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An Inventory Model for Non instantaneous Deteriorating items under the effect of Price with Partial Backlogging and shortage

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ABSTRACT

This paper presents an inventory model for deteriorating item having price dependent demand and partial backlogging. The study of inventory system with deteriorating item has a special significance as it influences the profit and revenue of a retailer. It has been observed that some time item does not start to deteriorates as it comes to the storage so we have consider non instantaneous deteriorating item and its rate of deterioration varies linearly with time. The proposed model has been developed to determine optimal order quantity and optimal selling price for which profit can be maximized. The model is solved analytically by maximizing the total average profit. Sensitiveness of the parameter has been tested by sensitivity analysis and numerical example has been illustrated to explore the solution of the model.

Introduction

It's important to study the inventory model for deteriorating item because now a day it is quite difficult to sustain in a market in a competitive era of business. It becomes a necessity to make the proper strategy to maintain the inventory economically. Ghare and Schrader [1] were the first to describe the optimal ordering policy for exponentially decaying inventory. Deb and chaudhari [2] derived inventory model with time dependent deterioration rate. Min et al.[3] established a deteriorating inventory model considering stock dependent demand. Mandol et al.[4] worked on a single period inventory model of deteriorating item in which they considered that demand is function of time. In all these literature researchers assumed that the deterioration of the item in inventory starts from the instant of their arrival but there are some items that do not starts to deteriorate as it come to stocking point these items are named as non instantaneous deteriorating items. Wu et. al.[5] established an optimal policy for non instantaneous deteriorating items with partial backlogging considering stock dependent demand. Yang et.al.[6] considered non instantaneous deteriorating item with price dependent demand and partial backlogging. Chang and Lin [7] have studied non instantaneous deteriorating item with stock dependent consumption rate under inflation.

Furthermore, when the shortage occurs, it is generally assumed that demand is either completely backlogged or completely lost. But practically some customers are willing to wait for backorder and other would turn to buy for other retailer this situation is termed as partial backlogging. There are some researcher who work in this direction, for instance, Chang et.al[8] discussed a inventory model for trapezoidal type of demand with partial backlogging. Abolfazl mirzazadeh [9] developed a mathematical model for optimal production for an inventory control system of deteriorating item under time varying and stochastic inflation environment considering the partial backlogging rate is exponentially decreasing function of waiting time. Ahmed et al.[10] derived the EOQ for an inventory model with ramp type demand rate and partial backlogging they have developed a new method for finding the EOQ policy.

In most of the cases demand has been considered as a function of time and other one as a function of stock level but it is practically noticed in supermarket that the demand rate is usually influenced by the price of the product. The present paper has been developed considering the demand rate is linearly and exponentially decreasing function of price. We have considered the item is non instantaneous and also assumed that the backlogging rate is inversely proportional to waiting time of next replenishment. We have also carried out the sensitivity analysis and some special cases which include model with no shortage, completely backlogged, instantaneous deterioration etc.The most important purpose of modeling the inventory is to maximize the profit of the retailer and another one is to facilitate the uninterrupted service to the consumer or manufacturing division. Numerical examples also have been

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illustrated which are solved with the help of mathematica software using Newton Rapshon method to solve the system of non linear equation.

Notations and Assumptions

The following fundamental notations and assumptions are used to derive the model.

A	Ordering cost per period
c_h	Holding cost per unit per unit time
c_d	Cost of deteriorated unit
c_s	Shortage cost per unit per unit time
c_l	Lost sale cost per unit
t_d	Period of time during which there is no deterioration
t_1	Time when inventory level reaches to zero
T	The replenishment cycle
$PF\left(t_1,T,p\right)$	Average profit of the system
I_0	Maximum inventory level
$I_1\left(t\right)$	Inventory level at any time t, $0< t< t_1$
$I_2\left(t\right)$	Inventory level at any time t, $t_1 \le t \le T$
S	Shortage level
Q	Ordering quantity

(a) A fraction of on hand inventory deteriorates per unit time and deteriorated amount of item are not replaced.

(b) The replenishment is infinite and lead time is zero.

(c) The demand rate $D\left(p\right)$ defined as decreasing function of price p .

(d)The deterioration rate is defined as $\theta(t)=\alpha t,$ where $\alpha << 1$

(f) Shortage are allowed and partially backlogged. Backlogging rate is inversely proportional to waiting time that is longer the waiting time, smaller the backlogging rate and vice versa and it is defined as $B(T-t)=\dfrac{1}{1+\delta(T-t)}$,where $\delta>0$ and $T-t$ is waiting time up to next replenishment.

(e) It is assumed that the item starts to deteriorate after a certain period of time t_d that is items are non instantaneous deteriorating item.

Mathematical model and its solution

We consider the non instantaneous deteriorating item with exponentially increasing demand with partial backlogging .The variation of inventory level during the given cycle is depicted in fig.1. Initially I_0 amount of item are arrived at time $t=0$ which is the maximum level of inventory during each cycle. Inventory level start to decreases up to t_d due to demand only and during the interval $[t_d,t_1]$ inventory level depleted to zero due to demand as well as deterioration. Shortage is allowed to occur during the time interval $[t_1,T]$ and all the demand during this shortage period is partially backlogged.

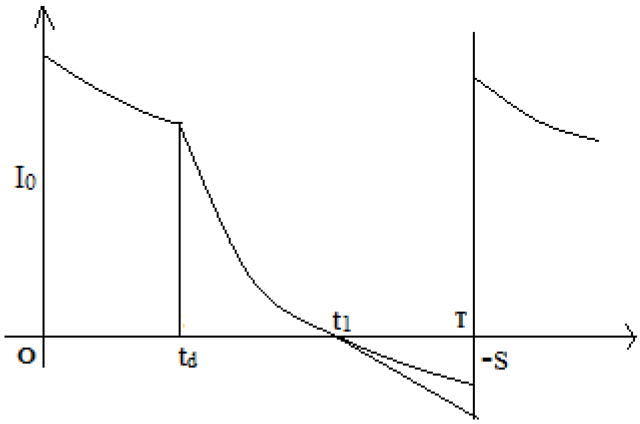

Fig 1. Geometry of the problem

On the basis of above discussion the dynamic of inventory level represented by following differential equations.

$$\frac{dI_1(t)}{dt} = -D(p), \quad 0 \le t \le t_d \tag{1}$$

With boundary condition $I_1(0) = I_0$, the solution of equation (1) can be given as

$$I_1(t) = I_0 - D(p)t, \quad 0 \le t \le t_d \tag{2}$$

During the time interval $[t_d, t_1]$ the inventory level depleted due to demand as well as deterioration and it is governed by the following differential equation

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(p), \quad t_d \le t \le t_1 \tag{3}$$

With boundary condition $I_2(t_1) = 0$, the solution of equation (3) is given by

$$I_2(t) = D(p)e^{-\frac{\alpha t^2}{2}}\left[t_1 - t + \frac{\alpha}{6}(t_1^3 - t^3)\right], \quad t_d \le t \le t_1 \tag{4}$$

Considering the continuity of $I(t)$ at $t = t_d$, then from equation (2) and (4)

$$I_1(t_d) = I_2(t_d)$$

$$or, I_0 - D(p)t_d = D(p)e^{-\frac{\alpha t_d^2}{2}}\left[t_1 - t_d + \frac{\alpha}{6}(t_1^3 - t_d^3)\right]$$, then the maximum inventory level is given by

$$I_0 = D(p)\left[t_d + e^{-\frac{\alpha t_d^2}{2}}\left(t_1 - t_d + \frac{\alpha}{6}(t_1^3 - t_d^3)\right)\right] \tag{5}$$

Using equation (5) in equation (2) we get,

$$I_1(t) = D(p)\left[t_d - t + e^{-\frac{\alpha t_d^2}{2}}\left(t_1 - t_d + \frac{\alpha}{6}(t_1^3 - t_d^3)\right)\right], \quad 0 \le t \le t_d \tag{6}$$

The variation of shortage level at time t during the time interval $[t_1, T]$ is described by the following differential equation.

$$\frac{dI_3(t)}{dt} = -\frac{D(p)}{1+\delta(T-t)}, \quad t_1 \le t \le T \tag{7}$$

With the boundary condition $I_3(t_1) = 0$, then the solution of the equation (7) is represented by

$$I_3(t)=\frac{D(p)}{\delta}\left[\log\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right],\quad t_1\le t\le T \tag{8}$$

Now at time $t=T$ we get the maximum amount of demand backlogged per period is given by

$$S\equiv -I_3(T)=\frac{D(p)}{\delta}\left[\log\left(1+\delta(T-t_1)\right)\right] \tag{9}$$

The order quantity can be obtained from equation (5) and(9) that is represented by

$$Q=I_0+S$$

$$or,\ Q=D(p)\left[t_d+e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\frac{\alpha}{6}(t_1^3-t_d^3)\right)+\frac{\log\left(1+\delta(T-t_1)\right)}{\delta}\right] \tag{10}$$

The total relevant inventory cost per cycle involves following factors.

(a) Ordering cost per cycle is A.

(b) Inventory holding/storage cost of the system is given by

$$HC=c_h\left[\int_0^{t_d}I_1(t)dt+\int_{t_d}^{t_1}I_2(t)dt\right]$$

$$=c_h\left[\int_0^{t_d}D(p)\left\{t_d-t+e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\frac{\alpha}{6}(t_1^3-t_d^3)\right)\right\}dt+\int_{t_d}^{t_1}D(p)e^{-\frac{\alpha t^2}{2}}\left\{t_1-t+\frac{\alpha}{6}(t_1^3-t^3)\right\}dt\right]$$

$$=D(p)c_h\left[e^{-\frac{\alpha t_d^2}{2}}\left\{t_1-t_d+\alpha(t_1^3-t_d^3)/6\right\}t_d+\frac{t_1^2}{2}+\frac{\alpha}{12}t_1^4-\frac{\alpha^2}{72}t_1^6-t_1t_d+t_d^2-\frac{\alpha t_1^3t_d}{6}+\frac{\alpha t_1t_d^3}{6}-\frac{\alpha}{12}t_d^4+\frac{\alpha^2t_1^3t_d^6}{36}-\frac{\alpha^2t_d^6}{72}\right] \tag{11}$$

(c) The deterioration cost per cycle is represented by

$$DC=c_d\left[I_0-\int_0^{t_1}D(p)dt\right]$$

$$=c_d\left[D(p)\left(t_d+e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\frac{\alpha}{6}(t_1^3-t_d^3)\right)\right)-D(p)t_1\right]$$

$$=D(p)c_d\left[e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\frac{\alpha(t_1^3-t_d^3)}{6}\right)+t_d-t_1\right] \tag{12}$$

(d)The shortage cost in the entire cycle is described by

$$SC=-c_s\int_{t_1}^{T}I_3(t)dt$$

$$=-\frac{D(p)c_s}{\delta}\int_{t_1}^{T}\log\frac{1+\delta(T-t)}{1+\delta(T-t_1)}dt$$

$$=\frac{D(p)c_s}{\delta}\left[(T-t_1)-\frac{\log\left(1+\delta(T-t_1)\right)}{\delta}\right] \tag{13}$$

(e)The cost due to lost sales is given by

$$LC = c_l \int_{t_1}^{T} D(p)\left(1 - \frac{1}{1+\delta(T-t)}\right)dt$$

$$= D(p)c_l\left[(T-t_1) - \frac{\log\left(1+\delta(T-t_1)\right)}{\delta}\right] \tag{14}$$

(f) Purchase cost for the total order quantity is expressed as

$$PC = cQ$$

$$= cD(p)\left[t_d + e^{-\frac{\alpha t_d^2}{2}}\left(t_1 - t_d + \frac{\alpha}{6}(t_1^3 - t_d^3)\right) + \frac{\log\left(1+\delta(T-t_1)\right)}{\delta}\right] \tag{15}$$

(g) Total sales revenue is represented by

$$SR = p\left[\int_0^{t_1} D(p)dt - I_3(T)\right]$$

$$= p\,D(p)\left[t_1 + \frac{\log\left(1+\delta(T-t_1)\right)}{\delta}\right] \tag{16}$$

Total average profit of the system per cycle is given by

$$PF\left(t_1, T, p\right) = \frac{1}{T}\left[SR - (A + HC + DC + SC + LC + PC)\right]$$

The necessary condition for maximizing the profit are given by

$$\frac{\partial PF}{\partial t_1} = 0, \quad \frac{\partial PF}{\partial T} = 0 \quad and \quad \frac{\partial PF}{\partial p} = 0 \tag{17}$$

Provided that the optimal value for t_1, T, p obtained from equation (17) satisfy the sufficient condition that the given Hessian matrix should be negative definite.

$$\text{H}=\begin{pmatrix} \frac{\partial^2 PF}{\partial t_1^2} & \frac{\partial^2 PF}{\partial t_1 \partial T} & \frac{\partial^2 PF}{\partial t_1 \partial p} \\ \frac{\partial^2 PF}{\partial T \partial t_1} & \frac{\partial^2 PF}{\partial T^2} & \frac{\partial^2 PF}{\partial T \partial p} \\ \frac{\partial^2 PF}{\partial p \partial t_1} & \frac{\partial^2 PF}{\partial p \partial T} & \frac{\partial^2 PF}{\partial p^2} \end{pmatrix} \tag{18}$$

Profit function for some special cases

Case 1: Model with completely backlogging, for this case δ=0 or B(T-t)=1 then $LC = 0$ and

$Q = D(p)\left[t_d + e^{-\frac{\alpha t_d^2}{2}}\left(t_1 - t_d + \frac{\alpha}{6}(t_1^3 - t_d^3)\right) + T - t_1\right]$, the relevant profit per unit time is represented by

$$PF=\frac{D(p)}{T}\left[\begin{array}{l} pT-\dfrac{A}{D(p)}-c_h\left\{e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\dfrac{\alpha}{6}(t_1^3-t_d^3)\right)t_d+\dfrac{t_1^2}{2}+\dfrac{\alpha}{12}t_1^4-\dfrac{\alpha^2}{72}t_1^6-t_1t_d+t_d^2-\dfrac{\alpha t_1^3t_d}{6}+\dfrac{\alpha t_1t_d^3}{6}-\dfrac{\alpha}{12}t_d^4+\dfrac{\alpha^2t_1^3t_d^6}{36}-\dfrac{\alpha^2t_d^6}{72}\right\} \\ -c_d\left\{e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\dfrac{\alpha}{6}(t_1^3-t_d^3)\right)+t_d-t_1\right\}-c_s\dfrac{(T-t_1)^2}{2}-c\left\{T-t_1+t_d+e^{-\frac{\alpha t_d^2}{2}}\left(t_1-t_d+\dfrac{\alpha}{6}(t_1^3-t_d^3)\right)\right\}\end{array}\right]$$

Case 2: Model without shortage that is $\delta\to\infty$ then $T=t_1$, $SC=0, LC=0$ and

$$Q=D(p)\left[t_d+e^{-\frac{\alpha t_d}{2}}\left(T-t_d+\frac{\alpha}{6}(T^3-t_d^3)\right)\right]$$

, the total relevant profit per unit time for this case is given by

$$PF=\frac{D(p)}{T}\left[\begin{array}{l} pT-\dfrac{A}{D(p)}-c_h\left\{e^{-\frac{\alpha t_d^2}{2}}\left(T-t_d+\dfrac{\alpha}{6}(T^3-t_d^3)\right)t_d+\dfrac{T^2}{2}+\dfrac{\alpha}{12}T^4-\dfrac{\alpha^2}{72}T^6-Tt_d+t_d^2-\dfrac{\alpha T^3t_d}{6}+\dfrac{\alpha Tt_d^3}{6}-\dfrac{\alpha}{12}t_d^4+\dfrac{\alpha^2T^3t_d^6}{36}-\dfrac{\alpha^2t_d^6}{72}\right\} \\ -c_d\left\{e^{-\frac{\alpha t_d^2}{2}}\left(T-t_d+\dfrac{\alpha}{6}(T^3-t_d^3)\right)+t_d-T\right\}-c\left\{t_d+e^{-\frac{\alpha t_d^2}{2}}\left(T-t_d+\dfrac{\alpha}{6}(T^3-t_d^3)\right)\right\}\end{array}\right]$$

Case 3: Model with instantaneous deterioration and completely backlogging.

In this situation $t_d=0$ *and* $\delta=0$ then $Q=D(p)\left(\frac{\alpha}{6}t_1^3+T\right)$

$$PF=\frac{D(p)}{T}\left[pT-\frac{A}{D(p)}-c_h\left(\frac{t_1^2}{2}+\frac{\alpha t_1^4}{12}-\frac{\alpha^2t_1^6}{72}\right)-c_d\frac{\alpha t_1^3}{6}-c_s(T-t_1)^2-c\left(T+\frac{\alpha t_1^3}{6}\right)\right]$$

Case 4: Model with instantaneous deterioration and without shortage

In this situation $t_d=0$, $\delta\to\infty$ *then* $T=t_1$ and order amount of quantity is given by

$$Q=D(p)\left(T+\frac{\alpha T^3}{6}\right)$$

$$PF=\frac{D(p)}{T}\left[pT-\frac{A}{D(p)}-c_h\left(\frac{T^2}{2}+\frac{\alpha T^4}{12}-\frac{\alpha^2T^6}{72}\right)-c_d\frac{\alpha T^3}{6}-c\left(T+\frac{\alpha T^3}{6}\right)\right]$$

Illustration of the model using Numerical examples

Example 4.1

For the numerical illustration we consider an inventory system with the following parameter in proper unit.

$A=10,\ c_d=1.0,\ c_h=1.0,\ c_s=5.0,\ a=100.0,\ b=2.0,\ \alpha=0.040,\ c_l=1.0,\ \delta=0.1,\ t_d=2,\ c=1$ Then using $D(p)=a-bp$

equation (17), we get,

The optimal order quantity $Q^*=56.5081,$

Optimal time of on hand inventory $t_1^*=0.9846,$

Optimal cycle $T^*=1.0905,$

Optimal price $p^*=24.8774,$

Optimal average Profit of the system $PF^* = 1160.32$

Example 4.2

$A = 10,\ c_d = 1.0,\ c_h = 1.0,\ c_s = 5.0,\ a = 200.0,\ b = 0.05,\ \alpha = 0.0050,\ c_l = 1.0,\ \delta = 0.1,\ t_d = 3/12,\ c = 2$ Then $D(p) = a\, e^{-bp}$

using equation (17), we get,

The optimal order quantity $Q^* = 38.7002$

Optimal time of on hand inventory $t_1^* = 0.514225,$

Optimal cycle $T^* = 0.587222,$

Optimal price $p^* = 22.1954$

Optimal average Profit of the system $PF^* = 1297.41$

Sensitivity Analysis

Effect of various parameters used in the model can be seen from the table given below. This table shows the sensitiveness of the various parameter on optimal average profit PF^*, optimal ordering quantity Q^*, optimal price p^*,optimal cycle T^* and optimal time t_1^* of on hand inventory.

Table 1

Parameters	% Chang in	t_1^*	T^*	p^*	Q^*	PF^*
a	-50	+58.38	+62.05	-38.90	-42.11	-99.18
	-25	+18.07	+19.05	-20.35	-17.83	-61.59
	+25	-11.14	-11.70	+20.91	+15.05	+84.63
	+50	-18.94	-19.88	+42.09	+28.41	+191.96
b	-50	-2.91	-4.29	+87.59	+3.54	+173.96
	-25	-1.61	-2.12	+29.13	+1.94	+57.57
	+25	+1.86	+2.12	-17.40	-2.15	-34.02
	+50	+3.97	+4.61	-28.92	-4.50	-56.25
α	-50	+1.58	+1.30	+0.09	+1.11	+0.09
	-25	+0.75	+0.62	+0.04	+0.53	+0.04
	+25	-0.69	-0.57	-0.04	-0.49	-0.04
	+50	-1.33	-1.09	-0.08	-0.94	-0.08
δ	-50	-0.30	+0.34	+0.02	+0.40	+0.16
	-25	-0.15	+0.17	+0.01	+0.20	+0.07
	+25	+0.14	-0.16	-0.01	-0.19	-0.07
	+50	+0.28	-0.32	-0.02	-0.37	-0.14
	-50	-0.40	+0.05	+2.18	-2.32	-0.27

	-25	-0.39	-0.11	+1.25	-1.52	-0.10
t_d	+25	+1.01	+0.66	-1.51	+2.51	-0.09
	+50	+2.92	+2.12	-3.19	+6.27	-0.51

Sensitivity analysis is carried out by changing the specified parameter by -50%, -25%, +25% +50% keeping the remaining parameter at their standard value. The study manifested the following facts.

1. Optimal average profit PF^* is highly sensitive to the change in the value of parameters a and b ,while it is slightly sensitive to the change in the parameters α and δ and it is moderately sensitive to changes in t_d
2. Study reflects that optimal price p^* is highly sensitive to the change in parameter a and b where as low sensitiveness of p^* is observed to the parameter α and δ while it is moderately sensitive to change in the parameter t_d .
3. Q^* is slightly sensitive to the change in the parameter δ and α also it is observed that it is highly sensitive to the change in the parameter a whereas it is moderately sensitive to t_d and b .
4. Optimal cycle T^* is highly sensitive to parameter a where as slight sensitiveness is exhibited due to change in the parameter t_d, α and δ and it is moderately sensitive to changes in b
5. t_1^* is slightly sensitive to the change in parameter a and t_d where as moderately sensitive to change in b and α and highly sensitive to a .

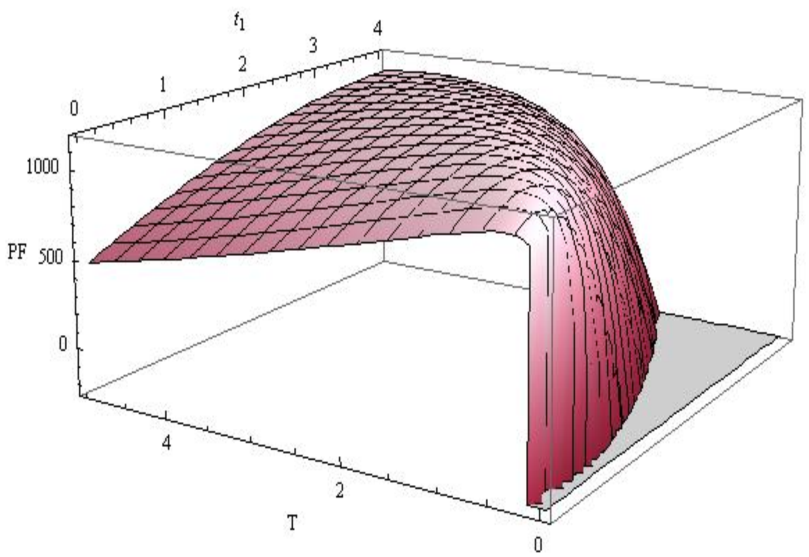

Fig 2. Variation of average profit PF with respect to t_1 and T for linear demand

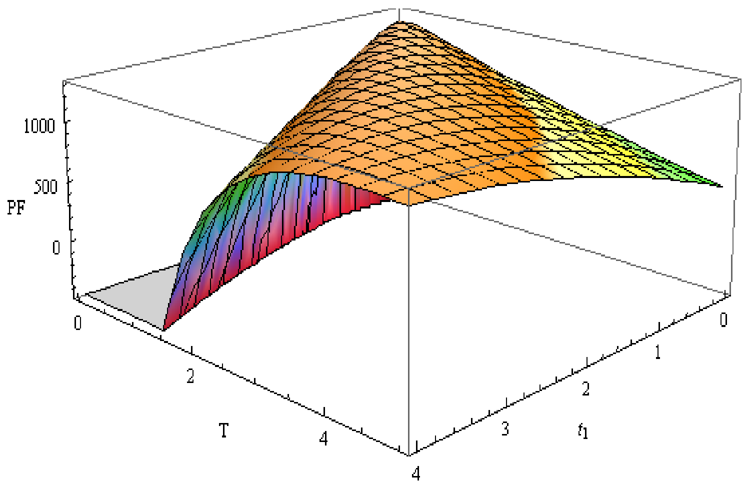

Fig. 3: Variation of average profit PF with respect to t_1 and T for exponential demand

Conclusions

The present paper deals a deterministic inventory system model for non instantaneous deteriorating item for exponentially and linearly decreasing demand under the effect of price with partial backlogging. It is observed from the sensitivity analysis o f the model that longer the fresh life time of product or lesser the deterioration parameter profit of the system increases and order quantity increases while decreasing the backlogging rate the order quantity decreases.

Furthermore, sensitivity analysis with respect to various parameters of the system has been carried out. Numerical examples a re illustrated considering two types of demand, first one for the linearly decreasing demand with price and another one with exponentially decreasing demand with price. Thus, this model incorporates realistic features that are likely to be associated with some kinds of inventory. For instance Sometime it is observed that some goods when initially received to market retailer avail the product at low price for making the goodwill of the company so demand is initially high but as product get its recognition in the market its price increases and accordingly their demand decreases due to hike in price of product. Hence the model is good agreement with these situations of market and prompted by this we modeled the inventory problem considering demand is decreasing function of price , so the model is very useful in the retail business. Stochastic demand rate may be considered in the problem, which needs to be f urther studied in the future research.

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