



Evaluation the Drift Velocity to Diffusion Coefficient Ratio for Low Energy Electrons in Air and He Gas

Ibrahim Kaïttan Fayyadh*, Kareem Khalaf Mohammad, Ali Hussein Jawad and Mohammad Mohsen Shneter
Department of Mechanical, Ministry of Science and Technology, Nahrain University, College of Engineering, Baghdad, Iraq.

ARTICLE INFO

Article history:

Received: 28 November 2013;

Received in revised form:

26 January 2014;

Accepted: 10 February 2014;

Keywords

Electron Transport Coefficient,
E-N values,
Discharge,
Thin-Film-Transistor
Density Ratios,
Electron Swarms.

ABSTRACT

In this work had be determine the ratio of the drift velocity to the diffusion coefficient W/D to calculate the motion of free electrons in air and helium gas. The following parameters of the electronic motion were determined as a function of the electric field strength E, apparent energy factor, K₁ and Townsend energy factor, K_T for Helium gas and air at 300°K and 288°K respectively in case of Maxwell and Druyvesteyn distribution law. The transport equation solved numerically to obtain the transport coefficients values and had be fed to the equations to calculate the above parameters. The gas parameters are obtained and compared with available experimental data.

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Introduction

A determination of the ratio W/D of electron drift velocity to diffusion coefficient have found application, more especially those dealing with collision phenomena for electrons with mean energy of several electron-volts, the accuracy of existing data is sufficient good.

The relation between low energy electrons and gas molecules are one of the methods of examining collision phenomena is to measure the ratio of the drift velocity W to the diffusion coefficient D of an electron swarm moving under the influence of an electric field E in a gas at a pressure p. Since W/D is a pressure-dependent quantity, the experimental results are always given either in terms of D/μ, where μ defines as the ratio W/E, or of K_T, a parameter closely related to the Townsend energy factor, K_e, which is defined as the ratio of the mean energy of agitation of the electrons to the mean thermal energy of the molecules of the gas through which the electron swarm moves. These quantities are related by the expression [1-7]:

$$K_1 = \frac{eE / KT}{W / D} = \frac{e}{KT_g} \frac{D}{\mu} \quad (1)$$

where, K, e and T_g respectively the Boltzmann's constant, electronic charge and gas temperature, which is K₁, D/μ as a functions of the E/N and T_g.

This field a benefit in arc which is used in nuclear fusion systems to generate plasma state so to reach a certain collision between ions. The collision should be enough to trigger the fusion and to find the drift velocity to electrons diffusion coefficient ratio which ionize the gas.

Theory

The partial differential equation for the electrons concentration n, in the d/dt=const. within the diffusion chamber is [1]:

$$\nabla^2 n = \frac{W}{D} \frac{\partial n}{\partial Z} = 2\lambda \frac{\partial n}{\partial Z} \quad (2)$$

Where n is the number of electros per unit volume, W the drift velocity in the Z direction, and D the diffusion coefficient of electrons.

Eq.(2) describes the distribution of electrons in a stream moving of agitational motion in a uniform electric field Z parallel to Oz in a coordinate system.

Calculation of W/D in terms of E, K_1 and K_T

The electron drift velocity W of the centre of mass of a group of electrons moving through a gas at constant and uniform electric field E is [8]:

$$W = \frac{2}{3} \frac{Ee}{m} \frac{\overline{\lambda}}{\overline{U}} \quad (3)$$

where e , m and λ are respectively the electronic charge, mass and the mean free path of an electron whose agitation velocity is U . In diatomic gases λ is not depend on U and no detectable error is introduced in Eq.(2) which is:

$$W = \frac{2}{3} \frac{e}{m} E l(U^{-1}) = \frac{2}{3} \frac{e}{m} \left(\frac{E}{p} \right) L(U^{-1}) \quad (4)$$

whereas P is the gas pressure, L the mean free path at unit pressure ($p=1\text{mm}$ of mercury) and $\overline{(U^{-1})}$ is the mean reciprocals of U along the free paths.

From the above Eqs.(3,4) the changes in the velocities U along a free path through the action of the force Ee , are small compared with $\overline{U}$, and that all directions of motion are equally probable after collision.

The simulation formula for the diffusion coefficient D according to the w is:

$$D = \frac{1}{3} \lambda \overline{U} \quad (5)$$

From Eqs.(2) obtained:

$$2\lambda = \frac{W}{D} \quad (6)$$

Substitute Eqs.(4,5) into Eq.(6) yields:

$$\begin{aligned} 2\lambda &= \frac{2}{3} \frac{e}{m} E l(U^{-1}) \frac{3}{\lambda \overline{U}} \\ 2\lambda &= 2 \frac{eE}{m} \frac{\overline{(U^{-1})}}{\overline{U}} = \frac{W}{D} \end{aligned} \quad (7)$$

From definition of Townsend energy factor K_T , which is:

$$K_T = \frac{Q}{Q_t} \quad (8)$$

where

$$Q = \frac{1}{2} m \overline{U^2} \quad (9)$$

$$Q_t = \frac{1}{2} M \overline{\Omega^2} \quad (10)$$

whereas m and $\overline{U^2}$ defines the mass and mean-square velocity of a agitation of an electron, and M , $\overline{\Omega^2}$ the corresponding quantities for a gas molecule.

from Eq.(8) obtained:

$$Q = K_T Q_t = K_T \frac{3R_0 T}{2N} \quad (11)$$

Where Q is the mean energy of agitation of an electron, and Q_t is the mean energy of thermal agitation of a gas molecule.

Multiplying the numerous and dunumerous by $\overline{U^2}$ for right hand of (9) and substitute into Eq.(7):

$$2\lambda = \frac{W}{D} = \frac{eE}{\frac{1}{2}m\overline{U^2}} \left[\frac{(\overline{U^{-1}})\overline{U^2}}{\overline{U}} \right] \quad (12)$$

$$\frac{W}{D} = \frac{eE}{K_T Q_t} \left[\frac{(\overline{U^{-1}})\overline{U^2}}{\overline{U}} \right]$$

$$\frac{W}{D} = \frac{eE}{K_T \frac{3R_0 T}{2N}} \left[\frac{(\overline{U^{-1}})\overline{U^2}}{\overline{U}} \right]$$

$$\frac{W}{D} = \frac{2NeE}{K_T 3R_0 T} \left[\frac{(\overline{U^{-1}})\overline{U^2}}{\overline{U}} \right]$$

$$\frac{W}{D} = \frac{2}{3} \frac{NeE}{R_0 T K_T} \left[\frac{(\overline{U^{-1}})\overline{U^2}}{\overline{U}} \right] = \frac{NeE}{\frac{3}{2} R_0 T K_T} \left[\frac{\overline{U}}{(\overline{U^{-1}})\overline{U^2}} \right] \quad (13)$$

where:

$$A = \frac{3}{2} \left[\frac{\overline{U}}{(\overline{U^{-1}})\overline{U^2}} \right] \quad (14)$$

$$K_1 = AK_T \quad (15)$$

Substitute Eqs.(14,15) into Eq.(13) yields:

$$\frac{W}{D} = \frac{NeE}{R_0 T K_1} \quad (16)$$

where N is Avogadro's number = $6.023 \times 10^{23} \text{ atom mol}^{-1}$, R_0 is the gas constant = $8.3143 \text{ J } ^\circ\text{K}^{-1} \text{ mol}^{-1}$ and e is columbic charge = $1.602 \times 10^{-19} \text{ C}$, it follows for temperature $T = (273 + 27)^\circ \text{Eq.}(16)$ becomes:

$$\begin{aligned} \frac{W}{D} &= \frac{6.023 \times 10^{23} \text{ atom} \cdot \text{mol}^{-1} \times 1.602 \times 10^{-19} \text{ C}}{8.3143 \text{ J } ^\circ\text{K}^{-1} \text{ mol}^{-1} \times 300 ^\circ\text{K}} \frac{E}{K_1} \\ \frac{W}{D} &= 38.683 \frac{E}{K_1} = 38.683 \frac{E}{AK_T} \end{aligned} \quad (17)$$

Eq.(17) represents the present work where E expresses in unit of V/cm. When the velocities U are distributed according to the law of Maxwell obtained.

$$K_1 = K_T \quad (18)$$

but for distributed according to the law of Druyvesteyn, which is K_T notably exceeds units as show bellow:

$$K_1 = 1.14 K_T \quad (19)$$

Theoretical procedure

Assume an electron swarm in a ionized gas. The velocity distribution function $g(r, v, t)$ in the present of a uniform electric field E can be referred by the Boltzmann equation [9,10,11]:

$$\frac{\partial g}{\partial t} + v \cdot \frac{\partial g}{\partial r} + \frac{eE}{m} \frac{\partial g}{\partial v} = J_{\text{elas}}(g) + J_{\text{exci}}(g) + J_{\text{ion}}(g) \quad (20)$$

Since v refers the velocity of the electron. J_{elas} , J_{exci} and J_{ion} are respectively, the collision integral relative to elastic, excitation and ionization collisions.

After prepared the excitation, electronical, ionization and momentum transfer cross-section respectively, had been fed to the computer code solved numerically transport equation using Finite-Difference method as in Ref.[12]. Wherefore obtained the electron transport motion parameter, namely electric field E and the ratio of the diffusion coefficient to the electron mobility (D/μ). This parameters used as input to the Eqs.(1-17).

The obtained data are tabulated and graphically in tables (1, 2) and figures (1-6) respectively.

Table (1): The input and output data for He				
W/D (cm ⁻¹) (Druyvesteyn)	W/D (cm ⁻¹) (Maxwell)	K _T (eV)	K _I (eV)	E (V/cm)
101.63	115.85	1.018	1.160	3.05
114.45	130.48	1.357	1.547	4.58
285.88	325.91	1.357	1.547	11.44
435.97	497	2.376	2.708	30.53
457.51	521.57	3.394	3.869	45.77
519.84	592.61	7.467	8.513	114.4
586.82	668.98	17.65	20.121	305.3
601.94	686.21	25.79	29.408	457.7
604.76	689.42	34.28	39.082	611.1
609.29	694.59	42.42	48.369	761.9

Table (2): The input and output data for Air				
W/D (cm ⁻¹) (Druyvesteyn)	W/D (cm ⁻¹) (Maxwell)	K _T (eV)	K _I (eV)	E (V/cm)
0.876	0.999	3.36	3.83	0.099
1.195	1.362	4.95	5.64	0.199
1.464	1.669	6.07	6.92	0.299
1.698	1.936	6.99	7.97	0.399
2.012	2.294	8.85	10.09	0.599
2.166	2.469	10.96	12.49	0.798
2.261	2.577	13.13	14.97	0.998
2.435	2.776	18.29	20.85	1.497
2.664	3.037	22.33	25.46	1.999
3.209	3.659	27.79	31.69	2.998
3.770	4.298	31.49	35.90	3.990
4.376	4.989	33.94	38.69	4.991
7.418	8.457	40.05	45.66	9.983

Results and Discussion

Figs.(1, 4) are showing the increasing in the ratio of the drift velocity to the diffusion coefficient, W/D of an electron swarm moving under the influence of an electron field E , in Helium gas and air, from Fig.(1), at $E = (3.05 - 45.77)$ V/cm the ratio W/D was rapidly increasing then after these values the ratio was stable with increasing of E for both Maxwell and Druyvesteyn, but the Fig.(4) at $E = (0.099 - 0.599)$ V/cm, the ratio W/D was rapidly increasing then after these values, the increasing between W/D and E is approximately linear for both Maxwell and Druyvesteyn distribution law in air.

Figs.(2, 5) are appearing the ratio W/D as a function of the apparent energy factor K_I . From fig.(2), at $K_I = (1.16087 - 2.708696)$ eV, the ratio W/D was rapidly increasing then after this values, the ratio W/D could be increasing gradually to show stability with K_I increasing, but from Fig.(5) at $K_I = (3.83087 - 7.971304)$ eV, the relation between W/D and K_I is linear, after this values W/D increase exponentially with K_I for both Maxwell and Druyvesteyn law in air.

Figs.(3, 6) are showing the ratio W/D against K_I . From Fig.(3), at $K_T = (1.018307 - 2.376049)$ eV, the ratio W/D are rapidly increase, then after this values the ratio W/D could be stable with K_T , but from Fig.(6) at $K_T = (3.360412 - 6.992372)$ eV, the relation

between W/D and K_T is linear, then after these values the relation could be exponential. These results are in good agreement with the experimental data published by [13].

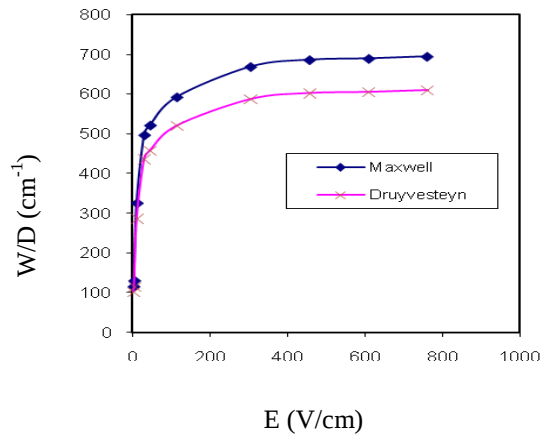


Fig.(1): The ratio of the electron drift velocity W to the diffusion coefficient D as a function of the applied electric field E in He gas.

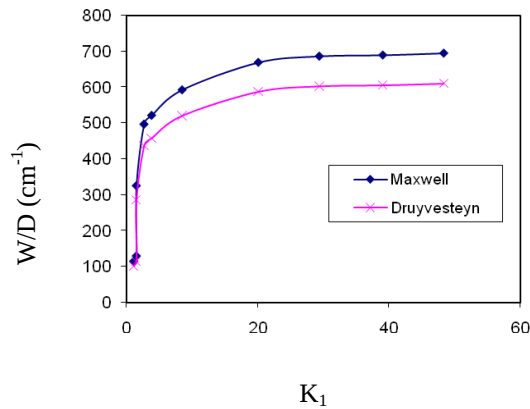


Fig.(2): The ratio of the electron drift velocity W to the diffusion coefficient D as a function of K_1 in He gas.

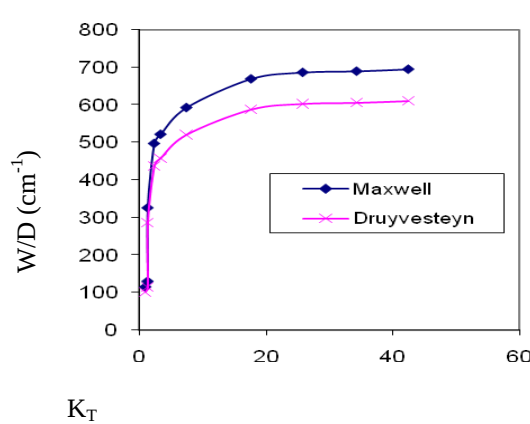


Fig.(3): The ratio of the electron drift velocity W to the diffusion coefficient D as a function of K_T in He gas.

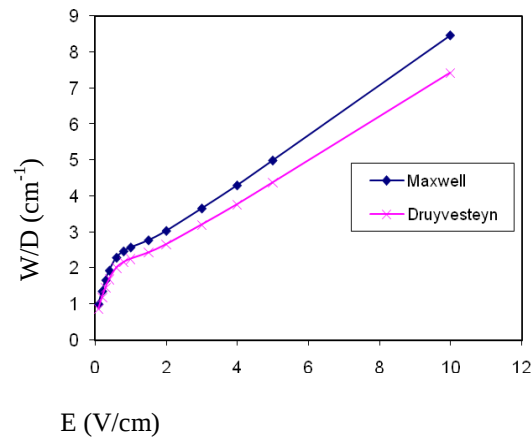


Fig.(4): The ratio of the electron drift velocity W to the diffusion coefficient D as a function of the applied electric field E in Air.

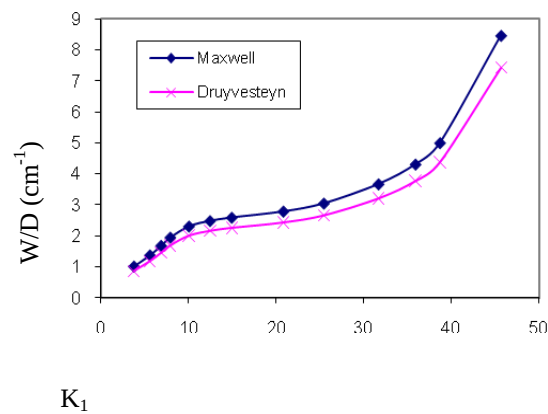


Fig.(5): The ratio of the electron drift velocity W to the diffusion coefficient D as a function of K_1 in Air.

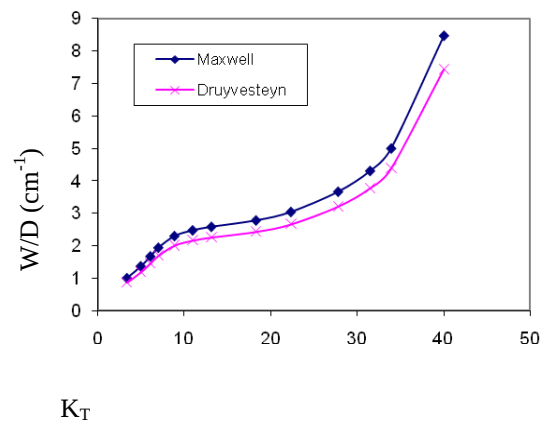


Fig.(6): The ratio of the electron drift velocity W to the diffusion coefficient D as a function of K_T in Air.

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