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Notes on interval valued fuzzy subhemirings of a hemiring

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ABSTRACT

In this paper, we study some of the properties of interval valued fuzzy subhemiring of a hemiring and prove some results on these.

Introduction

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (*R ; + ; .*) share the same properties as a ring except that (*R ; +*) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. Interval-valued fuzzy sets were introduced independently by Zadeh [10], Grattan-Guiness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function. Jun.Y.B and Kin.K.H[7] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju.A and Nagarajan.R[9] defined the charactarization of interval valued Anti fuzzy Left h-ideals over Hemirings. Azriel Rosenfeld[2] defined a fuzzy groups. We introduce the concept of interval valued fuzzy subhemiring of a hemiring and established some results.

Preliminaries

1.1 Definition: Let X be any nonempty set. A mapping [M] : X $\rightarrow$ D[0, 1] is called an interval valued fuzzy subset (briefly, IVFS) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and [M](x) = [$M^-(x)$, $M^+(x)$], for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus [M](x) is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

Definition: Let [M] = {⟨ x, [$M^-(x)$, $M^+(x)$] ⟩ / x$\in$X }, [N] = {⟨ x, [$N^-(x)$, $N^+(x)$] ⟩ / x$\in$X} be any two interval valued fuzzy subsets of X. We define the following relations and operations:

(i) [M] $\subseteq$ [N] if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$, for all x in X.

(ii) [M] = [N] if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$, for all x in X.

(iii) [M] $\cap$ [N] = { ⟨ x, [min { $M^-(x)$, $N^-(x)$}, min { $M^+(x)$, $N^+(x)$}] ⟩ / x$\in$X }.

(iv) [M] $\cup$ [N] = { ⟨ x, [max { $M^-(x)$, $N^-(x)$}, max { $M^+(x)$, $N^+(x)$}] ⟩ / x$\in$X }.

(v) $[M]^C$ = [1] – [M] = { ⟨ x, [1– $M^+(x)$, 1- $M^-(x)$] ⟩ / x$\in$X }.

Definition: Let (R, +, $\cdot$) be a hemiring. An interval valued fuzzy subset [M] of R is said to be an **interval valued fuzzy subhemiring(IVFSHR)** of R if the following conditions are satisfied:

(i) [M](x+y) $\geq$ min { [M](x), [M](y) },

(ii) [M](xy) $\geq$ min { [M](x), [M](y) }, for all x and y in R.

Definition: Let [M] and [N] be any two interval valued fuzzy subsets of sets R and H, respectively. The product of [M] and [N], denoted by [M]×[N], is defined as [M]×[N] = { ⟨ (x, y), [M]×[N](x, y) ⟩ / for all x in R and y in H}, where [M]×[N](x, y) = min { [M](x), [N](y) }.

Definition: Let [M] be an interval valued fuzzy subset in a set S, the **strongest interval valued fuzzy relation** on S, that is an interval valued fuzzy relation [V] with respect to [M] given by [V](x, y) = min { [M](x), [M](y) }, for all x and y in S.

Properties of interval valued fuzzy sub Hemirings:

Theorem: If [M] is an interval valued fuzzy subhemiring of a hemiring (R, +, $\cdot$), then [M](x) $\leq$ [M](0), for x in R, the zero element 0 in R.

Proof: For x in R and 0 is zero element of R. Now, [M](x) = [M](x+0) $\geq$ min {[M](x), [M](0)} and [M](0) = [M](x.0) $\geq$ min {[M](x), [M](0)}. If x+y = 0, then [M](0) = [M](x+y) $\geq$ min {[M](x), [M](y)}. Hence, [M](0) $\geq$ [M](x), for all x in R.

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Theorem: If [M] is an interval valued fuzzy subhemiring of a hemiring (R, +, ·), then H = { x / x∈R : [M](x) = [1] } is either empty or is a subhemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then [M](x+y) ≥ min { [M](x), [M](y) } = min { [1], [1] } = [1]. Therefore, [M](x+y) = [1]. We get x+y in H. And [M](xy) ≥ min { [M](x), [M](y) } = min { [1], [1] } = [1]. Therefore, [M](xy) = [1]. We get xy in H. Therefore, H is a subhemiring of R. Hence H is either empty or is a subhemiring of R.

Theorem: If [M] is an interval valued fuzzy subhemiring of a hemiring (R, +, ·), then H = { x∈R: [M](x) = [M](0) } is a subhemiring of R.

Proof: Let x and y be in H. Now, [M](x+y) ≥ min { [M](x), [M](y) } = min { [M](0), [M](0) } = [M](0). Therefore, [M](x+y) ≥ [M](0). Hence [M](0) = [M](x+y). Therefore, x+y in H. And, [M](xy) ≥ min { [M](x), [M](y) } = min { [M](0), [M](0) } = [M](0). Therefore, [M](xy) ≥ [M](0). Hence [M](0) = [M](xy). Therefore, xy in H. Hence H is a subhemiring of R.

Theorem: Let [M] be an interval valued fuzzy subhemiring of a hemiring (R, +, ·). If (i) [M](x+y) = [0], then either [M](x) = [0] or [M](y) = [0], for x and y in R.
(ii) [M](xy) = [0], then either [M](x) = [0] or [M](y) = [0], for x and y in R.

Proof: Let x and y in R. By the definition [M](x+y) ≥ min { [M](x), M](y) }, which implies that [0] ≥ min { [M](x), [M](y) }. Therefore, either [M](x) = [0] or [M](y) = [0]. By the definition [M](xy) ≥ min { [M](x), M](y) }, which implies that [0] ≥ min {[M](x), [M](y) }. Therefore, either [M](x) = [0] or [M](y) = [0].

Theorem: If [M] and [N] are two interval valued fuzzy subhemirings of a hemiring R, then their intersection [M]∩[N] is an interval valued fuzzy subhemiring of R.

Proof: Let x and y belong to R, [M] = {⟨ x, [M](x) ⟩ / x in R} and [N] = {⟨ x, [N](x) ⟩ / x in R}. Let [K] = [M]∩[N] and [K] = {⟨ x, [K](x) ⟩/ x in R}. (i) [K](x+y) = min { [M](x+y), [N](x+y) } ≥ min { min { [M](x), [M](y) }, min { [N](x), [N](y) } } = min { min {[M](x), [N](x) }, min {[M](y), [N](y) } } = min { [K](x), [K](y) }. Therefore, [K](x+y) ≥ min { [K](x), [K](y) }, for all x and y in R. (ii) [K](xy) = min { [M](xy), [N](xy) } ≥ min { min { [M](x), [M](y) }, min { [N](x), [N](y) } } = min { min { [M](x), [N](x)}, min { [M](y), [N](y) } } = min { [K](x), [K](y) }. Therefore, [K](xy) ≥ min { [K](x), [K](y) }, for all x and y in R. Hence [M]∩[N] is an interval valued fuzzy subhemiring of the hemiring R.

Theorem: The intersection of a family of interval valued fuzzy subhemirings of a hemiring R is an interval valued fuzzy subhemiring of R.

Proof: Let $\{[M_i]\}_{i\in I}$ be a family of interval valued fuzzy subhemirings of a hemiring R and [M] = $\bigcap_{i\in I}$ [M_i]. Then for x and y belongs to R, we have (i) [M](x+y) = $\inf_{i\in I} [M_i](x+y) \geq \inf_{i\in I}$ min { [M_i](x), [M_i](y) } ≥ min { $\inf_{i\in I} [M_i](x), \inf_{i\in I} [M_i](y)$ } = min { [M](x), [M](y) }. Therefore, [M](x+y) ≥ min { [M](x), [M](y) }, for all x and y in R. (ii) [M](xy) = $\inf_{i\in I} [M_i](xy) \geq \inf_{i\in I}$ min { [M_i](x), [M_i](y) }≥ min { $\inf_{i\in I} [M_i](x), \inf_{i\in I} [M_i](y)$ }= min { [M](x), [M](y)}. Therefore, [M](xy) ≥ min {[M](x), [M](y) }, for all x and y in R. Hence the intersection of a family of interval valued fuzzy subhemirings of the hemiring R is an interval valued fuzzy subhemiring of R.

Theorem: If [M] and [N] are interval valued fuzzy subhemirings of the hemirings R and H, respectively, then [M]×[N] is an interval valued fuzzy subhemiring of R×H.

Proof: Let [M] and [N] be interval valued fuzzy subhemirings of the hemirings R and H respectively. Let x_1 and x_2 be in R, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in R×H. Now, [M]×[N] [(x_1, y_1) + (x_2, y_2)] = [M]×[N](x_1+x_2, y_1+y_2) = min { [M](x_1+x_2), [N](y_1+y_2) }≥ min { min { [M](x_1), [M](x_2) }, min { [N](y_1), [N](y_2) } } = min { min { [M](x_1), [N](y_1)}, min { [M](x_2), [N](y_2)} }= min { [M]×[N](x_1, y_1), [M]×[N](x_2, y_2) }. Therefore, [M]×[N][(x_1, y_1)+(x_2, y_2)] ≥ min { [M]×[N](x_1, y_1), [M]×[N](x_2, y_2) }. And, [M]×[N][(x_1, y_1)(x_2, y_2)] = [M]×[N](x_1x_2, y_1y_2) = min { [M](x_1x_2), [N](y_1y_2) }≥ min {min { [M](x_1), [M](x_2) }, min { [N](y_1), [N](y_2) } } = min { min { [M](x_1), [N](y_1) }, min {[M](x_2), [N](y_2) } } = min { [M]×[N](x_1, y_1), [M]×[N](x_2, y_2) }. Therefore, [M]×[N][(x_1, y_1)(x_2, y_2)] ≥ min {[M]×[N](x_1, y_1), [M]×[N](x_2, y_2) }. Hence [M]×[N] is an interval valued fuzzy subhemiring of R×H.

Theorem: Let [M] and [N] be interval valued fuzzy subsets of the hemirings R and H, respectively. Suppose that 0 and 0^{I} are the identity element of R and H, respectively. If [M]×[N] is an interval valued fuzzy subhemiring of R×H, then at least one of the following two statements must hold.
(i) [N](0^{I}) ≥ [M](x), for all x in R, (ii) [M](0) ≥ [N](y), for all y in H.

Proof: Let [M]×[N] be an interval valued fuzzy subhemiring of R×H.

By contra positive, suppose that none of the statements (i) and (ii) holds. Then we can find a in R and b in H such that [M](a) > [N](0^{I}) and [N](b) > [M](0). We have, [M]×[N](a, b) = min { [M](a), [N](b) } > min { [M](0), [N](0^{I}) } = [M]×[N](0, 0^{I}). Thus [M]×[N] is not an interval valued fuzzy subhemiring of R×H. Hence either [N](0^{I}) ≥ [M](x), for all x in R or [M](0) ≥ [N](y), for all y in H.

Theorem: Let [M] and [N] be interval valued fuzzy subsets of the hemirings R and H, respectively and [M]×[N] is an interval valued fuzzy subhemiring of R×H. Then the following are true:
(i) if [M](x) ≤ [N](0^{I}), then [M] is an interval valued fuzzy subhemiring of R.
(ii) if [N](x) ≤ [M](0), then [N] is an interval valued fuzzy subhemiring of H.
(iii) either [M] is an interval valued fuzzy subhemiring of R or [N] is an interval valued fuzzy subhemiring of H.

Proof: Let [M]×[N] be an interval valued fuzzy subhemiring of R×H and x, y be in R. Then (x, 0^{I}) and (y, 0^{I}) are in R×H. Now, using the property [M](x) ≤ [N](0^{I}), for all x in R, we get, [M](x+y) = min{ [M](x+y), [N]($0^{I}0^{I}$) } = [M]×[N]((x+y), ($0^{I}0^{I}$)) = [M]×[N][(x, 0^{I}) +(y, 0^{I})] ≥ min { [M]×[N](x, 0^{I}), [M]×[N](y, 0^{I}) } = min { min { [M](x), [N](0^{I})}, min { [M](y), [N](0^{I}) }} = min { [M](x), [M](y) }. Therefore, [M](x+y) ≥ min {[M](x), [M](y) }, for all x, y in R. And, [M](xy) = min { [M](xy), [N]($0^{I}0^{I}$) } = [M]×[N]((xy), ($0^{I}0^{I}$)) = [M]×[N][(x, 0^{I})(y, 0^{I})] ≥ min { [M]×[N](x, 0^{I}), [M]×[N](y, 0^{I})} = min {min { [M](x), [N](0^{I}) }, min { [M](y), [N](0^{I}) } } = min { [M](x), [M](y) }. Therefore, [M](xy) ≥ min { [M](x), [M](y) }, for all x, y in R. Hence [M] is an interval valued fuzzy subhemiring of R. Thus (i) is proved. Now, using the property [N](x) ≤ [M](0), for all x in H, we get, [N](x+y) = min { [N](x+y), [M](00) } = [M]×[N](

(00), (x+y)) = [M]×[N][(0, x)+(0, y)] ≥ min { [M]×[N](0, x), [M]×[N](0, y)} = min {min {[N](x), [M](0) }, min { [N](y), [M](0) } } = min { [N](x), [N](y) }. Therefore, [N](x+y) ≥ min { [N](x), [N](y) }, for all x and y in H. And, [N](xy) = min { [N](xy), [M](00) } = [M]×[N]((00), (xy)) = [M]×[N][(0, x)(0, y)] ≥ min { [M]×[N](0, x), [M]×[N](0, y) } = min { min { [N](x), [M](0) }, min { [N](y), [M](0) } } = min { [N](x), [N](y) }. Therefore, [N](xy) ≥ min { [N](x), [N](y) }, for all x and y in H. Hence [N] is an interval valued fuzzy subhemiring of H. Thus (ii) is proved. (iii) is clear.

Theorem: Let [M] be an interval valued fuzzy subset of a hemiring R and [V] be the strongest interval valued fuzzy relation of R with respect to [M]. Then [M] is an interval valued fuzzy subhemiring of R if and only if [V] is an interval valued fuzzy subhemiring of R×R.

Proof: Suppose that [M] is an interval valued fuzzy subhemiring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R. We have, $[V](x+y) = [V][(x_1, x_2)+(y_1, y_2)] = [V](x_1+y_1, x_2+y_2) = \min \{ [M](x_1+y_1), [M](x_2+y_2) \} \geq \min \{ \min \{ [M](x_1), [M](y_1) \}, \min \{[M](x_2), [M](y_2) \}\} = \min \{ \min\{[M](x_1), [M](x_2) \}, \min \{ [M](y_1), [M](y_2) \}\} = \min \{[V](x_1, x_2), [V](y_1, y_2)\} = \min \{ [V](x), [V](y) \}$. Therefore, [V](x+y) ≥ min {[V](x), [V](y) }, for all x and y in R×R. And we have, $[V](xy) = [V][(x_1, x_2)(y_1, y_2)] = [V](x_1y_1, x_2y_2) = \min \{ [M](x_1y_1), [M](x_2y_2)\} \geq \min \{\min \{[M](x_1), [M](y_1)\}, \min\{[M](x_2), [M](y_2)\}\} = \min \{\min \{[M](x_1), [M](x_2) \}, \min \{[M](y_1), [M](y_2) \}\} = \min \{[V](x_1, x_2), [V](y_1, y_2)\} = \min \{[V](x), [V](y)\}$. Therefore, [V](xy) ≥ min {[V](x), [V](y)}, for all x and y in R×R. This proves that [V] is an interval valued fuzzy subhemiring of R×R. Conversely, assume that [V] is an interval valued fuzzy subhemiring of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R, we have $\min \{[M](x_1+y_1), [M](x_2+y_2) \} = [V](x_1+y_1, x_2+y_2) = [V][(x_1, x_2)+(y_1, y_2)] = [V](x+y) \geq \min \{[V](x), [V](y)\} = \min \{ [V](x_1, x_2), [V](y_1, y_2) \} = \min \{ \min \{[M](x_1), [M](x_2) \}, \min \{[M](y_1), [M](y_2) \} \}$. If we put $x_2 = y_2 = 0$, where 0 is the zero element of R. We get, $[M](x_1+y_1) \geq \min\{[M](x_1), [M](y_1) \}$, for all x_1, y_1 in R. And $\min \{ [M](x_1y_1), [M](x_2 y_2)\} = [V](x_1y_1, x_2y_2) = [V][(x_1, x_2)(y_1, y_2)] = [V](xy) \geq \min\{[V](x), [V](y)\} = \min\{[V](x_1, x_2), [V](y_1, y_2)\} = \min \{ \min \{[M](x_1), [M](x_2)\}, \min \{[M](y_1), [M](y_2)\} \}$. If we put $x_2 = y_2 = 0$, where e is the identity element of R. We get, $[M](x_1y_1) \geq \min \{[M](x_1), [M](y_1)\}$, for all x_1 and y_1 in R. Hence [M] is an interval valued fuzzy subhemiring of R.

Theorem: Let [M] be an interval valued fuzzy subset of a hemiring R. Then [M] is an interval valued fuzzy subhemiring of R if and only if M^- and M^+ are fuzzy subhemiring of R.

Proof: Let x and y belong to R and [M] be an interval valued fuzzy subhemiring of R, $[M] = \{\langle x, [M^-(x), M^+(x)] \rangle / x \in X \}$. So, $[M](x+y) \geq \min \{ [M](x), [M](y) \} = [\min \{M^-(x), M^-(y)\}, \min \{ M^+(x), M^+(y)\}]$. Thus $[M](x+y) \geq [\min \{ M^-(x), M^-(y)\}, \min \{M^+(x), M^+(y)\}]$. Therefore $[M^-(x+y), M^+(x+y)] \geq [\min \{M^-(x), M^-(y)\}, \min \{ M^+(x), M^+(y)\}]$. Thus $M^-(x+y) \geq \min \{M^-(x), M^-(y)\}$ and $M^+(x+y) \geq \min \{M^+(x), M^+(y)\}$. And $[M](xy) \geq \min \{ [M](x), [M](y) \} = [\min \{M^-(x), M^-(y)\}, \min \{ M^+(x), M^+(y)\}]$. Thus $[M](xy) \geq [\min \{ M^-(x), M^-(y)\}, \min \{M^+(x), M^+(y)\}]$. Therefore $[M^-(xy), M^+(xy)] \geq [\min \{M^-(x), M^-(y)\}, \min \{ M^+(x), M^+(y)\}]$. Thus $M^-(xy) \geq \min \{M^-(x), M^-(y)\}$ and $M^+(xy) \geq \min \{M^+(x), M^+(y)\}$. Hence M^- and M^+ are fuzzy subhemiring of R. Conversely, assume that M^- and M^+ are fuzzy subhemiring of R. So, $M^-(x+y) \geq \min \{M^-(x), M^-(y)\}$ and $M^+(x+y) \geq \min \{M^+(x), M^+(y)\}$ which implies that $[M^-(x+y), M^+(x+y)] \geq [\min \{M^-(x), M^-(y)\}, \min \{ M^+(x), M^+(y)\}]$ which implies that $[M](x+y) \geq [\min \{ M^-(x), M^-(y)\}, \min \{M^+(x), M^+(y)\}]$ which implies that $[M](x+y) \geq \min \{ [M](x), [M](y) \}$. And $M^-(xy) \geq \min \{M^-(x), M^-(y)\}$ and $M^+(xy) \geq \min \{M^+(x), M^+(y)\}$ which implies that $[M^-(xy), M^+(xy)] \geq [\min \{M^-(x), M^-(y)\}, \min \{ M^+(x), M^+(y)\}]$ which implies that $[M](xy) \geq [\min \{ M^-(x), M^-(y)\}, \min \{M^+(x), M^+(y)\}]$ which implies that $[M](xy) \geq \min \{[M](x), [M](y)\}$. Hence [M] is an interval valued fuzzy subhemiring of R.

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