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Advances in Pure Mathematics

Elixir Adv. Pure Math. 65 (2013) 19536-19538

ISSN: 2229-712X

Fuzzy Generalized Super Closed Sets

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ARTICLE INFO

Article history:

Received: 21 October 2013;

Received in revised form:

25 November 2013;

Accepted: 2 December 2013;

Keywords

Fuzzy topology,

Fuzzy super closure,

Fuzzy super interior,

Fuzzy super closed set,

Fuzzy super open set,

Fuzzy super generalized closed set.

ABSTRACT

In this paper we introduced the concept of fuzzy g- super closed and explore various properties fuzzy topological space.

Introduction

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha \in \Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y)=\beta$ for y=x and x(y) =0 for $y \neq x$, $\beta \in [0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x \in X$ such that $A(x) + B(x) > 1$.$A \le B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1[5]:- Let (X,τ) fuzzy topological space and $A \subseteq X$ then

1. Fuzzy Super closure $scl(A)=\{x \in X:cl(U) \cap A \neq \phi\}$
2. Fuzzy Super interior $sint(A) =\{x \in X:cl(U) \le A \neq \phi\}$

Definition 1.2[5]: -A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) Fuzzy super closed if $scl(A) \le A$.

(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy super closed.

Remark 1.3[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy super closed.

Definition 1.5[3,8,9,10, 11]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:

1. fuzzy g- super closed if $cl(A) \le G$ whenever $A \le G$ and G is super open.

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2. fuzzy g- super open if its complement 1-A is fuzzy g- super closed.

Definition 1.8. [3,8,9,10, 11]:- A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p qA$ iff p + A(x) > 1. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by $A_q B$ iff there exists x $\in$ X such that A(x) + B(x) > 1. If A and B are not quasi-coincident then we write $A_q B$. Note that A $\leq$ B , Aq(1−B).

Fuzzy g-super Closed Sets.

Definition 2.1.: A fuzzy set A of a fuzzy topological space (X, 'τ) is called fuzzy generalized super closed (fuzzy g- super closed) if Cl(A) $\leq$ O whenever A$\leq$ O and O is fuzzy super open.

Remark 2.1. : Every fuzzy closed set is fuzzy g- super closed but its converse may not be true. For,

Example 2.1. : Let X {a,b}and A and U be defined as follows:

A(a) = 0.3, A(b) = 0.2; UQ) = 0.5, U(b) 0.7; Let τ = {ϕ, U, X) be a fuzzy topology on X. Then A is fuzzy g- super closed but not fuzzy super closed.

Theorem 2.1. : If A and B are fuzzy g- super closed in a fuzzy topological space (X, τ) then A $\cup$ B is fuzzy g- super closed.

Proof: Let O be a fuzzy open set in X, such that A $\cup$ B $\leq$O then A $\leq$O and B $\leq$O so CI(A) $\leq$ O and Cl(B) $\leq$O. Therefore Cl(A) $\cup$ Cl(B) = Cl(A U B) $\leq$O. Hence A $\cup$ B is fuzzy g-super closed.

Remark 2.2.: The intersection of two fuzzy g- super closed sets in a fuzzy topological space (X, τ) may not be fuzzy g- super closed. For,

Example 2.2.: Let X = {a ,b} and U, A and B be defined as follows U(a) = 0.7, U(b) = 0.6; A(a) = 0.6, A(b) = 0.7; B(a) = 0.8, B(b) = 0.5; Let τ ={ϕ, U, X}, then A and B are fuzzy g- super closed in (X, τ) but A ∩ B is not fuzzy g- super closed.

Theorem 2.2.: Let A $\leq$ B$\leq$ Cl(B) and A is fuzzy g- super closed in' a fuzzy topological (X, τ). Then B is fuzzy g- super closed.

Proof. : Let O be a fuzzy super open set such that B$\leq$O then A$\leq$ O and since A is fuzzy g-super closed CI(A) $\leq$ O. Now B $\leq$CI(A) $\Rightarrow$ Cl(B) $\leq$ Cl(A) $\leq$O. Consequently B is fuzzy g- super closed.

Definition 2.2.: A fuzzy set A of a fuzzy topological space (X; -τ) is called fuzzy g- super open iff A^c is fuzzy g.- super closed.

Remark 2.3.: Every fuzzy open set is fuzzy g- super open. The converse may not be true.

Theorem 2.3.: A fuzzy set A of fuzzy topological space (X, τ) is fuzzy g- super open iff F $\leq$ lnt(A) whenever F is fuzzy closed and F $\subset$ A.

Theorem 2.4. : Let A and B are Q-separated fuzzy g- super open subsets of a fuzzy topological space (X, τ) then A $\cup$B is fuzzy g- super open.

Proof.: Let F be a fuzzy super closed subset of A $\cup$ B. Then F ∩ Cl(A) $\leq$ (A $\cup$ B) ∩ Cl(A)=(A ∩ Cl) (A)) $\cup$ (B ∩ Cl(A)) $\leq$int (A). Similarly F ∩ Cl(B) $\leq$ int (B). Now F ∩ (A $\cup$ B) $\leq$ (F ∩ Cl (A)) $\cup$ (F ∩ Cl(B)) $\leq$ int (A) $\cup$ int (B) $\leq$int (A $\cup$ B). Hence F$\leq$ int(A $\cup$ B) and by theorem (2.2) A $\cup$B is fuzzy g- super open.

Theorem 2.5.: Let A and B be two fuzzy g- super closed sets of a fuzzy topological space (X, τ) and suppose that A^c and B^c are Q-separated, then A ∩ B is fuzzy g- super closed.

Theorem 2.6. : Let A be a fuzzy g- super open subset of a fuzzy topological space (X, τ) and int(A) $\leq$ B $\leq$ A then B is fuzzy g- super open.

Proof. : Since $A^c \subseteq B^c \subseteq Cl(A^c)$ and A^c is fuzzy g- super closed it follows that B^c is fuzzy g- super closed by theorem (2.2), thus B is fuzzy g- super open.

Theorem 2.7.: Lét(Y, τ_Y) be a subspace of a fuzzy topological space (X, τ) and A be a fuzzy set in Y. If A is fuzzy g- super closed in X then A is fuzzy g- super closed in Y.

Proof. : Let $A \leq O_Y$, where O_Y, is fuzzy super open in Y. Then there exists a fuzzy super open set O in X such that OY = O∩ Y. Therefore A ≤O and since A is fuzzy g- super closed in X, Cl(A)≤O . It follows that CI_Y (A) = Cl(A) ∩ Y ≤ O ∩ Y = O_Y. Hence A is fuzzy g- super closed in Y.

Theorem 2.8.: Let (X, τ) be a fuzzy topological space and $\mathcal{F}$ be the family of all fuzzy super closed sets of X. Then $\tau = \mathcal{F}$ iff every fuzzy subset of X is fuzzy g- super closed.

Proof: Necessity.: Suppose that $\tau = \mathcal{F}$ and that A ⊂O ∈τ then Cl(A)≤ Cl(O) = O and A is fuzzy g- super closed.

Sufficiency. Suppose that every fuzzy subset of X is fuzzy g- super closed. Let O ∈τ then since O≤O and O is fuzzy g- super closed where C1(O) ⊆O and O∈ $\mathcal{F}$. Thus t⊆ $\mathcal{F}$ If T∈ $\mathcal{F}$ then T^c ∈ t≤ $\mathcal{F}$ and hence T∈ τ consequently $\mathcal{F}$ ⊆τ and $\tau = \mathcal{F}$.

Theorem 2.12. Let A be a fuzzy g- super closed set in a fuzzy topological space (X, τ) and f: (X, τ) — (Y, τ*) is fuzzy super continuous and fuzzy super closed then f(A) is fuzzy g- super closed in Y.

Proof: If f(A)≤ G where G is fuzzy super open in Y then A $\leq f^{-1}(G)$ and hence Cl(A) ≤ $f^{-1}(G)$. Thus f(Cl(A)) ≤G and f(Cl(A)) is a fuzzy super closed set. It follows that Cl(f(A))≤ Cl(f(Cl(A))) = f(Cl(A)) ≤G. Then Cl(f(A)) ≤ G and f(A) is fuzzy g- super closed.

References

[1] Azad K.K on fuzzy semi continuity, fuzzy Almost continuity and fuzzy weakly continuity , J. Math. Anal. Appl. 82(1981),14-32.

[2] Balchandran K, Sundram P and Maki H., on Generalized Super continuous Map in Topological Spaces , Mem. Fac. sci. Kochi Univ.(Math)12(1991),5-13.

[3] Chang C.L. fuzzy Topological Spaces J. Math Anal. Appl.24(1968),182-190.

[4] El-Shafei M. E. and Zakari A. semi-generalized continuous mappings in fuzzy topological spaces J. Egypt. Math. Soc.15(1)(2007) , 57{67.

[5] Mishra M.K. ,et all on " Fuzzy super closed set" Accepted.

[6] Mishra M.K. ,et all on " Fuzzy super continuity" (Accepted)

[7] Mishra M.K., Shukla M. "Fuzzy Regular Generalized Super Closed Set" IJSRP, Dec(2011).

[8] Mishra M.K., Thakur S.S. "Fuzzy w-continuous mappings" IJSRP, Dec(2011).

[9] Pu P. M. and Liu Y. M. Fuzzy Topology I: Neighborhood structure of a fuzzy point and More Smith Convergence, J. Math. Anal Appl.76(1980)571-599.

[10] Pushpaltha A. studies on Generalization of Mappings in Topological Spaces Ph.D. Thesis Bharathiyar University Coimbotore (2000).

[11] Sundram P. and M. Shekh John, On w-super closed sets in topology, Acta Cinica Indica 4(2000) 389-392.

[12] Tapi U. D ,Thakur S S and Rathore G.P.S. Fuzzy sg –super continuous mappings Applied sciences periodical (2001) ,133-137.

[13] Tapi U. D., Thakur S. S. and Rathore G.P.S. Fuzzy semi generalized super closed sets, Acta Cien. Indica 27 (M) (3) (2001), 313-316.

[14] Tapi U. D., Thakur S. S. and Rathore G.P.S. Fuzzy sg- irresolute mappings stud. Cert. Stii. Ser. Mat. Univ. Bacu (1999) (9) ,203-209.

[15] Thakur S.S. & Malviya R., Generalized super closed sets in fuzzy topology Math. Note 38(1995),137-140.

[16] Yalvac T.H. fuzzy sets and Function on fuzzy spaces, J. Math. Anal. App. 126(1987), 409-423.

[17] Yalvac T.H. Semi interior and semi closure of fuzzy sets,. J. math. Anal.Appl. 132 (1988) 356-364.

[18] Zadeh L.A. fuzzy sets, Inform and control 18 (1965), 338-353.