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Hybridization of certain techniques in combinatorial optimization: an overview

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ABSTRACT

This paper dwells on combinatorial optimization with a view to unveiling various techniques for solving problems therein. We are interested in the combination of exact techniques (ET) and metaheuristics (MH) to provide optimal solutions or mainly to generate better heuristic solutions. In doing this, we were able to give a kind of categorization of the possible combinations, their usefulness and areas of applications.

Introduction

An overview of combinatorial optimization problems (COP) reveals that lots of real world applications exist in such fields as assignment, scheduling, cutting and packing and covering designs and many other areas of economic, industrial and scientific importance. The study also shows that there are two main categories of the available techniques for solving these problems; these are the exact techniques and the heuristic methods. The exact techniques (ET/EA) are guaranteed to locate an optimal solution and show its optimality for every instance of combinatorial optimization problem. In locating the optimal solution, the run-time often decreases dramatically with the size of the problem at hand, such that only small or moderately sized problems can be practically solved to prove optimality. In the light of this, if larger situations or problems are to be solved, we must trade optimally for run-time. That is the guarantee for locating optimal solution is sacrificed fore the sake of getting good solution in a limited time. When this happens, we have what we call heuristic techniques or algorithms.

In literature, we have two successful ways COP has been solved with significant success viz.

Integer Programming IP) as an exact technique used in Operations research (OR) and based on the concepts of Linear Programming (LP) [7].

Local Search with different extensions are independently developed variants called metaheuristics as a heuristic technique.

Branch-an-bound (B&B) dynamic programming, Lagrangian relaxation based methods, Linear and Integer programming based methods such as branch-and-cut, branch-and-price and branch-and-cut-and-price [19] are examples of exact techniques. Whereas metaheuristics methods include among others simulated annealing [13], tabu search [10], iterated local search variable neighbourhood search [12] and various populations based models such as evolutionary algorithm [2] – Genetic Algorithm (GA), Scatter search [11].

Due to the dynamic nature of man and his environment and the cross-breeding of ideas, there have been various attempts to combine ideas and methods from the two successful streams mentioned above. In discussing this area of this paper, mention must be made of Dumitrescu and Stutzle [9], who in describing existing hybridizations observed in local search approaches that they are strengthened by the use of exact algorithms. Infect they concentrated on integrating rather obvious union like preprocessing.

However, in metaheuristics algorithms will be considered. We present this in figure 1.1 below.

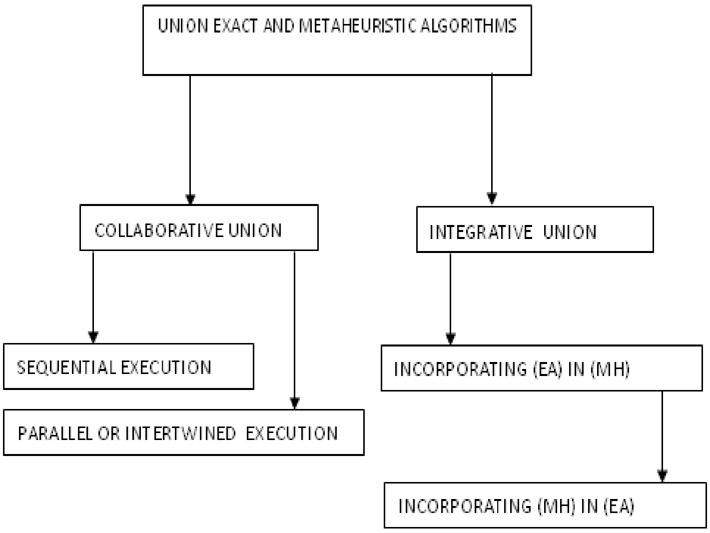

Figure 1.1

Definition 1.1 Collaborative Union

By this we imply that the algorithms exchange information relating to them, however they are NOT part of each other. The union can be formed sequentially, intertwine or in parallel mode.

Definition 1.2 Integrative Union

This means that one algorithm is a subordinate embedded component of the other algorithm. By this we declare clearly that there is a distinguished master algorithm, which can be either of the two and at last one integrated slaves.

Definition 1.3 Asynchronous Teams (A-teams)

An A-teams is a problem solving architecture consisting of a collection of agents and memories connected onto a strongly

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cyclic directed network. Each agent is an optimization algorithm and can work on the target problem, on a relaxation, super class or subclass of the problem.

The ensuring sections of this paper will focus on: Section two on the various ways of generating unions of the EA and MH, within the collaborative fold; section thre3e examines different ways of combining EA and MH under the Integrative approach and section four draws a conclusion and possible future work.

Formation of (partial or total) union of exact algorithms (ea) and Metaheuristic algorithms (MH)

As mentioned in the previous section, there are two main approaches of generating unions of (EA) and (MH), viz the collaborative and the integrative approaches. The aim behind a union may be better heuristic results. In the light of this, we examine the following approaches.

Collaborative Approach

In the collaborative approach, we examine the sequential, and parallel or intertwined mode of execution. All the algorithms and mode of formation discussed here are top-level unions of (MH) and (EA) techniques.

Sequential mode formation or execution

Since collaborative entails preprocessing, either the exact technique is executed as a pattern of preprocessing before meteheuristics or vice-versa. It is difficult to say if the first technique is chosen as initialization of the second or the second is a post processing of the solution(s) generated by the first.

In an attempt to solve a production-line scheduling problem Clements et al [5] proposed a column generation approach. First, the squeaky wheel optimization (SWO), heuristic is used to generate feasible solutions (each feasible solution of the problem consists of a line – schedule for each production line) to the problem. As a SWO, it uses a greedy algorithm to construct a solution which is thereafter analyzed to obtain the problematic compoenets. These components are assigned higher priorities in the greedy algorithm and the process restarts until a termination condition is encountered. In order to generate a set of diverse solutions, SWO is called several times at random. During the second phase of the solution process, the line-schedules in these solutions are used as columns of a set partitioning formulation for the problems; which is then solved using MINTO. The process described above always provides a solution which is at least as good as but usually better than the best solution yielded by SWP. However, SWO often performs better than a tabu-search algorithm.

Another instance of forming unions of (EA) and (MH_2) was discussed by Klau, et al [14], when a memetic algorithm was combined with integer programming (IP) to heuristically solve the prize collecting Steiner tree problem. The proposed algorithmic framework comprise of three parts viz, the extensive preprocessing, a memetic algorithm and an exact branch-and-cut algorithm applied as post-optimization procedure to the merged final solutions of the memetic algorithm.

Similarly Plateau et al [19] formed a union of interior point methods and (MH) for point method with early termination. By rounding and applying several different ascent population for a path-relinking (scatter search) algorithm. From literature, extensive computational experiments performed on standard multi-constrained knapsack benchmark instances showed results promising research direction.

Furthermore, by relaxing the original problem, we can solve it optimality and the ensuing results repaired to act as a promising starting point for a subsequent (MH). For instance in solving LP, relaxation is often used for this purpose, with only a simply rounding scheme needed. Feltl and Raidl [21] solve the generalized assignment problem using a hybrid Genetic Algorithm (GA). We recall that "(GA), which borrowed the ideas of Darwinian principle of nature selection, is a powerful global search and optimization technique. It is coined as" survival of the fittest" its basic idea is population evolution generation by generation. This begins from an initial population and the offspring of the current population are generated by applying three basic evolution operators-selection, crossover and mutation on the current population with different probability based on the fullness of each individual.

This process is repeated on the new generation until some satisfied individuals or population appears." The LP-relaxation of the problem is solved using CPLEX; its solution is applied in a randomized rounding procedure to generate population of promising integral solutions. However, three solutions are often infeasible, hence randomized repair and improvement operators are applied in addition to yield even more and more meaningful initial population for (GA). Results from computational experiments have so far been effective.

Another sequential mode of generating union of (B&B) and (GA) is discussed by nagar, et al [13]. In this a two-machine floechart scheduling problem is considered and solution candidates were taken as permutations of jobs. By this, prior to the running of the (GA) (B&B) is execute, down to a predetermined depth K and suitable bounds are calculated and recorded at each node of the explicitly stored B&B tree. While executing the (GA), the partial solutions up to the Kth position are transformed onto the correct tree node. If the suitable bounds calculate during the B&B execution indicate that no path below this node can lead to an optimal solution, then the permutation is subjected to a mutation operator that has been specifically designed to change the early part of the permutation in a favourable way.

In an attempt to solve a job-shop scheduling problem, Tamura, et al [23] created the union of IP and GA and started from its IP formation. For each variable we take the range of possible values and partition it into a set of subranges later indexed. In engaging the GA, the chromosomes of the GA are defined such that each position represents a variable and its value corresponds to the index of one of the subranges. We check the fitness of the chromosomes using the Lagrangian relaxation to obtain a bound on the optimal solution subject to the constraints that the values of the variables fall within the correct ranges. On the final note of the GA an exhaustive search of the region identified as the most promising is performed to generate the final solution.

Parallel or Intertwined Union or Execution

This is another way of generating unions or combinations of the exact and heuristic algorithms. It is quite different from the sequential approach, since it is not strictly generated as the sequential mode. It is generated or executed in a parallel technique; however such resultant techniques are less frequent. One fantastic design fondly associated with the asynchronous teams (A-Teams) for this task, was presented by Talukdar, et al [22]. As defined and explained in definition (1.3), the main thrust of the A-Teams is having these agents work asynchronously and autonomously on a set of shared memories. The memories consist of trial solutions for some problems (such as mentioned earlier), while the action of an agent consists of modifying the memory:- by adding a solutions, deleting a solution, or altering a solution. There is no doubt that the A-Teams have been successfully applied in a variety of combinatorial optimization problems, see [22].

A similar approach that comprises of multi-agents and meant for accomplishing cooperation between search systems with different search paradigms was proposed by Denzinmger and Offerman [7]. Denzinger and offerman method known as TECHS (Teams for Cooperative Heterogeneous Search) approach consists of teams of one or more agents adopting the same search paradigm. The link between the agents is controlled by so-called send-and receive-referees, with the intention to filter the exchanged data. However, how can we employ the services of TECHS in combining a GA and a B&B based system for job-shop scheduling?

Having seen what TECHS entails, the GA and B&B agents exchange only positive information (solutions), whereas the B&B agents can exchange negative information (closed sub-trees). Cooperation results emanating from computational experiment indicate that in finding better solutions given a fixed time-limit and in finding solutions comparable to the best individual system consume lee time.

Combining ea (et) and MH: integrative approach

A number of techniques exist in combining EA and MH under this approach; among these are incorporating exact algorithms in mateheuristics (MH) incorporating metaheuristics in exact algorithms. Within the first group, we observe that we can exactly solve relaxed problems and the usefulness of the solutions obtained is quite tremendous. This is besides exploiting them to generate promising initial solutions for a subsequent algorithm which can be of great value for heuristically guiding neighbourhood search recombination or one of the basic evolution operators.

Searching neigbourhoods in local search based on MH using exact algorithm is another way of combining EA & MH. In this approach, if neighourhoods are well chosen, they can be relatively large but nevertheless still yield an efficient search for the best neighourhood. These approaches are called VeryLarge Scale neighourhood (VLSN) search, Burke et al [3] examined an effective local variable neighourhood search heuristic for the asymmetric traveling salesman problem in which they have embedded and EA in the local search part called Hyper Opt, to assist in exhaustively search relating large promising regions of the solution space. They proposed a hybrid of HyperOpt and 3-Opt which take advantages of both approaches and gain better tours overall. Certain concepts become indispensable in the discussion of some methods; as a result the following definition is in order.

Definition 1.4 Independent within a Dynasearch Environment

This means that the independent moves do not interfere with each other such that dynamic programming can be best used to find the best combination of independent moves.

Dynasearch is a technique in which exponentially large neighourhoods are searched and the neigbourhood where the search is conducted consist of all possible combinations of mutually independent simple search steps and one Dynasearch move comprises of a set of independent moves which are executed in parallel in a single local search iteration. It should be noted that Dynasearch are restricted to problems where single search steps are independent. This has reduced its applicability so far.

Still within searching large neighourhoods, Purchinger, et al describe a combined GA and B&B approach for solving real world glass cutting problem. In this, the GA employs an order-based representation which is decoded using a greedy heuristic. Thereafter the B&B algorithm is applied proportionately to enhance the decoding phase of GA by generating locally optimal subpatterns. Reported results indicate that this method seldomly solve subpatterns to optimality, and may increase the overall solution quality.

We can also combine EA and MH by merging solutions. In this, subspaces defined by the merged attributes of two or more solutions can, like the neighbourhoods of single solutions, also be searched by exact techniques. An approach where merging is iteratively carried out within MH is presented below.

Marino et al [6], discussed an approach where a GA is combined with an exact method for the linear Assignment Problem (LAP) to solve the graph colouring problem. The Lap algorithm is infused into the crossover operator and through this generates the optimal permutation of colours within a cluster of nodes. In doing this, we try to avoid the offspring to be less fit than its parents. This algorithm has only produced comparable results with other approaches; however we can say that solving Lap through the crossover operator stronger improves the performance of the GA compared to the GA using crossover without Lap.

Similarly it is possible to incorporate MH in Ea in a number of ways. We shall discuss only two. Heuristic and MH are seldomly used to obtain bounds and incumbent selection in B&B approaches. For instance, Woodruff describes a chunking-based selection strategy to find at each node of the B&B tree whether or not reactive tabu search is called in order to eventually obtain a better incumbent solution. The strategy takes into cognizance the distance between the current nodes already examined by MH in order to bias the selection toward distant points. There are significant improvements in B&B performance when this strategy is applied. We also observed that in branch-and-cut and branch-in-price algorithms, the dynamic separation of cutting-planes and the pricing of columns respectively is occasionally carried through heuristics combined with MH in order to speed up the whole optimal process.

Puchinger and Raidl [20] presented new integer Linear programming formulation for the 3-stage 2-dimensional bin packing problem. Through the presentation a branch-and-price algorithm was developed in which fast column generation is performed by applying a hierarchy of fur approaches:

- ✓A greedy heuristic,
- ✓An evolutionary algorithm
- ✓Solving a restricted form of the pricing problem using CPLEX
- ✓And finally solving the full pricing problem using CPLEX as Mixed IP solver.

Combining these four approaches in branch-and-price algorithm yields best result in relation with the average objective value, average run-time and number of instances solved to show optimality. Several other approaches for combining EA and MH for strategic guidance of EA search, applying the spirit of MH etc.

Conclusions

A concise overview of the various, presently existing ways of hybridizing exact and metaheuristics algorithms have been presented in this paper. In the course of this, we were able to distinguish two main approaches of generating the unions of EA and MH viz the collaborative approach and the integrative approach. While discussing in some details the techniques in these two main approaches, we cited instances where these unions generated were applied to solved problems and observed how successful they were. There is no doubt in saying that there exists huge advantage in these combinations, most especially when they complement each other. Suitable unions are highly promising and vividly exhibit high performance indictors in solution quality and run-time. However, we must remark here that some of these techniques are still in their infancy and so they need some further research for them to enhance better performance.

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