



## The field strength above atmospheric surface duct

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### ABSTRACT

The duct model used here is that Kahan and Eckart [1950]. The source of the electromagnetic field is taken to a vertical magnetic dipole in the upper surface layer, with an arbitrary time – varying moment , we can determine the transient behaviour of the electric field strength at any distance above the duct .The paper presents a method which allows the calculation of the atmospheric distortion of radar pulse , provided that the influence of the atmosphere is to transfer the transmitted signal through a duct .the polarization of the primary source whose moment varies arbitrarily in time, is chosen in such a way that it allows the exact determination of the electric field strength at some field point above the duct layer .From the physical point of view. Cahniard s idea is applicable as it is based on evaluating the field in a series of image sources of the primary source. The step – function solution of the problem can then be determined as an infinite integrals over finite integrals. Two cases would be distinguished on the basis of the distance between the receiving and transmitting ends and whether it is greater or lesser than the total reflection distance.

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### Introduction

Historically, in the problem of electromagnetic radiation from a vertical magnetic dipole situated at a certain height  $h$  above a plane earth , all field quantities are usually assumed to vary harmonically in time . Sommerfeld [1] ,calculated the electromagnetic radiation from an electric vertical dipole , located above the plane interface of two media .

Many writers Wait [2] .Moore [3] and Durrani [4] have considered this problem, the aim of the present work is to extend the study – state to transient excitation when no restrictions on the distance between receiving and transmitting ends are made. two integral transforms are applied to analyze the transient field of vertical electric dipole above a dielectric layer the distinction of different cases where the distance between the receiving and transmitting end are greater and lesser than the total reflection distance studied Abo Seliem [5 ] .

The problem has been studied by Arutaki and Chiba [6] and Abo Seliem [7] .This Integral is estimated by using the steepest descent method , along the contour  $\Gamma$  and around the branch –cuts , from the obtained results the Saddle point method show that the reflected waves and integrals Abo Seliem [8], the component of electric field strength is also arbitrary for the excitation function  $F(t) = t$  at some fixed but arbitrary position from the point of observation in the half-space .The problem has been studied by Abo-seliem [7] and this integrad is extended by using the steepest descent method along the contour  $\Gamma$  and around the branch – cuts . From the obtained results ,the Saddle point method show that the reflected waves an integrals.

### Formulation of the problem

As shown in Fig. 1 , the duct model of Kahan and Eckart [9] . A dielectric layer is assumed of relative permittivity  $\epsilon_1$  over laying an infinitely conducting plane earth which is confined by the plane  $z = 0$  of a rectangular coordinate system . The source of the field is assumed to be a vertical electric dipole in the medium 1 at the point  $x = y = 0$  ,  $z = d > 0$  whose moment is given by

$\vec{P}_m = \{0, 0, F(t)\delta(x, y, z - d)\}$ ,  $t$  being the time variable and  $\delta$  the three dimensional –distribution. Regarding  $F(t)$  , we make the assumptions  $F(t) = 0$  for  $t \leq 0$  and  $\frac{dF(t)}{dt} = 0$  at  $t = 0$  the electric field  $\vec{E}(x, y, z; t)$  of this vertical magnetic dipole has only horizontal component and co -forms to following wave equation.

$$(\nabla^2 - v^{-2} \frac{\partial^2}{\partial t^2}) \vec{E}(x, y, z; t) = \nabla \cap \frac{\partial}{\partial t} \mathbf{P}_m \tag{1}$$

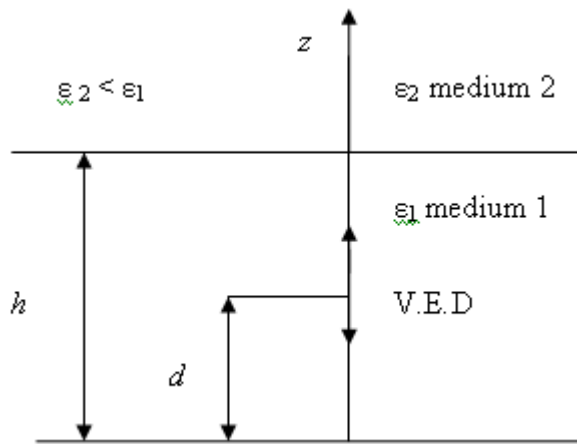
where  $v_i$  denotes the phase velocity in the medium  $i$ . The application of a Laplace transform in time and a two – dimensional Fourier transform in horizontal coordinates  $x, y$  leads under consideration of the initial, boundary and transform of  $\vec{E}(x, y, z; t)$  being

the variable in the transform space we get for  $h < z < \infty$

**Method of Solution**

The solution point is the wave equation for the electrical field  $\vec{E}(x, y, z; t)$  in the two media

$$[\frac{\partial^2}{\partial z^2} - s^2 \gamma_i^2] \vec{E}(\alpha, \beta, z, s) = \begin{cases} 0 & \text{for } i = 2 \\ jsf(s)\delta(z-d)(\beta, -\alpha, 0) & \text{for } i = 1 \end{cases} \tag{2}$$



**Fig 1. Geometry of the problem**

Where  $j^2 = -1, \gamma_i^2 = (\alpha^2 + \beta^2 + v_i^{-2}), i=1,2$  with  $Re(\gamma_i) \geq 0$  in the medium 1 This an integral representation results of the Laplace transform of electric field in terms of two – dimensional inverse Fourier integral .

$$\begin{aligned} E_x^{(i)}(x, y, z; s) = & -\frac{s^3 f(s)}{8\pi^2} \int_{-\infty}^{+\infty} \int j\beta \left[ \frac{e^{-s\gamma_1|z-d|}}{\gamma_1} + \frac{(1+c_{12}e^{-2s\gamma_1(h-d)})e^{-s\gamma_1(z+d)}}{\gamma_1(1+c_{12}e^{-2\gamma_1h})} + \right. \\ & \left. \frac{c_{12}(1+c_{12}e^{-2s\gamma_1d})e^{-s\gamma_1(2h-z-d)}}{\gamma_1(1+c_{12}e^{-2\gamma_1h})} \right] e^{js(\alpha x + \beta y)} d\alpha d\beta \end{aligned} \tag{3}$$

with the reflection coefficient at the upper duct boundary is  $C_{12} = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}$ . Here  $\alpha$  and  $\beta$  are variables in the transform space of the

two – dimensional Fourier transform  $f(s)$  is the Laplace transform of  $F(t)$ . The components  $E_y^{(i)}(x, y, z, s)$  can be determined from  $E_x^{(i)}(x, y, z, s)$  by substituting  $\frac{j\beta}{\gamma_1}$  which precedes the square brackets in (3), with  $\frac{j\alpha}{\gamma_1}$ , also, in the duct  $E_x^{(i)}(x, y, z, s)$  and

$E_y^{(i)}(x, y, z, s)$  can be determined, we discuss only  $E_x^{(i)}(x, y, z, s)$ , it follow that by using polar coordinate ,we get

$$\begin{aligned} E_x^{(i)}(p, \phi, z; s) = & -\frac{s^3 f(s)}{8\pi^2} \int_{-\infty}^{+\infty} \int j\lambda \sin' \phi' \left[ \frac{e^{-s\gamma_1|z-d|}}{\gamma_1} + \frac{(1+c_{12}e^{-2s\gamma_1(h-d)})e^{-s\gamma_1(z+d)}}{\gamma_1(1+c_{12}e^{-2\gamma_1h})} + \right. \\ & \left. \frac{c_{12}(1+c_{12}e^{-2s\gamma_1d})e^{-s\gamma_1(2h-z-d)}}{\gamma_1(1+c_{12}e^{-2\gamma_1h})} \right] e^{js\lambda p \cos(\phi-\phi')} \lambda d\lambda d\phi' \end{aligned} \tag{4}$$



We treat the far field, so that the Hankel function can be transform into the asymptotic expansion ,the  $E_x^{(i)}(x, y, z, s)$  will be as follows;

The above integral cannot be solved exactly , I we applied then the Saddle point method, in order to change the integral paths which it the steepest descent through  $\Gamma_1$

$$E_x^{(s)}(\rho, \phi, z; s) = -\frac{s^3 f(s)}{2\pi} e^{-\frac{3}{2}\pi} \sqrt{\frac{2}{\rho \pi s}} v_1^{-1} \int_{\Gamma_1} \phi(\alpha) \sin^2 \alpha \exp(-j v_i^{-1} \sin(\alpha + \theta_i)) d\alpha \tag{7}$$

Where

$$\phi = \left[ \frac{((N_1 + \cos \alpha) + (N_1 - \cos \alpha) \exp(-2js v_1^{-1} (h-d) \cos \alpha))}{((N_1 + \cos \alpha) + (N_1 - \cos \alpha) \exp(-2js v_1^{-1} (h) \cos \alpha))} \right] \tag{8}$$

and  $N_1 = \sqrt{-\sin^2 \alpha + n_1^2}$ ,  $n_1 = \frac{v_1}{v_2}$  and the integral in the complex  $r$ - plane corresponding to  $\lambda_i, -\infty \rightarrow \infty$ . By Taylor expansion ,

we get

$$\phi = \sum \left[ \frac{(-N_1 + \cos \alpha)}{(N_1 + \cos \alpha)} \right]^m \exp(-2js v_1^{-1} m(h) \cos \alpha) + \sum \left[ \frac{(-N_1 + \cos \alpha)}{(N_1 + \cos \alpha)} \right]^{m+1} \exp(-2js v_1^{-1} m + 1(h-d) \cos \alpha)$$

$$\phi = \sum_{m=0}^{\infty} \phi_u(m) \exp(-2js v_1^{-1} m(h) \cos \alpha) + \sum_{m=0}^{\infty} \phi_d(m) \exp(-2js v_1^{-1} m + 1(h-d) \cos \alpha) \tag{9}$$

Where

$$\exp(-2js v_1^{-1} (h) \cos \alpha) \left| \frac{-N_1 + \cos \alpha}{N_1 + \cos \alpha} \right| < 1 \text{ along } \Gamma_0, -N_1 + \cos \alpha = 0$$

$$E_x^s = E_u + E_d = \sum_{m=0}^{\infty} E_u(m) + \sum_{m=0}^{\infty} E_d(m) \tag{10}$$

where

$$E_*(m) = c \int_{\Gamma_1} \phi_*(\alpha) \sin^2 \alpha \exp(-j v_i^{-1} R_*(m) \sin(\alpha + \theta_i(m))) d\alpha \tag{11}$$

And  $R_u = \sqrt{r^2 + (z + d + 2mh)^2}$  ,  $R_d = \sqrt{r^2 + (z - d + 2(1+m)h)^2}$

$$\cos \theta_*(m) = \frac{r}{R_*(m)} , c = \frac{jf(s)s^3}{2\pi} \sqrt{\frac{2}{\rho \pi s v^2}} \exp(\frac{3j}{4} \pi) \text{ and } * = u, d$$

**Evaluation of the integral along the contour and around the branch cuts.**

As  $E_x(0)$  is the dominating term in  $E_x^s$  it will be initially estimated ,Jones [10]

$$E_u(0) = c \int_{\Gamma_1} \sin^2 \alpha \exp(-j v_i^{-1} R_u(0) \sin(\alpha + \theta_u(m))) d\alpha \tag{12}$$

$$E_d(0) = c \int_{\Gamma_1} \frac{-N_1 + \cos \alpha}{N_1 + \cos \alpha} \sin^2 \alpha \exp(-j v_i^{-1} R_d(0) \sin(\alpha + \theta_d(m))) d\alpha \tag{13}$$

The above integral cannot be solved exactly , we applied to the saddle point method ,in order to change the integral path  $\Gamma_i$  to  $\Gamma_1$  which is the steepest descent through  $\alpha_1 = \frac{\pi}{3} - \theta_u(0)$  and we must consider the poles and stride our branch cuts. If  $R_u(0)$  and  $R_d(0)$  are

large  $\theta_u(0) = \theta_1$  h, can be assumed to be nearly zero thus

$$E_u(0) = c \int_{\Gamma_1} \sqrt{\frac{\pi}{s v_1^{-1} R_u(0)}} \exp(-j s v_i^{-1} R_u(0)) \frac{\exp(-j s v_i^{-1} R_u)}{R_1} ds \tag{14}$$

$$E_d(0) = c \int_{\Gamma_1} \sqrt{\frac{\pi}{s v_1^{-1} R_d(0)}} \exp(-j s v_i^{-1} R_d(0)) ds \tag{15}$$

According  $E_{uc}(0)$  and  $E_{ds}(0)$  are vanishing rapidly, when r becomes large . generally , we suppose the following

$$E_x^s = \sum_{m=0}^{\infty} E_{uc}(m) + \sum_{m=0}^{\infty} E_{sd}(m)$$

$E_{uc}(m)$  and  $E_{ds}(m)$  can be estimated similarly to  $E_{uc}(0)$  and  $E_{ds}(0)$

$$E_{cu}(m) = c \int_{\Gamma_1} \sqrt{\frac{\pi}{s v_1^{-1} R_{cu}(m)}} \exp(-j s v_i^{-1} R_{cu}(m)) ds \tag{16}$$

$$E_{dsu}(m) = c \int_{\Gamma_1} \sqrt{\frac{\pi}{s v_1^{-1} R_{dsu}(m)}} \exp(-j s v_i^{-1} R_{dsu}(m)) ds \tag{17}$$

And the integral path strides over the two branch cuts in the complex plane , we obtain the  $E_{ucl}(0)$  and  $E_{dsl}(0)$ , where

$$E_{ucl}(0) = c \int_{\arcsin \frac{v_1}{v_2}}^{\frac{\pi-j\alpha}{4}} \sin^2 \alpha \exp(-j v_i^{-1} R_u(0) \sin(\alpha + \theta_u(0))) d\alpha \tag{19}$$

$$E_{dsl}(0) = c \int_{\arcsin \frac{v_1}{v_2}}^{\frac{\pi-j\alpha}{4}} \frac{-N_1 + \cos \alpha}{N_1 + \cos \alpha} \sin^2 \alpha \exp(-j v_i^{-1} R_d(0) \sin(\alpha + \theta_d(0))) d\alpha \tag{20}$$

The steepest descent through the branch point  $\alpha_{\beta 1}$  is considered Brekhovsikh[11], the trace of the steepest descent is obtained

from  $\text{Im}\{-j v_1^{-1} \sin(\alpha + \theta_u(0))\} = \text{Im}\{-j v_1^{-1} \sin(\alpha_{\beta 1} + \theta_u(0))\} = \frac{1}{v}$  and the real part of exponent decreases most rapidly

along this path, we deform the function of  $\alpha$  in the integral into a function of s using Tayler expansion .the new variable s through the

equation is  $\sin \alpha = n_i + i s^2$  where  $n_i = \sin \alpha_{\beta 1}$ ,  $\cos d\alpha = 2 j s d\tau$ , we get;

$$E_{ucl} = \sum_{m=0}^{\infty} E_{uc} = \sum_{m=0}^{\infty} \frac{j s^2 f(s)}{2\pi r^2} \frac{v_1^2}{v_2^2} \exp(-j v_i^{-1} - j s \sqrt{v_1^2 - v_2^2} (z + d + 2mh)(m+1) (\frac{1-n_1}{1+n_1})^m) \tag{21}$$

$$E_{dsl} = \sum_{m=0}^{\infty} E_{dsc} = \sum_{m=0}^{\infty} \frac{j s^2 f(s)}{2\pi r^2} \frac{v_1^2}{v_2^2} \exp(-j v_i^{-1} - j s \sqrt{v_1^2 - v_2^2} (z + d + (2m=1)h)(m+1) (\frac{1-n_1}{1+n_1})^{m+1})$$

where

$$n_i = \sin \alpha_{\beta 1}, \cos d\alpha = 2 j s d\tau \tag{22}$$

$E(r, z, s)$  is the transform of  $E(r, z, t)$ . By using the inverse Laplace transform of  $f(s) = \frac{1}{s}$  and equation (12), (13), (21 and

(22) we have the complete solution of  $E_x^{(1)}$

$$E_x^{(1)}(r, t) = E_x^{prim}(r, t) + E_x^{(s)}(r, t) \tag{23}$$

Where

$$E_x^{prim}(r, t) = \begin{cases} 0 & t > \frac{\tau}{v_1} \\ \frac{2}{\pi} \delta(1 - \frac{t}{v_1}) & t < \frac{\tau}{v_1} \end{cases} \tag{24}$$

And

$$E_x^{(s)}(r, t) = \sum_{m=0}^{\infty} E_{uc1}(m) + \sum_{m=0}^{\infty} E_{dc1}(m) + \sum_{m=0}^{\infty} E_{uc}(m) + \sum_{m=0}^{\infty} E_{ds}(m) \tag{25}$$

Where

$$E_{uc} = \frac{-2j}{\pi} \sum_{m=0}^{\infty} \frac{v_1^2}{2r^2 v_2} (2m+1) \left( \frac{1-n_1}{1+n_1} \right)^m \delta(t-t_1) \tag{26}$$

Where

$$t_1 = \frac{r}{v_1} + \frac{(z = d + 2mh)}{v_2}$$

And,

$$E_{dc} = \frac{j}{\pi} \sum_{m=0}^{\infty} \frac{v_1^2}{2r^2 v_2} (2m+1) \left( \frac{1-n_1}{1+n_1} \right)^{m+1} \delta(t-t_2) \tag{27}$$

where

$$t_2 = \frac{r}{v_1} + \frac{(z - d + (2m + 1)h)}{v_2}$$

And

$$E_{us}(r, t) = \frac{j}{\pi} \sum_{m=0}^{\infty} \frac{v_1^{-2}}{r^{\frac{1}{2}}} (r^2 + (z + d + 2mh)^2)^{-\frac{1}{2}} (2m+1) \left( \frac{1-n_1}{1+n_1} \right)^m \delta(t-t_3) \tag{28}$$

where

$$t_3 = \frac{(r^2 + (z + d + 2mh)^2)^{\frac{1}{2}}}{v_1}$$

$$E_{dss}(r, t) = \frac{j}{\pi} \sum_{m=0}^{\infty} \frac{v_1^{-2}}{r^{\frac{1}{2}}} (r^2 + (z - d + 2(m+1)h)^2)^{-\frac{1}{2}} (m+1) \left( \frac{1-n_1}{1+n_1} \right)^{m+1} \delta(t-t_4) \tag{29}$$

where

$$t_4 = \frac{(r^2 + (z - d + 2(m+1)h)^2)^{\frac{1}{2}}}{v_1}$$

Where

$$\left( \frac{1-n_1}{1+n_1} \right)$$

is the coefficient equivalent to the reflection one at the surface in the duct .If the height duct is h=20m and the difference of relative permutations in the boundary  $\Delta\varepsilon = 4.10^{-4}$ ,the height of the primary source and the point of observation are taken as z=d=15m ,the ,if r=4000m.

$$n_1 = \frac{V_1}{V_2} = 1.00033$$

Figures describes the relationship between  $Im|y|$  and  $t$  at  $t < 6 \times 10^{-3} s$  .The absolute value of the z-component of the electric field strength increasing with increase the time . The absolute value of the z-component of the electric field strength is increasing when the spherical distance between the source and the point of observation is very small.(  $|E| \propto \frac{1}{R}$  ). We note that The absolute value of the of the electric field strength is dependent of  $R$  . At  $t > 6 \times 10^{-3} s$  .The absolute value of the electric field strength is constant for each values of  $R = 5 km ,10 km , 15 km , 20 km$  and  $25 km$  .

We note that :- the saturation curve when the time increase.

Figure (3-6): describes the relationship between  $\text{Log } |Abs|y||$  and  $t$ .

- 1- At time  $< 5 \times 10^{-3} s$  .The value of  $\text{Log}$  of the electric field strength is negative (-ve) value .
- 2- At time  $(t) < 6 \times 10^{-3} s$  .  $\text{Log}$  value of the electric field strength increasing by increase the time .
- 3- At time  $(t) > 6 \times 10^{-3} s$  .  $\text{Log}$  value of the electric field strength is constant (+ve) for all value of .

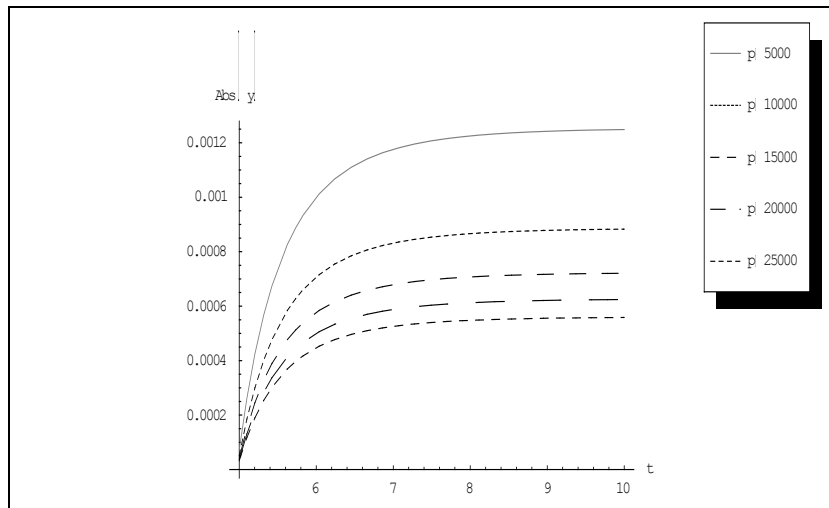


Fig 3 Describes the relationship between  $Abs|y|$  and  $t$

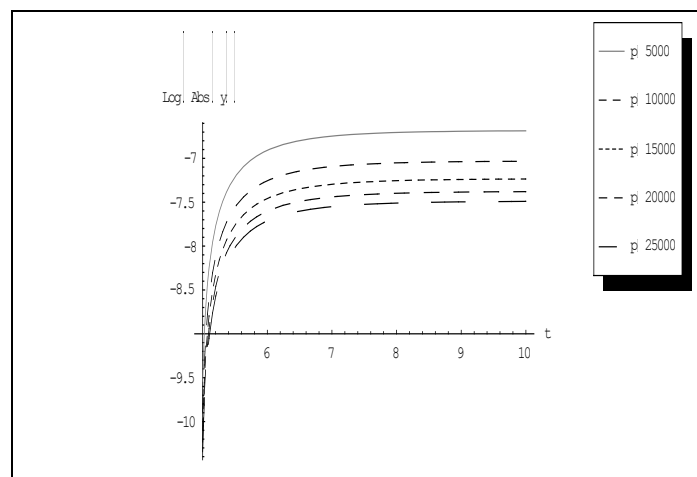


Fig 4. Describes the relationship between  $\text{Log } |Abs|y||$  and  $t$

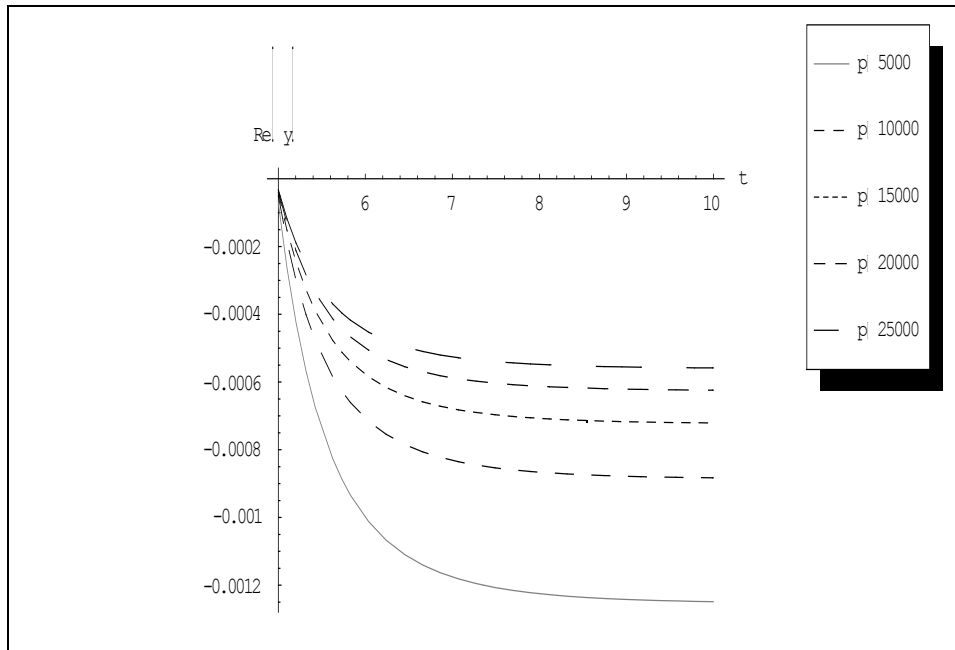


Fig.(5) describes the relationship between  $Re|y|$  and  $t$

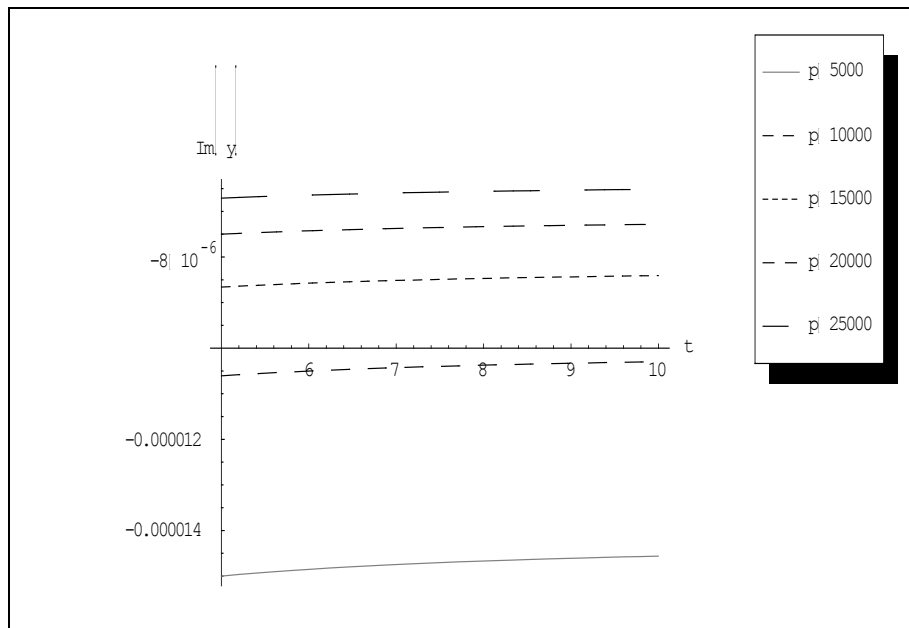


Fig.(6) describes the relationship between  $Im|y|$  and  $t$

The real value of the electric field strength is constant taken negative (-ve) value for all values of  $R$ . Figure (3-6): describes the relationship between imaginary value of the electric field strength and the time. In this figure (saturation relationship) where value of imaginary is constant at varying time for each value of  $R$  but we note that at  $R = 5$  km and 10 km the saturation curves are negative (-ve) value. But for  $R = 15$  km, 20 km and 25 km the saturation curve are positive (+ve) value.

**Conclusion**

We give the exact solution of the electric field strength above a two layer medium. The integral represent of the physical point of view, also, the integral is evaluated by two mathematical methods: Residue and Saddle point method. A disadvantage of the method is not it cannot be used to calculated the potential in the dielectric half-space outside the layer in similar manner.

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