



Diffusion Verification of Slow Electrons in Gases

Nada Mohammad Hussan, Ibrahim Kaittan Fayyadh, Farhan Lafta Rashid and Ali Kadhim Aziz
Ministry of Science and Technology/ Materials Researches Directorate, Baghdad- Iraq.

ARTICLE INFO

Article history:

Received: 18 May 2013;

Received in revised form:

19 December 2013;

Accepted: 28 December 2013;

Keywords

Boltzmann transport equation,
Electron energy distribution,
Liquid state physics,
Kinetic theory,
Plasma.

ABSTRACT

The diffusion of slow electrons in Nitrogen ,Argon and Helium gases in uniform electric fields has been verified for ratios of electric field to pressure from 1.611 to 16.115,0.0322 to 0.3223 and 3.9131×10^{-3} to 0.9767 (V/cm .Torr⁻¹) respectively. These are calculations lead to a determination of the ratio of electron drift velocity to diffusion coefficient . By assuming a distribution in velocity of the electrons in the swarm, the Townsend energy factor K_T and the mean electron velocity can be computed as a function of E/P, where E is electric field and P is the gas pressure, where the mean free path at unit pressure, the average energy loss per collision, and the gas kinetic cross section can be calculated. The results are presented in Figures forms. The obtained results appeared a good agreement with the experimental and theoretical data.

© 2013 Elixir All rights reserved

Introduction

The lateral spreading by diffusion of a steam of electrons moving through a gas under the action of a uniform electric field can lead to a calculation of the ratio of the electron drift velocity to the diffusion coefficient . For an assumed energy distribution function of the electrons in the swarm , the average agitational energy of the electrons and the mean velocity of agitation can be inferred[1-3].

If the electron drift velocity is know ,values of the mean free path at unit pressure , the mean proportion of energy lost by the electron per collision , and the gas kinetic cross section can be calculated . More recently ; Huxley and Zaazou [4-5] have investigated the diffusion of electrons in H₂ , N₂ and air while these swarm experiments lead only to average values of the various parameters and while certain of the assumptions required for the interpretation of the data are more arbitrary than realistic, they still offer the most convenient means of investigation in the ranges of low electron energies (Less than a few electron volts). In the present work the diffusion of electrons in N₂, Ar, and He, has been investigated for a range of E/P=1.611 to 16.115 , 0.0322 to 0.3223 and 3.9131×10^{-3} to 0.9767 (V/cm Torr⁻¹) respectively, where E is the electric field strength and P is the gas pressure[6-15].

Theoretical Background

In the steady state of agitation motion for steam of electrons moving under the action of uniform electric field E, along Z axis, the distribution of electrons for this motion written as [16].

$$\nabla^2 n = \frac{V_d}{D} \frac{\partial n}{\partial z} \quad (1)$$

Whereas n refers to the total gas number density, V_d refers to the drift velocity of electron a long the direction of the field and D is the electrons diffusion coefficient.

Solution of Equation(1)

The uniform electric field E, enter the diffusion chamber through a small hole in the upper plate. After drifting through a gas of the chamber they are collected by the receiving electrode system consisting of a central disk and a surrounding annular electrode, the two being separated by a narrow gap. If i_b and i_c are the electron currents to the contral disk and annular electrode respectively, the ratio R of the currents is [17] :

$$R = \frac{i_b}{i_b + i_c} = \frac{\left\{ 1 - \left(\frac{h}{d} \right) \exp \left[- \left(\frac{V_d}{2D} \right) (d - h) \right] \right\}}{\left\{ 1 - \left(\frac{h}{e} \right) \exp \left[- \left(\frac{V_d}{2D} \right) (e - h) \right] \right\}} \quad (2)$$

Where h is the depth of the diffusion chamber, b is the radius of the central disk, c is the outer radius of the annular electrode ,

$d = \sqrt{h^2 + b^2}$ and $e = \sqrt{h^2 + c^2}$, the boundary conditions of this solution are :

$Z = 0$ (The upper electrode).

$Z = h$ (Over the entire electrode).

The current ratio R , as in Eq.(2) represents the experimental measurement in term of V_d/D .

It could be write in terms of parameter K_1 related to the Townsend energy factor K_T [18] :

$$K_T = \frac{vge}{vem}$$

vge : Average agitation energy of the electrons.

vem : Average energy of the gas molecules

Where,

$$K_1 = A K_T \quad (3)$$

$$K_1 = \frac{e D}{K T_g \mu} \quad (4)$$

Where e is the electron charge, K is the Boltzmann constant, T_g is the absolute temperature in Kelvin, ($T_g = 300^\circ\text{K}$) , D/μ is the diffusion coefficient to the electron mobility ratio and A is a constant depending on the energy distribution function of the electrons in the swarm. The average electron energy factor K_1 , may be obtained from V_d/D as follows [10] :

$$\frac{V_d}{D} = 38.92 \frac{E}{K_1} \quad (5)$$

Where, V_d in unit of cm/sec , D in unit of cm and E are the electron drift velocity , diffusion coefficient and the electric field in unit of Volt/cm respectively. This consider the drift velocity is small compared with the electron mean velocity of agitation , that the molecules may be considered as fixed elastic scattering centers with all direction of motion of the electrons equally probable after collision, and that the electron mean free path does not vary significantly with the electron agitation velocity . Eq.(2) can be written [19] :

$$R = \frac{1 - \left(1 + \left(\frac{b}{h}\right)^2\right)^{-\frac{1}{2}}}{1 - \left(1 + \left(\frac{c}{h}\right)^2\right)^{-\frac{1}{2}}} \times \frac{\exp\left\{-19.46\left(\frac{Eh}{K_1}\right)\left[\left(1 + \left(\frac{b}{h}\right)^2\right)^{\frac{1}{2}} - 1\right]\right\}}{\exp\left\{-19.46\left(\frac{Eh}{K_1}\right)\left[\left(1 + \left(\frac{c}{h}\right)^2\right)^{\frac{1}{2}} - 1\right]\right\}} \quad (6)$$

Where, values of R for a constant ratio of $c/h=1.5$. The different values of the ratio b/h are plotted in Fig.(1).

Computational Diagnostics

From the above the distribution of velocities in the electron stream is not known, the two distributions most commonly considered are the classical Maxwellian distribution and the Druyvestyn distribution which can be obtained from the solution of the Boltzmann transport equation for constant collision cross sections Eq.(7). The calculation of the following parameters for these two velocity distributions are [19] :

1 – Townsend energy factor, K_T :

$K_T = K_1$ (Maxwell)

$K_T = 0.875 K_1$ (Druyvestyn)

$$K_1 = \frac{e}{K T_g} \frac{D}{\mu}$$

Where,

2-Root-mean - square velocity, $\langle u^2 \rangle^{\frac{1}{2}}$:

$$\langle u^2 \rangle^{\frac{1}{2}} = 1.16 \times 10^7 \sqrt{K_1} \quad (\text{Maxwell Velocity Distribution})$$

$$= 1.09 \times 10^7 \sqrt{K_1} \quad (\text{Druyvestyn Velocity Distribution})$$

3 - Mean electron Velocity, \bar{u} :

$$\bar{u} = 1.07 \times 10^7 \sqrt{K_1} \quad (\text{Maxwell})$$

$$= 1.04 \times 10^7 \sqrt{K_1} \quad (\text{Druyvestyn})$$

4 - Mean free path at unit pressure, L:

$$L = 7.2 \times 10^{-9} \frac{V_d \sqrt{K_1}}{E/P} \quad (\text{Maxwell})$$

$$= 7.47 \times 10^{-9} \frac{V_d \sqrt{K_1}}{E/P} \quad (\text{Druyvestyn})$$

5 - Average energy loss per

Collision, η :

$$\eta = 1.74 \times 10^{-14} \frac{V_d^2}{K_1} \quad (\text{Maxwell})$$

$$= 2.14 \times 10^{-14} \frac{V_d^2}{K_1} \quad (\text{Druyvestyn})$$

6-Gas kinetic cross section, σ :

$$\sigma = 4.26 \times 10^{-9} \frac{E/P}{V_d \sqrt{K_1}} \quad (\text{Maxwell})$$

$$= 4.14 \times 10^{-9} \frac{E/P}{V_d \sqrt{K_1}} \quad (\text{Druyvestyn})$$

Calculation of the transport coefficients V_d , E , D/μ

We can find the transport coefficients namely, drift velocity V_d , electric field, E , and the ratio of the diffusion coefficient to the electron mobility, D/μ , by solution the Boltzmann transport equation numerically which is given by [20,21]:

where v is the electron velocity, ∇ is the operator, ν is momentum transfer collision frequency, f_0 is the electron distribution function, e is the electronic charge, m is the electron mass, M is the ion or neutral particle mass, δ is equal to $2m/M$, $E = E_a + E_i$, E is the sum of an applied field E_a , and an induced field E_i , K is the Boltzmann's constant, ω is the angular frequency, $S_{o,e}$ is the zeroth-order electron-electron collision term.

$$-\frac{1}{3} v \nabla \cdot \left[\frac{v \nabla f_o}{\nu} + \frac{e E_i}{m v} \frac{\partial f_o}{\partial v} \right] - \frac{e}{3 m v^2} \frac{\partial}{\partial v} \left\{ v^2 \left[\frac{e}{m} \frac{\partial f_o}{\partial v} \frac{1}{\omega^2 + v^2} \frac{E_1^2 + E_2^2}{2} + \frac{E_i}{v} \cdot \left(v \nabla f_o + \frac{e E_i}{m} \frac{\partial f_o}{\partial v} \right) \right] \right\}$$

$$= \frac{1}{2 v^2} \frac{\partial}{\partial v} \left\{ v^2 \delta v \left[\frac{K T_g}{m} \frac{\partial f_o}{\partial v} + v f_o \right] \right\} + S_{o,e} \quad (7)$$

Results and Discussion

The quantity determined from the experimental measurement of the current ratio is V_d/D , its more common to write the value of R in terms of parameter K_1 as shown in equation (3). The average electron energy factor K_1 may be obtained from V_d/D as shown in equation (5), from equation (6) the values of R for a constant ratio of $c/h = 1.5$ and for several values of the ratio b/h are reflected in Fig.(1).

Fig.(2-3) are reflect the relation between the Townsend energy factor, K_T and mean electron velocity, \bar{u} versus the applied electric field to the gas pressure ratio, E/P , for Ar, He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwellian and Druyvestyn types, this figure appeared a good agreement between the theoretical and experimental data [16]. Fig.(4,5) are represent the Root-mean-square velocity, $\langle u^2 \rangle^{1/2}$ and the mean electron velocity, \bar{u} , as a functions of the parameter K_1 related to the Townsend energy factor Ar, He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwellian and Druyvestyn velocity distributions, these figures appeared the linear increasing of the Root-mean-square, $\langle u^2 \rangle^{1/2}$ and the mean electron velocity, \bar{u} , with K_1 for both Maxwellian and Druyvestyn distributions.

Fig(6) refers to the mean free path at unit of pressure, L , as a function of the applied electric field to the gas pressure ratio, E/P , for Ar, He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of both Maxwellian and Druyvestyn velocity distributions. These curves brought out a linear reducing for L with increasing of E/P in case of theoretical and experimental data, [16].

Fig.(7) represented the electron drift velocity to the diffusion coefficient ratio V_d/D , as a function of the applied electric field to the gas pressure ratio E/P , for argon(Ar), Helium(He), Nitrogen (N_2) and Ar(5%)- H_2 (95%) mixture in case of Maxwellian and Druyvestyn velocity distributions. This figure show the increasing of V_d/D with E/P in linear function form for both experimental and theoretical values and a good agreement [16].

Fig(8) reflects the linear increasing for the average energy loss per collision, η , with $V_d^2 K_1$ for Ar, He, N_2 gases and Ar(5%)- H_2 (95%) mixture.

Fig(9) exhibits the linear increasing for the gas kinetic cross section with E/P , for Ar, He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwellian and Druyvestyn velocities distribution. These curves brought out a good agreement with the experimental and theoretical data, [16].

Conclusions

- 1 – Experimentally calculated values of V_d/D , for Hydrogen and Nitrogen are given in Fig.(7).
- 2 – Theoretically determined values of V_d/D for other gases are given in the figures.
- 3 – The Townsend energy factor K_1 had been calculated for assumed velocity distributions of both the Maxwellian and Druyvestyn types.
- 4 – The mean electron velocity \bar{u} , mean free path at unit pressure L , average energy loss per collision η , and gas kinetic cross section, σ , are calculated for both Maxwellian and Druyvestyn velocity distribution.
- 5 – The values of drift velocity, V_d , electric field E , and diffusion coefficient to the electron mobility ratio D/μ are calculated by solution the Boltzmann equation numerically Eq.(7).
- 6 – The experimental and theoretical values in good agreement with literature [16]

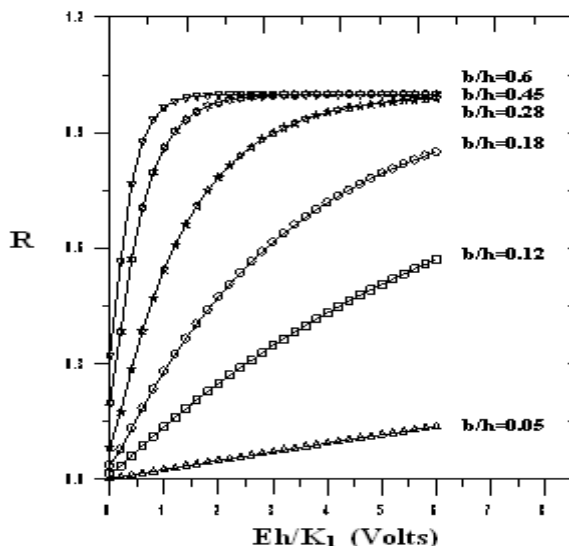


Fig 1. The current ratio R as a function of Eh/K_1 for several values of b/h

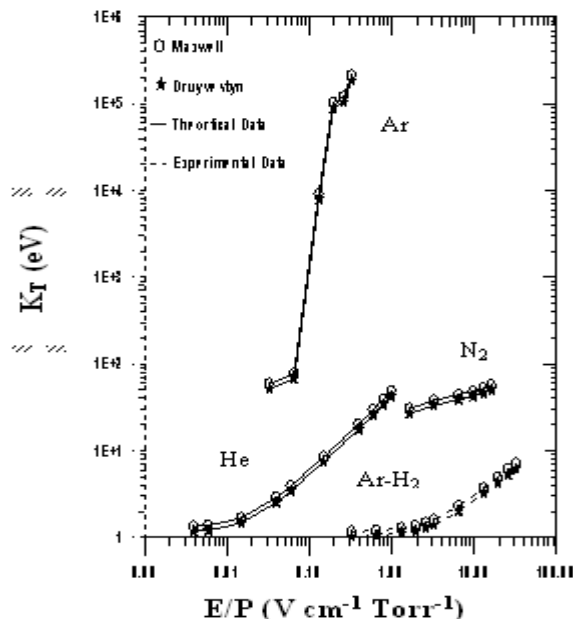


Fig 2. The Townsend energy factor, K_T as a function of the applied electric field to the gas Pressure ratio, E/P for Ar,He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwell and Druyvestyn velocities distribution

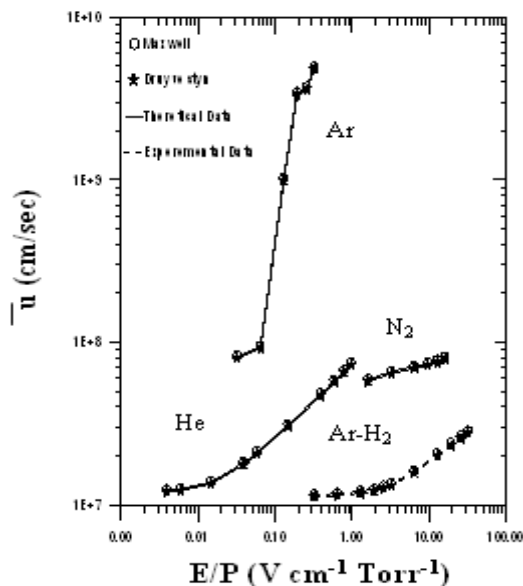


Fig 3. The mean electron velocity, \bar{u} as a function of the applied electric field to the gas pressure ratio E/P , for Ar, He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case case of Maxwell and Druyvestyn velocities distribution

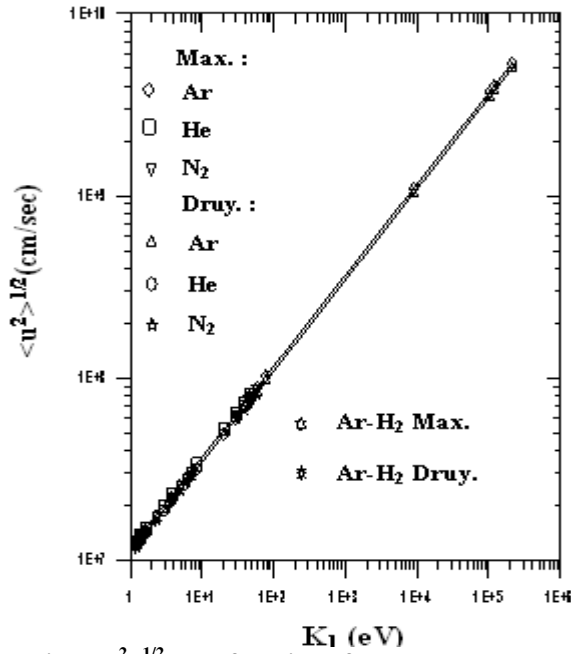


Fig 4. The Root-mean-square velocity, $\langle u^2 \rangle^{1/2}$ as a function of the Townsend energy factor, K_1 , for Ar, He, N_2 gases and Ar(5%)-H₂(95%) mixture for both Maxwell and Druyvestyn velocities distribution

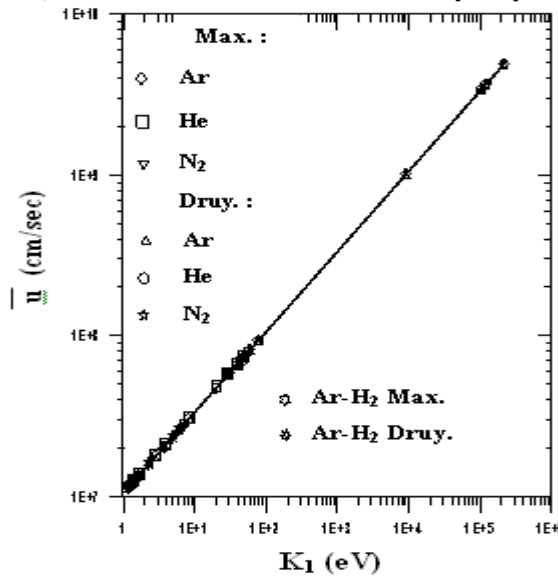
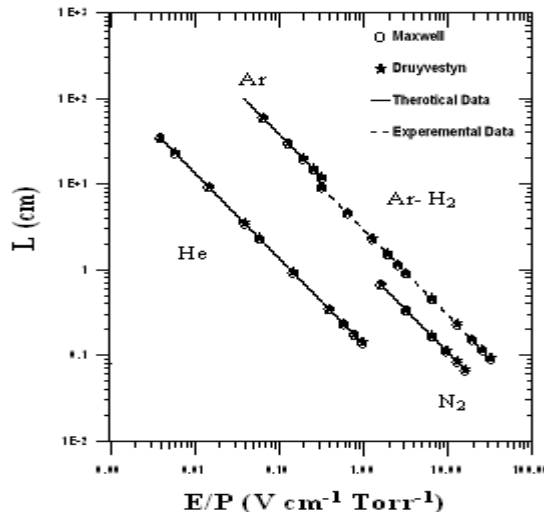


Fig 5. The mean electron velocity, \bar{u} , as a function of the Townsend energy factor, K_1 , for Ar, He, N_2 gases and Ar(5%)-H₂(95%) mixture in case of Maxwell and Druyvestyn velocities distribution



Fig(6) The mean electron free path, L , as a function of the applied electric field to the gas pressure ratio E/P , for Ar, He, N_2 gases and Ar(5%)-H₂(95%) mixture in case of Maxwell and Druyvestyn velocities distribution

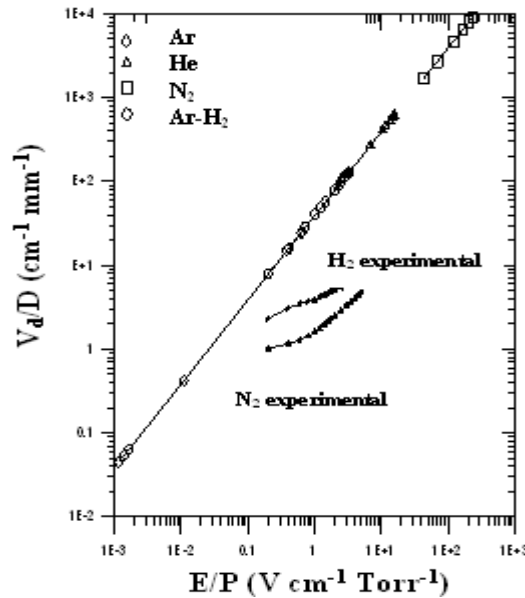


Fig 7. The electron drift velocity to the diffusion coefficient ratio, V_d/D , as a function of the applied electric field to the gas pressure ratio E/P , for Ar,He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwell and Druyvestyn velocities distribution

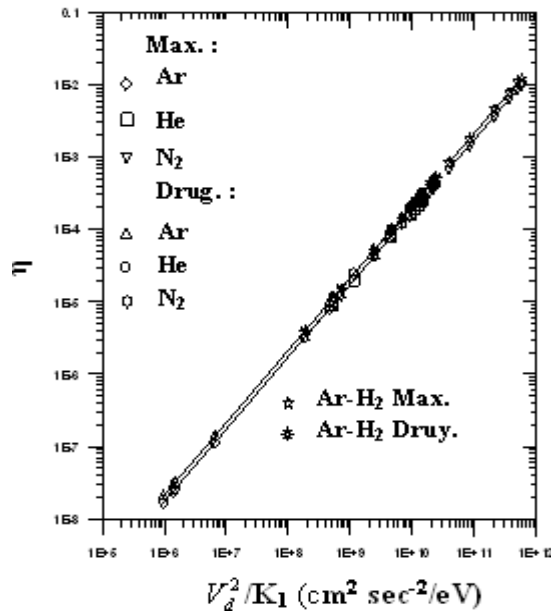


Fig 8. The average energy loss per collision, η , as a function of the drift velocity square to the Townsend energy factor ratio K_1 , for Ar,He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwell and Druyvestyn velocities distribution

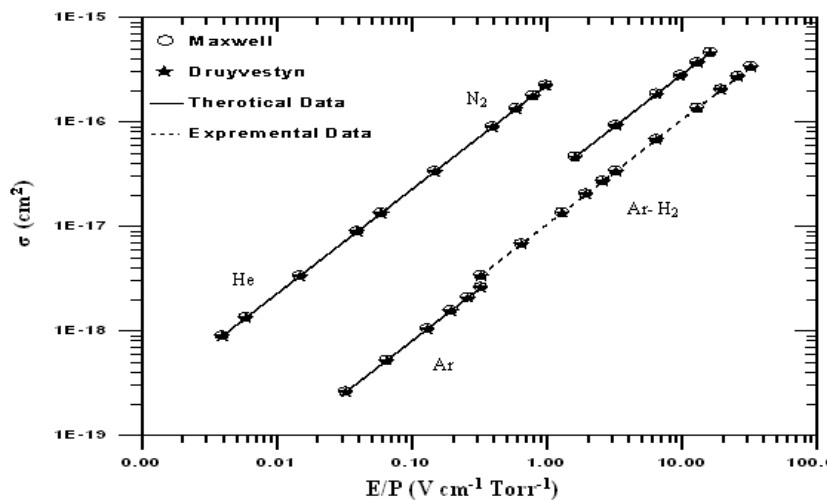


Fig 9. The gas kinetic cross section, σ , as a function of the applied electric field field to the gas pressure ratio, E/P , for Ar,He, N_2 gases and Ar(5%)- H_2 (95%) mixture in case of Maxwell and Druyvestyn velocities distribution

- 1 - Tkachev A. N. and Yakovlenko S. I., (2006), "Electron Energy Distribution Function and Dense – Gas Ionization in the Presence of a Strong Field", *Laser Physics*, Vol. 16, No. 9, PP. 1308-1310.
- 2 - Hideaki Matsuura, Yasuyuki Nakao, (2008), "Distortion of Bulk - Electron Distribution Function and its Effect on Core Heating in Fast Ignition Plasma", *J. of Physics: Conference Series*, Vol. 112, PP. 1-4.
- 3-Tezcan SS, Akcayol MA, Ozerdem OC, Dincer MS,(2010)Calculation of Electron Energy Distribution Functions From Electron Swarm Parameters Using Artificial Neural in SF₆ and Argon, *IEEE Transactions on Plasma Science*,Vol.38,No.9,PP.2332-2339, Part 1.
- 4 - Huxley L. H. G., D. Phil., and Zaazou A.A., (1949), "Experimental And Theoretical Studies of the Behavior of Slow Electrons in Air", *Proc. Roy. Soc. Vol., A*, 169, PP.402-425.
- 5-Stano M, Pinhao N, Loffhagen D, Kucera M, Donko Z, Matejcek S,(2011)Effect of Small Admixtures of N-2,H-2 or O-2 on the Electron Drift Velocity in Argon: Experimental Measurements and Calculations, *European physical J. D*,Vol.65,No.3,PP.489-498.
- 6-Maiorion JR,(2007)Boltzmann Transport Equation and its Importance to Nuclear Reactor Physics, *Revista Brasileira De Ensino De Fisica*,Vol.29,No.1,PP.1-2.
- 7-Atrazhev VM, Dmitrenko W, Chernysheva IV,(2004)Electron Transport in Xenon-ydrogen Gas Mixtures *Technical Physics Letters*,Vol.30,No.4,PP.301-303.
- 8-Urquijo J De, Mitrani A, Ruiz-Vargas G and Basurto E(2011) Limiting Field Strength and Electron Swarm Coefficients of the CF₃L-SF₆ Gas Mixture, *J. Phys. D: Appl. Phys.*, Vol.44,No.34,PP.22-37.
- 9-Boyle G J, White R D, Robson R E, Dujko S and Petrovic Z LJ,(2012) On the Approximation of Transport Properties in Structured Materials Using Momentum-Transfer Theory, *New J. Phys.*,Vol.14,PP1-25.
- 10 - Crompton R. W. and Sutton D. J. ,(1952), "Experimental Investigation of the Diffusion of Slow Electrons in Nitrogen and Hydrogen", *Proc. Roy. Soc. Vol., A*, 215, PP. 467-480.
- 11 - Charles Chien, (2001), "Digital Radio Systems on a Chip", Kinwer Academic Publishers, U.S.A, PP.40.
- 12-Tuan, D. A. and Jeon, B. H. (2012). Electron Collision Cross Sections for the Tetraethoxysilane Molecule and Electron Transport Coefficients in Tetraethoxysilane-O₂ and Tetraethoxysilane-Ar Mixtures, *J. Physical Society of Japan*, Vol. 81, No. 6, PP. 1-8.
- 13-Xaplanteris, C. L.; Filippaki, E. D.; Mistakidis I. S. (2012). Electron Drift by rf Field Gradient Creates Many Plasma Phenomena: An Attempt to Distinguish the Cause and the Effect, *J. Plasma Physics*, Vol. 78, PP. 165-174, Part 2.
- 14-Hien, P.X., Tuan, D.A., Jeon, B.H.,(2012) Electron Collision Cross Sections for The TMS Molecule and Electron Transport Coefficients in TMS-Ar and TMS-O-Mixtures, *J.The Korean Physical*,Vol.61,No.1,PP.62-72.
- 15-Pancheshnyi, S., Biagi, S., Bordage, M.C., Hagelaar, G.J.M., Morgan. WL, Phelps, A.V., Pitchford, L.C.,(2012) The LXCat Project: Electron Scattering Cross Sections and Swarm Parameters for Low Temperature Plasma Modeling, *Chemical Physics*, Vol.398,PP.148-153.
- 16 - Cochran L. W. and Forester D. W., (1962), "Diffusion of Slow Electrons in Gases", *Phys. Rev.*, Vol. 126, No. 5, PP.1785-1788.
- 17 - Aldo Gilardini, (1972), "Low Energy Electron Collisions in Gases: Swarm and Plasma Methods Applied to their Study", John Wiley & Sons, Now York, .S.A., pp.183.
- 18 - Faiadh I. G., Ismael M. I., Hamudy F.G. and Dawood S. S.,(2011), "Applicable Studies of the Slow Electrons Motion in Air with Application in the Ionosphere", *Eng. & Tech. Journal*, Vol. 29, No. 3, PP. 442-461.
- 19 - Cronson H. M., (1966), "Spatial Variations of Plasma Electron Temperature in a Standing Wave at Microwave Frequencies", *The Physics of Fluids* Vol. 9, No. 3, PP. 581-586.
- 20- Rockwood S. D., Greene A. E.,(1980), "Numerical Solutions of the Boltzmann Transport Equation", *Computer Physics Communications* 19, PP. 377-393.
- 21- Peter Bloomfield, Vic Barnett, (2006), "Numerical Methods in Finance and Economics Matlab-Based Introduction", Second Edition, John Wiley, Sons, INC, Publication, (Wiley-Interscience), U. S. A., PP.297.