970	0.99984626
	9	0.98437683	0.99973892	0.99999743
	10	--	--	0.98259676

Stage 1: Because in the first stage of a PMU must be installed on the network, equation (10) compares the candidate buses for PMU installation and chooses a bus that will maximize network APO, that was chosen bus 8 at this stage (in this stage, bus 8 is chosen); so $S^1 = \{8\}$.

Buses 2, 7, 8, and 9 are observable because of the PMU installed at bus 8. The values of $PO_i^1; \forall i \in I^1$ and APO^1 are given in Table II in which PO_i^1 associated with observable buses 2, 7, 8, and 9 are non-zero. The comparison of non-zero values of PO_i^1 reveals that PO_8^1 is the highest since this bus is directly observable by its respective PMU. Other PO_i^1 are not identical because the corresponding transmission lines have different availability values.

Stage 2: At this stage, a new transmission line between buses 4 and 6 is added to the network. The set of candidate buses at stage 2 is {4, 6, 9}. Bus 8 is omitted since this bus is equipped with PMU at stage 1. At Stage 2, two PMUs are assigned to buses 4 and 6 based on the optimization of (10)-(12) ; so S^2 = {4,6}. Buses 1, 3, 4, 5, and 6 remained unobservable during Stage 1. At Stage 2, they are made observable when the PMUs are installed at buses 4 and 6. Table II presents $PO_i^2; \forall i \in I^2$ and APO^2. The highest PO_i^2 are at buses 4 and 6 because line 4-6 is added to the network at this stage and these buses are made observable through two PMUs installed at buses 4 and 6. In other words, buses 4 and 6 have redundant measurements and would be unobservable only when the measurements would fail simultaneously. Other highest values of PO_i^2 are related to buses 5, 7, and 9 which all exploit redundant measurements. Buses 1, 2, 3, and 8 are observable based on one PMU; however, PO_8^2 is higher than PO_1^2, PO_2^2, and PO_3^2 because bus 8 is made observable by its own PMU unlike buses 1, 2, and 3 which are made observable by PMUs at adjacent buses.

Stage 3: The last stage does not need any optimization task because the number of candidate buses and remaining number of PMUs are equal. So, the candidate bus 9 at Stage 3 is equipped with the remaining PMU; that is S^3 = {9}.

The values of $PO_i^3; \forall i \in I^3$ and APO^3 are given in Table II. At this stage, bus 10 and line 9-10 are added to the network. Since bus 9 has not been equipped with a PMU at earlier stages, bus 10 would not be observable. At this stage, new bus 10 will be observable through remaining PMU installed at bus 9, and also makes a redundant measurement for buses 4 and 8. In Table II, the lowest value of $PO_i^3; \forall i \in I^3$ belongs to buses 1, 2, and 3 with no PMUs installation which are made observable by PMUs at adjacent buses.

IEEE 57-Bus System

A similar process is performed for the IEEE 57-Bus system that depicted in Fig. 5 The system includes 80 transmission lines and load are added to all buses in Fig. 5. The time span for the multi-stage placement problem is assumed to be 10 years divided into three stages. The planning results include lines 8-31 and 8-57 at Stage 1, lines 2-53 and 8-50 at Stage 2, and lines 1-33 and 1-25 at Stage 3. So, the number of transmission lines associated with Stages 1, 2, and 3 would be 82, 84, and 86, respectively [11]. These lines are depicted in Fig. 5. As no new buses are added to the system, the set of buses associated with the three stages are the same, i.e.,

$$I^1 = I^2 = I^3 = \{1, 2, \ldots, 57\}$$

The availabilities of PMU, communication link, and current and voltage measurements are assumed to be the same as those of the nine-bus system that is shown in Table V in Appendix. The reliability data of transmission lines are given in the Appendix.
Table III shows the solutions set of the PMU placement problem for the 57-Bus system with the expansion scenarios.

Table III. Full set of solutions for the 57-Bus system with the expansion scenarios

Test System	No. of PMUs	Number	Locations	SORI	APO
IEEE 57-Bus	16	1	1, 4, 13, 19, 22, 26, 29, 31, 34, 36, 41, 44, 47, 51, 54, 57	65	0.98794313
		2	1, 4, 13, 20, 22, 26, 29, 31, 34, 36, 41, 44, 47, 51, 54, 57	65	0.98788457
		3	1, 4, 10, 20, 23, 26, 29, 31, 34, 36, 41, 44, 46, 49, 54, 57	63	0.98737376

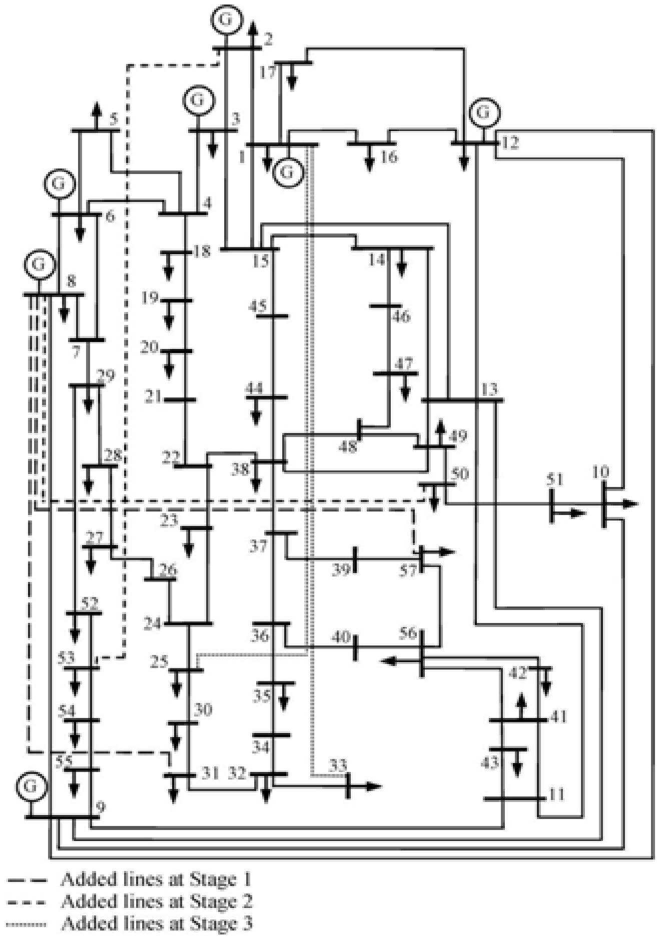

Fig. 5 IEEE 57-bus system with the expansion scenarios [11]

Consider the values of Table III, first and second set of solutions have equal amounts of SORI. But, first set of solution than the second set of solution has a higher value of APO. So, first set of solution has a higher reliability in the system observability. So, first set of solution is chosen in order to install PMUs.

$$\tilde{S}^3 = \begin{Bmatrix} 1, 4, 13, 19, 22, 26, 29, 31, \\ 34, 36, 41, 44, 47, 51, 54, 57 \end{Bmatrix}$$

The installation of the 16 PMUs is scheduled in three stages; they will be installed in the network with 5 PMUs in first Stage, 6 PMUs in second Stage and 5 PMUs in third Stage. That means, *n* will be: n_1=5, n_2=6, n_3=5. In order to determine S^1 and S^2, APO^1 and APO^2 are maximized, respectively. Subtracting $S^1 \bigcup S^2$, the remaining buses of $\tilde{S}^3$ would be S^3.

Table IV shows the placement results and the average probability of observability associated with the three stages. As expected, an increase in the number of PMUs would result in a significant improvement in the average probability of the network observability. Table V presents the bus indices for the three placement stages.

Table IV. Results of the multistage PMU placement for the 57-Bus system

Stage number *(t)*	Number of PMUs at each stage	PMU installation buses at each stage	APO^t
1	5	1, 4, 13, 22, 41	0.41554992
2	6	26, 29, 31, 36, 47, 51	0.77875960
3	5	19, 34, 44, 54, 57	0.98794313

Stage 1: At this stage, the PO_i^1 indices of 24 buses are nonzero; so, the corresponding buses are observable and the remaining 33 buses are unobservable. Since at the Stage 1, the buses with PMUs do not have redundant measurements, PO_i^1 of buses 1, 4, 13, 22 and 41 are identical and equal to 0.99015970 whereas PO_i^1 indices of must other buses, which are not equipped with PMUs, are

smaller. The indices of buses 11 and 15 are greater than 0.99015970. Hence, we conclude that these two buses are observable by more than one measurement. As depicted in Fig. 5, bus 11 is connected to PMU-equipped buses 13 and 41. Bus 15 is connected to the PMU-equipped buses 1 and 13.

Table V. Bus Indices of the Multistage PMU Placement For The 57-Bus System

Stage 3				Stage 2				Stage 1			
PO_i^3	Bus No.	PO_i^3	Bus No.	PO_i^2	Bus No.	PO_i^2	Bus No.	PO_i^1	Bus No.	PO_i^1	Bus No.
0.98150894	30	0.99015970	1	0.98150894	30	0.99015970	1	0	30	0.99015970	1
0.99015970	31	0.98497018	2	0.99015970	31	0.98497018	2	0	31	0.98497018	2
0.99972747	32	0.98655247	3	0.98388237	32	0.98655247	3	0	32	0.98655247	3
0.98141005	33	0.99015970	4	0	33	0.99015970	4	0	33	0.99015970	4
0.99015970	34	0.98131116	5	0	34	0.98131116	5	0	34	0.98131116	5
0.99972637	35	0.98150894	6	0.98566243	35	0.98150894	6	0	35	0.98150894	6
0.99015970	36	0.98378347	7	0.99015970	36	0.98378347	7	0	36	0	7
0.98279455	37	0.99967290	8	0.98279455	37	0.98249787	8	0	37	0	8
0.99975943	38	0.98625579	9	0.98684914	38	0.98625579	9	0.98684914	38	0.98625579	9
0.98417904	39	0.98724471	10	0	39	0.98724471	10	0	39	0	10
0.98269565	40	0.99976589	11	0.98269565	40	0.99976589	11	0	40	0.99976589	11
0.99015970	41	0.98299233	12	0.99015970	41	0.98299233	12	0.99015970	41	0.98299233	12
0.98576133	42	0.99015970	13	0.98576132	42	0.99015970	13	0.98576133	42	0.99015970	13
0.98289344	43	0.98605800	14	0.98289344	43	0.98605800	14	0.98289344	43	0.98605800	14
0.99015970	44	0.99979277	15	0	44	0.99979277	15	0	44	0.99979277	15
0.98417904	45	0.98328901	16	0	45	0.98328901	16	0	45	0.98328901	16
0.98279455	46	0.98417904	17	0.98279455	46	0.98417904	17	0	46	0.98417904	17
0.99015970	47	0.99967731	18	0.99015970	47	0.98309123	18	0	47	0.98309123	18
0.98724471	48	0.99015970	19	0.98724471	48	0	19	0	48	0	19
0.98427794	49	0.98378347	20	0.98427794	49	0	20	0.98427794	49	0	20
0.98121227	50	0.98655247	21	0.98121227	50	0.98655247	21	0	50	0.98655247	21
0.99015970	51	0.99015970	22	0.99015970	51	0.99015970	22	0	51	0.99015970	22
0.98447572	52	0.98328901	23	0.98447572	52	0.98328901	23	0	52	0.98328901	23
0.98220119	53	0.98556354	24	0	53	0.98556354	24	0	53	0	24
0.99015970	54	0.98150894	25	0	54	0	25	0	54	0	25
0.98516797	55	0.99015970	26	0	55	0.99015970	26	0	55	0	26
0.99980974	56	0.98625579	27	0.98615690	56	0.98625579	27	0.98615690	56	0	27
0.99015970	57	0.98437683	28	0	57	0.98437683	28	0	57	0	28
APO=0.98794313		0.99015970	29	**APO=0.77875960**		0.99015970	29	**APO=0.41554992**		0	29

Stage 2: 6 PMUs are added to the network at this stage. Indices at Stages 1 and 2 reveal that the number of unobservable buses decreases from 33 to 12. So, 21 buses are made observable at this stage. Note that bus 8 is made observable by the added line between buses 8 and 31 at Stage 1 and PMU-equipped bus 31 at Stage 2.

Stage 3: Based on Stage 2, we have 12 unobservable buses which are buses 19, 20, 25, 33, 34, 39, 44, 45, 53, 54, 55 and 57. Table IV shows that the remaining 5 PMUs are installed at 19, 34, 44, 54 and 57 buses. So, the network is made observable at this stage. Table V shows that the PO_i^3 of buses 8, 11, 15, 21, 32, 35, 38 and 56 are very close to 1 because these eight buses are observable by two PMUs. The indices that are less than 0.99015970 are related to buses which are not equipped with PMUs and are made observable by a single PMU at other locations. The buses 25 and 33 are made observable by the transmission lines added at this stage.

Conclusions

In this paper, method of observability of power system by using PMU is explained. Optimal PMU placement has multiple solutions. This paper a model in PMU placement problem by using integer linear programming (ILP) is presented. The proposed model is flexible to provide all solutions instead of one or partial solutions. This model increase the reliability of monitoring of power system and it makes power engineers convinced, in practical decisions. In addition, algorithm for optimal multistage scheduling of PMU placement according to a set of probabilistic criteria has been devised. A suitable and linear objective function is introduced in each stage to determine the PMUs, which make a large area of network observable and increase average probability of observability indices, to install on power network. The presented results for the nine-bus and the IEEE 57-bus systems showed that the proposed formulation works effectively.

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Appendix

The reliability data for the nine-bus system are presented in Table VI. Table VII shows the A_{ij}^{Line} values associated with the IEEE 57-bus system.

Table VI. Reliability Data of Nine-Bus System

Parameter	Value	Parameter	Value
A_i^{PMU}	0.99549768	A_{4-9}^{Line}	0.9943
A_i^{Vm}	$(0.99854238)^3$	A_{5-6}^{Line}	0.9905
A_{ij}^{Cm}	$(0.99958447)^3$	A_{6-7}^{Line}	0.9976
A_i^{Link}	0.999	A_{7-8}^{Line}	0.9944
A_{1-4}^{Line}	0.9907	A_{8-9}^{Line}	0.9954
A_{2-8}^{Line}	0.9965	A_{4-6}^{Line}	0.9945
A_{3-6}^{Line}	0.9942	A_{9-10}^{Line}	0.9936
A_{4-5}^{Line}	0.9952	-	-

Table VII. Availabilities of transmission lines of the IEEE 57-bus system

Line		Value	Line		Value	Line		Value
From	To		From	To		From	To	
1	2	0.9960	14	15	0.9935	41	42	0.9968
2	3	0.9944	18	19	0.9919	41	43	0.9939
3	4	0.9976	19	20	0.9948	38	44	0.9927
4	5	0.9923	21	20	0.9962	15	45	0.996
4	6	0.9925	21	22	0.9976	14	46	0.9931
6	7	0.9929	22	23	0.9943	46	47	0.9938
6	8	0.9966	23	24	0.9931	47	48	0.9983
8	9	0.9944	24	25	0.9955	48	49	0.9943
9	10	0.9982	24	25	0.9977	49	50	0.9958
9	11	0.9955	24	26	0.9966	50	51	0.9922
9	12	0.9962	26	27	0.9973	10	51	0.9983
9	13	0.9973	27	28	0.9965	13	49	0.9953
13	14	0.9971	28	29	0.9954	29	52	0.9955
13	15	0.9955	7	29	0.9948	52	53	0.9953
1	15	0.9977	25	30	0.9953	53	54	0.9932
1	16	0.9943	30	31	0.9925	54	55	0.9962
1	17	0.9952	31	32	0.9949	11	43	0.9931
3	15	0.9937	32	33	0.992	44	45	0.9952
4	18	0.9937	34	32	0.9941	40	56	0.9981
4	18	0.9941	34	35	0.9919	56	41	0.9972
5	6	0.9981	35	36	0.9967	56	42	0.9935
7	8	0.9979	36	37	0.9938	39	57	0.9952
10	12	0.9962	37	38	0.9964	57	56	0.9973
11	13	0.9948	37	39	0.9939	38	49	0.9961
12	13	0.9940	36	40	0.9937	38	48	0.9943
12	16	0.9956	22	38	0.9979	9	55	0.9971
12	17	0.9929	11	41	0.9966	-	-	-