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# Study on Heat Transfer Area of a Plate-Fin Heat Exchanger with Wavy

Surfaces

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# ABSTRACT

Heat Exchangers are widely used in many industries, such as chemical, food, airconditioning system, petrochemical, oil, and thermal power plant. In the air-to-air and air-toliquid heat exchangers for air-side heat transfer applications, special surfaces are often employed to obtain high rates of heat transfer. One geometry that can be used to enhances heat exchanger performance is a sinusoidally curved wavy passage. Wavy channels are easy to fabricate, and can provide significant heat transfer enhancement in the appropriate Reynolds number regime. This article offers a new method for calculating total heat transfer area in a plate-fin heat exchanger with waviness surfaces. Kays and London presented some experimental data to determine heat transfer area in wavy surfaces, based on the ratio of total heat transfer area / total volume. One of the important weakness of the method presented by Kays and London is calculating total heat transfer area is a chain process and depends on the other thermal properties of the heat exchanger. So, existence of a direct method can be helpful and, of course, a strong tool in optimization process.

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#### Introduction

Fins are surface extensions widely used in different types of heat exchangers for increasing the rate of heat transfer among a solid surface and surrounding fluid. Often geometrically modified fins are incorporated, which beside increasing the surface area density of the heat exchanger, also improve the convection heat transfer coefficient. some examples of such enhanced surface compact cores include Offset-strip fins, Louvered fins, Wavy fins, Plain fins and Pin fins. Of these, wavy fins are particularly attractive for their simplicity of manufacture, potential for enhanced thermal-hydraulic performance and ease of usage in both plate-fin and tube-fin type exchangers[1].

The most voluminous body of work concerning flow through wavy passage is encompassed by a series of flow visualization and mass transfer experiments performed by Japanese Group Research headed by T.Nishimura [2]. He investigated the relationship among flow structure and mass transfer for fully-developed flow in a sinusoidally curved Wavy channel for laminar, transitional and turbulent flows. Nishimura [3] examined the occurrence and structure of longitudinal vortices in wavy channels, at relatively low Reynolds numbers  $(50 \le \text{Re} \le 500)$ . They observed that the developing flow becomes less stable as it proceeds downstream, and that flow can become three-dimensional for Reynolds numbers as low as 100.

Oyakawa [4] studied how heat transfer and fluid flow is effected by varying the height of a wavy sinusoidal passage. From these experiments, they obtained an optimum spacing for fully-developed flow using the chosen wall geometry. A study with similar goals conducted by Gschwind [5] using flow visualization and mass transfer experiments with air. Besides, demonstrate the qualitative effects of varying the Reynolds number and the channel spacing, they also provide a stability diagram that describes the exist ing range of longitudinal vortices.

Wang and Vanka [6] analyzed convective heat transfer for fully-developed flow in a periodic wavy passage for several Reynolds numbers. Patel [7-8] published a pair of papers dealing with flow through a channel with a single wavy wall. One paper dealt with steady laminar flow over a wall with six waives , and the other addressed turbulent flow over the same wall. For the laminar case, the pressure and friction coefficients are plotted over the course of the wavy wall. It should be noted, however, that these calculations are performed for a channel Reynolds number of 10760, a value that is considerably higher than the expected transition Reynolds number.

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Therefore, the results are not indicative of what would actually occur in an analogous physical situation. Garg and Maji [9-10] used a finite-difference methodology to solve the governing equations for steady laminar flow and heat transfer in a furrowed wavy channel. Calculations were performed using various wall amplitudes for  $100 \le \text{Re} \le 500$ . Both the developing and the fully-developed flow regions were analyzed. The local Nusselt number was observed to fluctuate 21 sinusoidally in the fully-developed region. Moreover, the Nusselt number increased with the Reynolds number, unlike what happens for laminar flow through a strait channel. Vajrave lu [11] solved the governing equations for fluid flow and heat transfer in a wavy passage by separating the solution into two parts: a mean part and a perturbed part. The mean part of the solution was found to correspond closely with that of plane Poiseuille flow, while the perturbed part of the solution represents the contribution from the waviness of the walls. Four different values of  $\phi$  were considered: 0°, 90°, 180°, 270°. The mean, perturbed, and total solutions were evaluated numerically for each configuration.

M.Quzzane and Z.Aidoun [12] conducted a numerical study on wavy fin and tube  $CO_2$  evaporator coil. They addressed the development of a mathematical model for design and analysis of  $air - CO_2$  wavy fin evaporators coils.  $CO_2$  flows inside horizontal tubes while air flows across the coil and over the fins, on the outside the tubes. C.Wang, W.Tao and Y.Du [13] investigated effect of waffle height on the air-side performance of wavy fin-and-tube heat exchangers under dehumidifying conditions. A total of 12 samples of heat exchangers, including eight having wavy fins and four having plain fins configurations were examined by them. The results showed that the effect of waffle height on the heat transfer enhancement ratio, compared to the plain-fin counterpart, is pronounced only for smaller fin pitch and larger waffle height, while its effect on the pressure drop is very pronounced throughout the test range, and is almost two times higher than in dry conditions.

Although many researchers have studied about the thermal performance of the wavy surface, information about the method of calculating total heat transfer area is sparse. Naturally, the effects of the total heat transfer area on thermal performance of the heat exchanger was remained unexplored. The main reason is that the only source of information for calculating heat transfer area in compact heat exchanger is a experimental graphs, which were presented by Kays and London. In this study, our objective is introducing a new method to calculate total heat transfer area. Unlike data presented by Kays and London this method is exact, and much more important is a direct method independence from calculating other thermal and geometrical properties of the heat exchanger. So, it can be used widely in the thermodynamical optimization (Minimizing entropy generation).

#### Mathematical Description

Heat transfer data are correlated based on the following dimensionless numbers:

Colburn factor: j=St.Pr <sup>2/3</sup>	(1)
Stanton number: $St = \frac{Nu}{Re.Pr}$	(2)

Nusselt number: 
$$Nu = \frac{h.D_h}{K}$$
 (3)

Prandtl number: 
$$Pr = \frac{C_p \cdot \mu}{K}$$
(4)

Reynolds number: 
$$Re = \frac{G.D_h}{\mu}$$
 (5)

Here, ε-NTU method was used for predicting the heat exchanger performance. The effectiveness of a heat exchanger when both fluids are unmixed is:

$$\varepsilon = 1 - \exp\left[\left(\frac{1}{C^*}\right) NTU^{0.22} \left\{ \exp\left[-C^*.NTU^{0.78}\right] - 1 \right\} \right]$$
(6)

Also, numbers of transfer units and heat capacity ratio are respectively,

$$NTU = \frac{UA_{tot}}{C_{min}}$$
(7)

$$C^* = \frac{C_{max}}{C_{min}}$$
(8)

The fin efficiency correlation for both streams is:

$$\eta_{fi} = \frac{Y_{o,fi}}{2(b_i + S_i)Lh} \frac{\tanh m_{fi}b_i + (2Y_{o,s} / Y_{o,f}) \tanh m_s S_i}{1 + (2Y_{o,s} / Y_{o,f}) \tanh m_s S_i \tanh m_{fi} b_i}$$
(9)

On the other hands, overall passage efficiency is:

$$\eta_{w,1} = 1 - \left(1 - \eta_{f,1}\right) \times \frac{S_{f,1}}{S_1} \tag{10}$$

So, now calculating heat transfer coefficient is possible by,

$$\frac{1}{U} = \frac{1}{h_1 \eta_{f,1}} + \frac{S_1}{h_2 \eta_{f,2} S_2}$$
(11)

 $A_{tot}$  is total heat transfer area, which can be computed from:

$$S_{\text{tot}} = \alpha_c \cdot \mathbf{V} + \alpha_h \cdot \mathbf{V} \tag{12}$$

Where  $\alpha$  is the ratio of the total surface on one side to the total surface on both sides.

$$\alpha_i = \frac{b_1}{b_1 + b_2 + 2a} \beta_i \tag{13}$$

In this formula,  $\beta$  is the ratio of total surface area to the total volume on one side of the heat exchanger. The mass velocity through the minimum frontal flow area is:

$$G_i = \frac{m_i}{A_{f,i}}$$
(14)

The heat transfer coefficient and pressure drop are:

$$h=j.G.C_{p}.Pr^{\frac{-2}{3}}$$
(15)

$$\Delta \mathbf{P}_{1} = \frac{\mathbf{G}_{1}^{2}}{2\rho_{\text{in},1}} \left[ \left(1 + \mathbf{K}_{\text{C},1} - \sigma_{1}^{2}\right) + 2\left(\frac{\rho_{\text{in},1}}{\rho_{\text{out},1}} - 1\right) + \left(\mathbf{f}_{1} \times \frac{\mathbf{S}_{1}}{\mathbf{A}_{1}} \times \frac{\rho_{\text{in},1}}{\rho_{\text{m},1}}\right) - \left(1 - \sigma_{1}^{2} - \mathbf{k}_{\text{e},1}\right) \times \frac{\rho_{\text{in},1}}{\rho_{\text{out},1}} \right]$$
(16)

Here, the Fanning friction factor, f, and Colburn factor, j, are extracted from Kays and London diagrams [14].



Figure 1. Schematic of Wavy plate-fin

The wavelength  $\lambda$  of the chevron pattern is the corrugation pitch as shown in Figure.(1). The amplitude of the corrugation is denoted as 2a, where a is the amplitude of the sinusoidal corrugation with the plate thickness  $\delta$ . Due to sinusoidal curve of wavy surfaces, the length of a sine curve is:

$$L_{\lambda} = \int_{0}^{\lambda} \sqrt{1 + \left(\frac{2\pi a}{\lambda}\right) \cos^{2}\left(\frac{2\pi x}{\lambda}\right)} dx$$
<sup>(17)</sup>



#### Figure.2. Plate-fin heat exchanger

If we assume that Fluid, 1 is hot stream, then the number of wavelength per plate " $N_{\lambda}$ " is :

$$N_{\lambda} = \frac{W}{\lambda}$$
(18)

Also, the number of channels for hot stream can be calculated from,

$$N_{c} = \frac{D-S}{2a}$$
(19)

So, based on the new method, the heat transfer area for hot and cold streams are respectively:

$$S_{hot} = 2.N_{\lambda}.N_{c}.L_{\lambda} \left[ b.N_{L} + a.(N_{L} + 2) \right]$$
<sup>(20)</sup>

$$S_{\text{cold}} = 2.N_{\lambda}.N_{\text{c}}.L_{\lambda} \left[ b.N_{\text{L}} + a.(N_{\text{L}} + 1) \right]$$
<sup>(21)</sup>

Here,  $N_L$  is the number of layers and b fin height. The equation of (5) is a elliptical integral, which its solution is:

$$L_{\lambda} = \int_{0}^{\lambda} \sqrt{1 + \left(\frac{2\pi a}{\lambda}\right)} \cos^{2}\left(\frac{2\pi x}{\lambda}\right)} dx = \frac{\sqrt{\left(\frac{2\pi a}{\lambda}\right) + 1}}{\left(\frac{2\pi}{\lambda}\right)} E\left(\left(\frac{2\pi x}{\lambda}\right)\right) \left(\frac{\left(\frac{2\pi a}{\lambda}\right)}{\left(\frac{2\pi a}{\lambda}\right) + 1}\right)$$
(22)

Terry.L Brown [15] offered a solution for incomplete elliptical integral of the first and second kind. Based on his research,

$$E(\phi|m) = \int_{0}^{\phi} \sqrt{1 - m\sin^{2}(\theta)} d\theta = \frac{2\phi}{\pi} E(m) + \sin(\phi) \cdot \cos(\phi) \cdot \left[\frac{1}{2}A_{2}m + \frac{1}{2.4}A_{4}m^{2} + \frac{1.3}{2.4.6}A_{6}m^{3} + ...\right]$$

$$A_{2} = \frac{1}{2} , A_{4} = \frac{3}{2.4} + \frac{1}{4}\sin^{2}(\phi) , A_{6} = \frac{3.5}{2.4.6} + \frac{5}{4.6}\sin^{2}(\phi) + \frac{1}{6}\sin^{4}(\phi)$$

$$(23)$$

Here, E(m) is the complete elliptical integral of second kind. However, A.Narayan [16] suggested a numerical approximation methods, based on series expansions. Based on his research a estimation of the incomplete elliptical integral of second type is:

$$E(\phi|m) = \int_0^{\phi} \sqrt{1 - m\sin^2(\theta)} d\theta = \int_{t_2}^{t_1} R(t) dt = R(t) \cdot (t_1 - t_2)$$
(25)

$$R(t) = \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{2.\sin\left(\frac{(t_1 - t_2)}{2}\right)}$$

# Method Validation

## Case Study

To validate new method a case study has been done. A plate-fin heat exchanger with wavy fins has been designed for heat recovery from exhaust gases in a Microturbine cycle. The operating conditions and fin properties are according to Table of (1) and (2) respectively.

Variables	Data
Allowable pressure drop for hot side (%)	6
Allowable pressure drop for hot side (%)	3
Outlet gas temperature(k)	694
Inlet gas temperature(k)	865
Outlet air temperature(k)	670
Inlet air temperature(k)	475
Outlet pressure from compressor(bar)	4
Outlet pressure from turbine(bar)	1.06
Gas mass flow rate (kg/s)	1.4676
Air mass flow rate(kg/s)	1.45

Table 1. Operating Conditions

Table	2.Fin	Geometric	Properties

	11.44-3/8 W	11.5-3/8 W	17.8-3/8 W
Fin Height(mm)	10.49	9.53	10.49
Hydraulic diameter(mm)	3.23	3.02	2.12
Fin thickness(mm)	0.152	0.254	0.152
Wavelength(mm)	9.53	9.53	9.53
Spacing(mm)	1.96	1.98	1.96
Double wave amplitude(mm)	2.21	2.20	1.42
Heat transfer area/ volume between plates	1152	1138	1686
Fin area/total area	0.847	0.822	0.892

# **Result and Discussion**

Two broad categories of problem specification are as follows. Given the core geometry, the flow rates, and the entering fluid temperatures. The main question in the sizing problem is that what is the size of the core, given the flow rates and their entering and leaving temperatures. These in turn establish the desired heat transfer rate and exchanger effectiveness. Here, it is assumed that not only are the performance characteristics completely established but also that the general type of heat exchanger has been selected, the flow arrangement has been chosen, and the heat transfer surface configurations for the two fluid sides have been s elected. The result of designing heat exchanger is presented in Table of (3). The note is that how calculating the dimensions of this heat exchanger is out of scope this paper(Refer to Design of Plate-Fin Heat Exchanger by M.Asadi and R.H.Khoshkhoo[17]).

	11.44-3/8 W	11.5-3/8 W	17.8-3/8 W
Height(mm)	1075	961	1075
Width(mm)	700	700	700
Depth(mm)	500	500	500
Total volume( m <sup>3</sup> )	0.375	0.336	0.375

Table 4.Vari	ables of	Heat	Transfer	Area (	Correl	lation
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	11.44-3/8 W		11.5-3/8 W		17.8-3/8 W	
	Air	Gas	Air	Gas	Air	Gas
$L_{\lambda}(mm)$	21.93	21.93	21.93	21.93	21.93	21.93
N <sub>λ</sub>	26.25	36.75	26.25	36.75	26.25	36.75
N <sub>c</sub>	315.85	225.35	317.28	226.37	489.50	349.25

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(26)

NL	51	50	51	50	51	50
b(mm)	10.49	10.49	9.25	9.25	10.49	10.49
S	194.54	190.51	172.32	168.75	301.50	295.26

In order to calculate heat transfer area Kays and London introduced a parameter which is the ratio of total heat transfer area to total volume. Figures of (3) and (4) have compared total heat transfer area for both fluids based on Kays and London research and this new method. As it is clear from these figures the error for hot stream is a little more than cold stream. The main reason is that the number of layers for cold stream is one more than hot. However, from Kays and London data there are equal value for both of fluids because the heat transfer area is only a function of the total volume and  $\alpha$  parameter (Equations of (12) and (13)). In the other words, the number of layers have not any effect on the heat transfer area. If be assumed that the number of layers for hot st ream is "N<sub>L</sub>", to transfer maximum available heat, the layer numbers for cold stream will be "N<sub>L</sub> +1". Here, the first and last layers are cold stream, and hot fluid flow between them. Therefore, it is reasonable that the heat transfer area for cold stream be a little more than hot, and this is what it is seen in this method. What is different for the first and last layers is temperature distribution and heat transfer coefficient.



Figure 3. Heat Transfer Area for Cold Stream



Figure 4. Heat Transfer Area for Hot Stream

Table 5. Comparing the error of the presented method with Kays and London Data

	11.44-3/8 W	11.5-3/8 W	17.8-3/8 W
Hot Stream	0.69%	7%	4%
Cold Stream	1.1%	9.4%	2.5%

One of the important application of the Equations of (20) and (21) is in the optimization process, where it does based on the entropy generation minimization. Bejan et.al, [18] presented some correlations to assign the rate of entropy generation in the heat exchanger with counter flow arrangement. Equation of (27) represent the number of entropy generation for this heat exchanger. In this

formula,  $T_1$  and  $T_2$  are inlet temperature fluid for both fluids. Optimum heat transfer area and heat exchanger dimensions can be calculated using minimizing this equation. Here, the heat exchanger efficiency is introduced by equs(6) through (8). As it is clear having a direct description from heat transfer area in order to use in the equation (7) is necessary, and presented method and correlation can be a strong tool in the optimization process.

$$N_{s} = (1-\varepsilon)\frac{(T_{1}-T_{2})^{2}}{T_{1}T_{2}} + \frac{R}{C_{p}} \left[ \left(\frac{\Delta P}{P}\right)_{1} + \left(\frac{\Delta P}{P}\right)_{2} \right]$$
(27)

Finally, the important note to calculate incomplete elliptical integral is that :

$$E\left(360^{\circ} \left| K\right.\right) = 4.E\left(90^{\circ} \left| K\right.\right)$$
<sup>(28)</sup>

Table of (6) gives required information to calculate this integral.

Table 6. Function of Incomplete Elliptical Integral Versus K

K	E(90 K)	Κ	E(90 K)
0.05	1.569	0.55	1.441
0.1	1.566	0.60	1.418
0.15	1.561	0.65	1.388
0.2	1.554	0.70	1.355
0.25	1.545	0.75	1.318
0.30	1.534	0.80	1.276
0.35	1.521	0.85	1.228
0.40	1.505	0.90	1.171
0.45	1.487	0.95	1.102
0.50	1.467	1.00	1.00

#### Conclusion

In the air-to-air or air-to-liquid heat exchangers, overall heat transfer coefficient is small. One of the best solution to increase heat transfer coefficient is using extended surfaces. Wavy fins are one of the these surfaces, which are popular among other types of fins such as Louvered, Strip, and Plain fins, mainly because of their thermal performance. Kays and London presented experimental data to calculate total heat transfer area, but there are two problems. Firstly, this is an approximate method, and secondly ,calculating total heat transfer area is a chain process and when it is possible that we calculated other thermal and geometrical properties of the heat exchanger. On the other hand, existence of a direct method can be a strong tool in optimization process. Here, we introduced a new method to assign total heat transfer area and then compared its result with the experimental data presented by Kays and London. The results showed high level of accuracy.

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