



## Connections between Extenics and Refined Neutrosophic Logic

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### ABSTRACT

The aim of this presentation is to connect Extenics with new fields of research, i.e. fuzzy logic and neutrosophic logic.

We show herein:

- How Extenics is connected to the 3-Valued Neutrosophic Logic,
- How Extenics is connected to the 4-Valued Neutrosophic Logic,
- How Extenics is connected to the n-Valued Neutrosophic Logic, when contradictions occurs.

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### Introduction

In this paper we present a short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of  $n$ -symbol or numerical valued logic, and the way they are connected to Prof. Cai Wen's Extenics Theory (1983). We show generalizations of 2-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz' 3-symbol valued logics or Belnap's 4-symbol valued logic to the most general  $n$ -symbol or numerical valued refined neutrosophic logic. Two classes of neutrosophic norm ( $n$ -norm) and neutrosophic conorm ( $n$ -conorm) are defined. Examples of applications of neutrosophic logic to physics are listed in the last section.

Similar generalizations can be done for  $n$ -Valued Refined Neutrosophic Set, and respectively  $n$ -Valued Refined Neutrosophic Probability in connections with Extenics.

### Two-Valued Logic

a) The Two Symbol-Valued Logic.

It is the Chinese philosophy: *Yin* and *Yang* (or Femininity and Masculinity) as contraries:



Fig 1. Ying and Yang

It is also the Classical or *Boolean Logic*, which has two symbol-values: truth  $T$  and falsity  $F$ .

b) The Two Numerical-Valued Logic.

It is also the Classical or *Boolean Logic*, which has two numerical-values: truth  $1$  and falsity  $0$ .

More general it is the *Fuzzy Logic*, where the truth ( $T$ ) and the falsity ( $F$ ) can be any numbers in  $[0, 1]$  such that  $T + F = 1$ .

Even more general,  $T$  and  $F$  can be subsets of  $[0, 1]$ .

### Three-Valued Logic

#### The Three Symbol-Valued Logics:

- i) *Lukasiewicz's Logic*: True, False, and Possible.
- ii) *Kleene's Logic*: True, False, Unknown (or Undefined).

iii) Chinese philosophy extended to: *Yin, Yang, and Neuter* (or Femininity, Masculinity, and Neutrality) – as in Neutrosophy. Neutrosophy philosophy was born from neutrality between various philosophies. Connected with Extenics (Prof. Cai Wen, 1983), and Paradoxism (F. Smarandache, 1980).

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>).

The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Such contradictions involves Extenics.

Neutrosophy is the base of all neutrosophics and it is used in engineering applications (especially for software and information fusion), medicine, military, airspace, cybernetics, physics.

**The Three Numerical-Valued Logic:**

- i) Kleene's Logic: True (1), False (0), Unknown (or Undefined) (1/2), and uses "min" for  $\wedge$ , "max" for  $\vee$ , and "1-" for negation.
- ii) More general is the *Neutrosophic Logic* [Smarandache, 1995], where the truth (T) and the falsity (F) and the indeterminacy (I) can be any numbers in [0, 1], then  $0 \leq T + I + F \leq 3$ .

More general: Truth (T), Falsity (F), and Indeterminacy (I) are standard or nonstandard subsets of the nonstandard interval ]-0, 1+[.

When  $t + f > 1$  we have conflict, hence Extenics.

**Four-Valued Logic**

a) The Four Symbol-Valued Logic

i) It is *Belnap's Logic*: True (T), False (F), Unknown (U), and Contradiction (C), where T, F, U, C are symbols, not numbers.

Now we have Extenics, thanks to C = contradiction.

Below is the Belnap's conjunction operator table:

$\cap$	F	U	C	T
F	F	F	F	F
U	F	U	F	U
C	F	F	C	C
T	F	U	C	T

**Table 1.**

Restricted to T, F, U, and to T, F, C, the Belnap connectives coincide with the connectives in Kleene's logic.

ii) Let G = Ignorance. We can also propose the following two 4-Symbol Valued Logics: (T, F, U, G), and (T, F, C, G).

iii) *Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics* [Smarandache, 1995].

Let  $T_A$  be truth in all possible worlds (according to Leibniz's definition);

$T_R$  be truth in at last one world but not in all worlds;

and similarly let  $I_A$  be indeterminacy in all possible worlds;

$I_R$  be indeterminacy in at last one world but not in all worlds;

also let  $F_A$  be falsity in all possible worlds;

$F_R$  be falsity in at last one world but not in all worlds;

Then we can form several Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics

just taking combinations of the symbols  $T_A, T_R, I_A, I_R, F_A,$  and  $F_R$ .

As particular cases, very interesting would be to study the Absolute-Relative 4-Symbol Valued Logic ( $T_A, T_R, F_A, F_R$ ), as well as the Absolute-Relative 6-Symbol Valued Logic ( $T_A, T_R, I_A, I_R, F_A, F_R$ ).

b) *Four Numerical-Valued Neutrosophic Logic*: Indeterminacy  $I$  is refined (split) as  $U =$  Unknown, and  $C =$  contradiction.

$T, F, U, C$  are subsets of  $[0, 1]$ , instead of symbols;

This logic generalizes Belnap's logic since one gets a degree of truth, a degree of falsity, a degree of unknown, and a degree of contradiction.

Since  $C = T \wedge F$ , this logic involves the Extenics.

### Five-Valued Logic

a) *Five Symbol-Valued Neutrosophic Logic* [Smarandache, 1995]:

Indeterminacy  $I$  is refined (split) as  $U =$  Unknown,  $C =$  contradiction, and  $G =$  ignorance; where the symbols represent:

$T =$  truth;

$F =$  falsity;

$U =$  neither  $T$  nor  $F$  (undefined);

$C = T \wedge F$ , which involves the Extenics;

$G = T \vee F$ .

b) If  $T, F, U, C, G$  are subsets of  $[0, 1]$  then we get: a *Five Numerical-Valued Neutrosophic Logic*.

### Seven-Valued Logic

a) *Seven Symbol-Valued Neutrosophic Logic* [Smarandache, 1995]:

$I$  is refined (split) as  $U, C, G$ , but  $T$  also is refined as  $T_A =$  absolute truth and  $T_R =$  relative truth, and  $F$  is refined as  $F_A =$  absolute falsity and  $F_R =$  relative falsity. Where:

$U =$  neither ( $T_A$  or  $T_R$ ) nor ( $F_A$  or  $F_R$ ) (i.e. undefined);

$C = (T_A \text{ or } T_R) \wedge (F_A \text{ or } F_R)$  (i.e. Contradiction), which involves the Extenics;

$G = (T_A \text{ or } T_R) \vee (F_A \text{ or } F_R)$  (i.e. Ignorance).

All are symbols.

b) But if  $T_A, T_R, F_A, F_R, U, C, G$  are subsets of  $[0, 1]$ , then we get a *Seven Numerical-Valued Neutrosophic Logic*.

### n-Valued Logic

a) *The n-Symbol-Valued Refined Neutrosophic Logic* [Smarandache, 1995].

In general:

$T$  can be split into many types of truths:  $T_1, T_2, \dots, T_p$ , and  $I$  into many types of indeterminacies:  $I_1, I_2, \dots, I_r$ , and  $F$  into many types of falsities:  $F_1, F_2, \dots, F_s$ , where all  $p, r, s \geq 1$  are integers, and  $p + r + s = n$ .

All subcomponents  $T_j, I_k, F_l$  are symbols for  $j \in \{1, 2, \dots, p\}$ ,  $k \in \{1, 2, \dots, r\}$ , and  $l \in \{1, 2, \dots, s\}$ .

If at least one  $I_k = T_j \wedge F_l =$  contradiction, we get again the Extenics.

b) *The n-Numerical-Valued Refined Neutrosophic Logic*.

In the same way, but all subcomponents  $T_j, I_k, F_l$  are not symbols, but subsets of  $[0, 1]$ , for all

$j \in \{1, 2, \dots, p\}$ , all  $k \in \{1, 2, \dots, r\}$ , and all  $l \in \{1, 2, \dots, s\}$ .

If all sources of information that separately provide neutrosophic values for a specific subcomponent are independent sources, then in the general case we consider that each of the subcomponents  $T_j, I_k, F_l$  is independent with respect to the others and it is in the non-standard set  $J^0, I^+J$ . Therefore per total we have for crisp neutrosophic value subcomponents  $T_j, I_k, F_l$  that:

$$-0 \leq \sum_{j=1}^p T_j + \sum_{k=1}^r I_k + \sum_{l=1}^s F_l \leq n^+ \tag{1}$$

where of course  $n = p + r + s$  as above.

If there are some dependent sources (or respectively some dependent subcomponents), we can treat those dependent subcomponents together. For example, if  $T_2$  and  $I_3$  are dependent, we put them together as  $-0 \leq T_2 + I_3 \leq I^+$ .

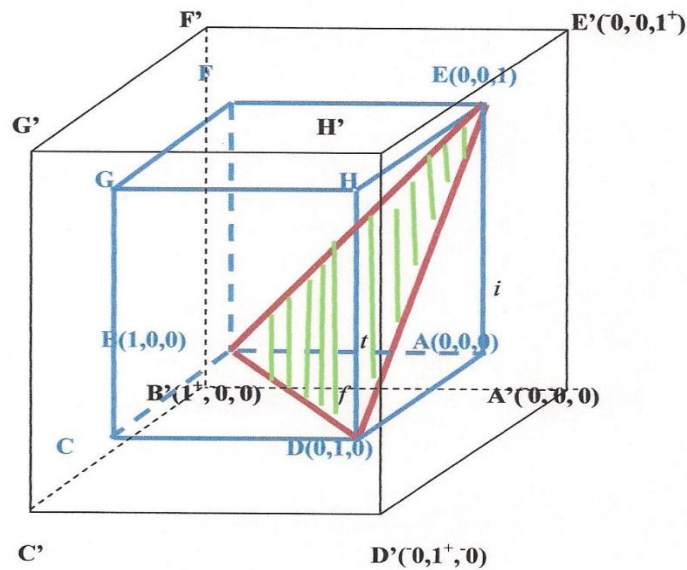
The non-standard unit interval  $]0, 1^+[$ , used to make a distinction between *absolute* and *relative* truth/indeterminacy/falsehood in philosophical applications, is replaced for simplicity with the standard (classical) unit interval  $[0, 1]$  for technical applications.

For at least one  $I_k = T_j \wedge F_l = \text{contradiction}$ , we get again the **Extenics**.

**Neutrosophic Cube and its Extenics Part**

The most important distinction between IFS and NS is shown in the below **Neutrosophic Cube**  $A'B'C'D'E'F'G'H'$  introduced by J. Dezert in 2002.

Because in technical applications only the classical interval is used as range for the neutrosophic parameters, we call the cube the **technical neutrosophic cube** and its extension the **neutrosophic cube** (or **absolute neutrosophic cube**), used in the fields where we need to differentiate between *absolute* and *relative* (as in philosophy) notions.



**Fig. 2. Neutrosophic Cube**

Let's consider a 3D-Cartesian system of coordinates, where  $t$  is the truth axis with value range in  $]0, 1^+[$ ,  $i$  is the false axis with value range in  $]0, 1^+[$ , and similarly  $f$  is the indeterminate axis with value range in  $]0, 1^+[$ .

We now divide the technical neutrosophic cube  $ABCDEFGH$  into three disjoint regions:

- 1) The equilateral triangle  $BDE$ , whose sides are equal to  $\sqrt{2}$  which represents the geometrical locus of the points whose sum of the coordinates is 1.
- If a point  $Q$  is situated on the sides of the triangle  $BDE$  or inside of it, then  $tQ+iQ+fQ=1$  as in Atanassov-intuitionistic fuzzy set (A-IFS).
- 2) The pyramid  $EABD$  {situated in the right side of the triangle  $EBD$ , including its faces triangle  $ABD$ (base), triangle  $EBA$ , and triangle  $EDA$  (lateral faces), but excluding its face: triangle  $BDE$  } is the locus of the points whose sum of coordinates is less than 1 (Incomplete Logic).
- 3) In the left side of triangle  $BDE$  in the cube there is the solid  $EFGCDEBD$  ( excluding triangle  $BDE$ ) which is the locus of points whose sum of their coordinates is greater than 1 as in the paraconsistent logic. This is the **Extenics** part.

It is possible to get the **sum of coordinates strictly less than 1** (in Incomplete information), or **strictly greater than 1** (in **contradictory Extenics**). For example:

We have a source which is capable to find only the degree of membership of an element; but it is unable to find the degree of non-membership;

Another source which is capable to find only the degree of non-membership of an element; Or a source which only computes the indeterminacy.

Thus, when we put the results together of these sources, it is possible that their sum is not 1, but smaller (Incomplete) or greater (Extenics).

#### Example of Extenics in 3-Valued Neutrosophic Logic

About a proposition P, the first source of information provides the truth-value  $T=0.8$ .

Second source of information provides the false-value  $F=0.7$ .

Third source of information provides the indeterminacy-value  $I=0.2$ .

Hence  $NL_3(P) = (0.8, 0.2, 0.7)$ .

Got Extenics, since Contradiction:  $T + F = 0.8 + 0.7 > 1$ .

Can remove Contradiction by normalization:

$NL(P) = (0.47, 0.12, 0.41)$ ; now  $T+F \leq 1$ .

#### Example of Extenics in 4-Valued Neutrosophic Logic

About a proposition Q, the first source of information provides the truth-value  $T=0.4$ , second source provides the false-value  $F=0.3$ , third source provides the undefined-value  $U=0.1$ , fourth source provides the contradiction-value  $C=0.2$ .

Hence  $NL_4(Q) = (0.4, 0.1, 0.2, 0.3)$ .

Got Extenics, since Contradiction  $C = 0.2 > 0$ .

Since  $C = T \wedge F$ , we can remove it by transferring its value 0.2 to T and F (since T and F were involved in the conflict) proportionally w.r.t. their values 0.4, 0.3:

$xT/0.4 = yF/0.3 = 0.2/(0.4+0.3)$ , whence  $xT=0.11$ ,  $yF=0.09$ .

Thus  $T=0.4+0.11=0.51$ ,  $F=0.3+0.09=0.39$ ,  $U=0.1$ ,  $C=0$ .

#### Conclusion

Many types of logics have been presented above related with Extenics. Examples of Neutrosophic Cube and its Extenics part, and Extenics in 3-Valued and 4-Valued Neutrosophic Logics are given.

Similar generalizations are done for **n-Valued Refined Neutrosophic Set**, and respectively **n-Valued Refined Neutrosophic Probability** in connections with Extenics.

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