Available online at www.elixirpublishers.com (Elixir International Journal)

Computer Science and Engineering

Elixir Comp. Sci. & Engg. 67 (2014) 21617-21622

ISSN: 2229-712X

Singular Value Decomposition (SVD) Before Covariance Based PCA for Image De-noising

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ARTICLE INFO

Article history:

Received: 27 July 2013;

Received in revised form: 24 January 2014;

Accepted: 8 February 2014;

Keywords

SVD,

PCA,

LPG.

ABSTRACT

This paper describes an efficient approach of using (SVD) before covariance PCA. Singular Value Decomposition (SVD) is a method of finding insignificant pixel values from an image. Though covariance PCA is used to de-correlate the original data set by extracting principle components of data set but there also left some noises. Singular Value Decomposition (SVD) is a method that discards insignificant pixel values from an image without affecting the quality of the image. In this method a pixel and its nearest neighbors are considered as the training samples recognized by using block matching regarded as Local Pixel Grouping (LPG). Experimental result shows that the use of SVD before covariance based PCA demonstrate that the de-noising performance of image is improved compared with state-of-art de-noising algorithm.

Introduction

Today most important digital images are often corrupted by image acquisition process so de-noising is an essential step to improve the image quality. When the noise type is recognized the [1] fixed filter can be applied for image de-noising but for dynamic noise PCA give best pattern recognition of the noise. In dynamic noise the use of SVD before covariance PCA gives more enhanced image. In this paper we represent an efficient PCA based de-noising with Singular Value Decomposition (SVD). Singular Value Decomposition only reduces the type of signals or pixels that are just similar to trivial or noisy signal without changing the visualization quality of an image. By transforming the original data set into covariance PCA, the noise and trivial information can be removed by preserving only the several most significant principle components but there is left some noise. In this paper, a SVD enhanced PCA scheme is proposed for image de-noising by using a moving window.

The organization of the paper is as follows: Section (II) describes proposed system flowchart and overview; section (III) describes Principle Component Analysis; section (IV) describes the relation between SVD and PCA; section (V) describes enhancement of image using SVD; section (VI) SVD de-noising algorithm; section (VII) describes Experimental result; section (VIII) describes Conclusion and then the References are denoted.

Proposed System Flowchart and Overview

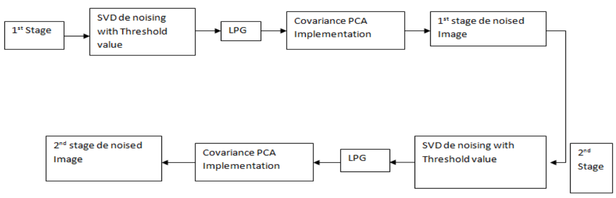

Fig.1. Flowchart of the proposed SVD based two-stage PCA-LPG de-noising scheme.

In this de-noising procedure the proposed SVD-based LPG-PCA method yields competitive with only covariance PCA method. SVD is a method used to identifying and ordering the dimension along with data points exhibit the most variation which tie into the third way to find SVD, if we have identified where the most variation is, it is possible to find the best approximation of the original data points using fewer dimensions. Hence, SVD can be seen as a method for data reduction.

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Let A be a matrix representing a given image where M×N represents the image row and column. We can obtain the truncated sum [6] after the first K term. Where K is the rank of matrix and K must be less than N which is very close to the original image and compression of the data represented by matrix A and the amount of storage necessary for the original matrix A was reduced if A is constructed by rank K. SVD method enhances the image, reducing the lower Eigen value and reconstructs an image that is very close to the original image A.

Here we use a minimal threshold .06 before PCA transformation for reducing dynamic noise before first stage .Thus the procedure will go two stage and it will get more enhanced image. In dynamic noise the threshold value is .01 is more efficient.

Principle Component Analysis (PCA)

Let $A = [A = [a_1\ a_2 \dots\dots . . a_m]^T]^T$ an m-component vector variable and denoted by

$$A = \begin{bmatrix} a_1^1 & a_1^2 \dots\dots . . & a_1^n \\ a_2^1 & a_2^2 \dots\dots\dots & a_2^n \\ \vdots & \vdots & \vdots \\ a_m^1 & a_m^2 & a_m^n \end{bmatrix}$$

The sample matrix of A, where a_i^j ; j=1, 2……….n are the discrete sample of variable a_i,i=1, 2………n. The i^{th} row of sample matrix A, denoted by

$A_i = [a_i^1\ a_i^2 \dots\dots\dots a_i^n]$ is called by sample vector a_i. The mean value of A_i is calculated as

$\mu_i = \frac{1}{n} \sum_{j=1}^{n} A_i\,(j)$

And the sample vector A_i is [3] centralized matrix of A is $\vec{A}_i = A_i - \mu_i = [\bar{a}_i^1\ \bar{a}_i^2 \dots\dots\dots\dots\dots\dots . . \bar{a}_i^n]$

where $\bar{a}_i^j = a_i^j - \mu_i$. Accordingly, the centralized matrix of A is $\vec{A} = [\vec{A}_1^T\ \vec{A}_2^T \dots\dots\dots\dots\dots . . \vec{A}_m^T]^T$

Compute the SVD of the matrix $\vec{A}$ and obtain the singular values $\vec{A}_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$

Here k is the rank. Reduce the singular values by a chosen rank k of the matrix $\vec{A}_k$ denoted by

$\vec{A} = \sum_{i=1}^{k} (\vec{A}_k + \sum_{i=1}^{k} \sigma_i u_i v_i^T)$

Now the co-variance matrix of the Singular Value reduced matrix is calculated as $\Omega = \frac{1}{N} \vec{A}\vec{A}^T$

The goal of PCA is to find orthonormal transformation matrixes P represents $\vec{A}$ de-correlate, i.e. $\vec{Z} = P\vec{A}$ for that the co-variance matrix of Z is diagonal. Since the covariance matrix Ω is symmetrical, it canbe written as $\Omega = \emptyset \cap \bar{\emptyset}$

Where $\emptyset = [\emptyset_1\ \emptyset_2 \dots\dots\dots\dots \emptyset_m]$ is the m × m orthonormal eigenvector matrix and $\cap = diag\{\gamma_1\ \gamma_2 \dots\dots\dots\dots . . \gamma_m\}$ is the diagonal eigenvalue matrix with $\gamma_1 \geq \gamma_2 \geq \dots\dots\dots\dots . \geq \gamma_3$. The terms $\emptyset_1\ \emptyset_2 \dots\dots\dots\dots \emptyset_m$ and $\gamma_1\ \gamma_2 \dots\dots\dots\dots . . \gamma_m$ are the eigenvectors and eigenvalues of Ω respectively.

By setting $P = \emptyset^T$

$\vec{A}$ Can be de-correlated, i.e. $\vec{Z} = P\vec{A}$ and $\cap = \frac{1}{N} \vec{A}\vec{A}^T$

The use of SVD in covariance based PCA reduce the low eigenvalues that is the type of noisy value and the energy of signal will concentrate on a small subset of the PCA transformed dataset, while the energy of the noise will evenly spread over the whole dataset i.e. it fully de-correlates the original dataset $\vec{A}$, separating signal from noise.

SVD and PCA

With similar computations it is [5] evident that the two methods are intimately related. Let us return to the original $m \times n$ data matrix X. We can define a new matrix Y as a $n \times m$ matrix

$Y = \frac{1}{\sqrt{n-1}} X^T$

where each column of Y has Zero mean. The definition of Y becomes clear by analyzing $Y^T Y$

$Y^T Y = \left(\frac{1}{\sqrt{n-1}} X^T\right)^T \left(\frac{1}{\sqrt{n-1}} X^T\right)$

$$= \frac{1}{n-1} X^{TT} X^T$$

$$= \frac{1}{n-1} X X^T$$

$$Y^T Y = C_x$$

By construction $Y^T Y$ equals the covariance matrix of X. We know that the principal components of X are the eigenvectors of C_x. If we calculate the SVD of Y, the columns of matrix V contain the Eigen vectors of $Y^T Y = C_x$. Therefore, the columns of V are the principle components of X.

Enhance Image Using SVD and Reducing Low Eigen value

SVD is a method for identifying and ordering the dimension along which data points exhibit the most variation. This ties into the third way to find SVD, which means once we have identified where the most variation is, it is possible [4] to find the best approximation of the original data points using fewer dimensions. Hence, SVD can be seen as a method for data reduction.

Let A be matrix representing a given image and A=$U \sum V^T$, then A can be written as

A=$\sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \ldots\ldots\ldots\ldots + \sigma_n u_n v_n^T$

We can obtain the truncated sum [6] after the first K term.

$$A_k = \sigma_1 u_1 v_1^T + \sigma_2\, u_2 v_2^T + \ldots\ldots\ldots\ldots + \sigma_k u_k v_k^T$$

Where k is the rank of matrix A_k and K must be less than n. A_k is very close to the original image.

A_k is a compression of the data represented by matrix A, where the amount of storage necessary for the original matrix A was reduced. If A_k has rank K by construction, then

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

This type of SVD method enhances the image, reducing the lower Eigen value and reconstructs an image A_k that is similar to the original image A.

DE-noising Algorithm

Using SVD before PCA de-noising algorithm

Variances are always related with the principle components of a dataset. Often we can find that large variances are associated with Noise by having the first K<M principal components .We can remove that most insufficient pixels by having first K dimensions in N dimensions and find the main principle components by using SVD [].

Variances are always related with the principle components of a dataset. Often we can find that large variances are associated with Noise by having the first K<M principal components .We can remove that most insufficient pixels by having first K dimensions in N dimensions and find the main principle components by using SVD [7].

Let A be an M×N matrix then we know

[U,S,V] = SVD (A);

Where

$U^T U$ = I and $V^T V$ = I

Here U and V are orthogonal and I is Identity matrix,

If M ≥ N where U is an M × K matrix, V is an N × K matrix, and S is a K × K matrix, where k is the rank of the matrix The columns of U are the left singular vectors (generate coefficient vectors); S (the same dimensions as A) has singular values and is diagonal; and V^T has rows that are the right singular vectors. The SVD represents an expansion of the original data in a coordinate system where the covariance matrix is diagonal. Diagonalizing S variance and finding the values which value is below of predefined threshold value that has to be set to zero. For each value of S find them with threshold value and set them null and retrieve the main image with

$$A_1 = A_1 + S_2(i) \times U(:,i) \times V(:,i)^T$$

Here S_2 is the reduced S; Here A_1 is first stage image to go through second image step by step .First use in SVD rank K=.60 in first stage to send the image in the first step in random noise and image noise is just can be removed smoothly only PCA technique which used to find best component in image .If we use here more threshold in more than .60 then the image main component will be lose for that here use fixed low variance and in dynamic noise it is tolerable .Thus by LPG PCA covariance technique in the first stage before entering the 2nd stage processing and it also get through SVD .Here in 2nd stage the image contain little amount of noise that why used the threshold variance only 0.01.Thus the Image random noise in second stage could get reduced by .01 threshold variance for dynamic noise.

Local Pixel Grouping (LPG)

To solve the classification problem occurs when grouping the training samples similar to the central K×K block in the L×L training window, the block matching method is the simplest and efficient one; in this paper use it for LPG.

In A_v which is computed image there are $(l-k+1)^2$ possible trading blocks, within L×L training window, We denote by $\vec{a}_0^v$ the column sample vector containing the pixels in the central K×K block and denote by a_i^v, i=1,2 …..$(l-k+1)^2$-1 the sample vectors corresponding to the other blocks .Let $\bar{a}_0$ and $\bar{a}_i$ be the associated noiseless [3] sample vectors of $\vec{a}_0^v$ and $\vec{a}_i^v$ respectively and can be calculated that

$$e_i = \frac{1}{m}\sum_{k=1}^{m} \vec{a}_0^v(k) - \vec{a}_i^v(k)^2 \approx \frac{1}{m}\sum_{k=1}^{m} \bar{a}_0(k) - \bar{a}_i(k)^2 + 2\sigma^2$$

White noise will be uncorrelated with signal, of the following condition is satisfied, i.e. e_i <T+$2\sigma^2$

Where T is the preset threshold which is chosen by the programmer, for better result the T value in first stage is different from the value in second stage of de-noising.σ^2 represents the noise level of the noisy image. If σ increase, the image get more corrupted. The training data set $A_v = [\bar{a}_0^v\, \bar{a}_{01}^v \ldots\ldots\ldots\ldots \bar{a}_{n-1}^v]$.

We can calculate $A = [\bar{a}_0\, \bar{a}_1 \ldots\ldots\ldots\ldots\ldots\ldots \bar{a}_{n-1}]$ from the corrupted image A_v.Where A is the noiseless data set. The window K×K more over the while L×L training block so that whole image can be de-noised.

Experiment Result

To get proper response from Principle Component Analysis it is necessary to have proper number of training samples. Here we have performed the experiment over Lena, House and Cameraman.

Fig 3: The test data image Lena, House and Cameraman.

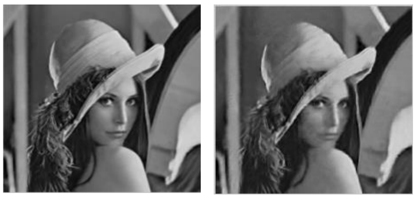
In the proposed SVD before PCA de-noising algorithm. With comparing the existing method [3], our proposed SVD based PCA de-noising has appear with higher PSNR and better image quality.

(a) Original image (b) Noisy Image

(c) 1st stage de-noised image (d) 2nd stage de-noised image

Fig 4: The steps of noise refinement using variance (σ=20) for Lena.

We see that the visual quality is much improved after the second stage refinement. Similarly for variance $\sigma = 10$ and $\sigma = 30$



Variance (σ=10) variance (σ=30)

Fig 5: Output image of Lena.

For image House the 2nd stage de-noised images are for different variances

(a) $\sigma = 10$ (b) $\sigma = 20$ (c) $\sigma = 30$

Fig 6: Output image of House.

For image Cameraman the 2nd stage de-noised images are for different variances

(a) $\sigma = 10$ (b) $\sigma = 20$ (c) $\sigma = 30$

Fig 7: Output Image of Cameraman.

Table I: The PSNR (dB) and SSIM results of the denoised images in the two stages by the proposed SVD based LPG-PCA method

Images	Lena	House	Cameraman
1st stage	PSNR(SSIM)	PSNR(SSIM)	PSNR(SSIM)
$\sigma = 10$	34.027(.9254)	35.921(.9074)	34.021(.9417)
$\sigma = 20$	30.217(.8466)	32.264(.8119)	29.909(.8799)
$\sigma = 30$	27.863(.7583)	29.801(.7095)	27.368(.7636)

Images	Lena	House	Cameraman
2nd stage	PSNR(SSIM)	PSNR(SSIM)	PSNR(SSIM)
$\sigma = 10$	34.119(.9292)	36.090(.9136)	34.391(.9817)
$\sigma = 20$	30.545(.8755)	33.122(.8674)	30.310(.9001)
$\sigma = 30$	28.363(.8236)	31.181(.8299)	27.903(.8675)

The table below represents the PSNR of existing system [3] using simple covariance based PCA method that is comparatively lower than our proposed SVD based PCA method.

Table II: The PSNR (dB) and SSIM results of the de-noised images in the two stages by Covariance based LPG-PCA method

Images	Lena	House	Cameraman
1st stage	PSNR(SSIM)	PSNR(SSIM)	PSNR(SSIM)
$\sigma = 10$	33.6(0.9218)	35.4(0.9003)	33.9(0.9261))
$\sigma = 20$	29.5(0.8346)	31.8(0.8084)	29.8(0.8320)
$\sigma = 30$	27.1(0.7441)	29.3(0.7225)	27.3(0.7395)

Images	Lena	House	Cameraman
2nd stage	PSNR(SSIM)	PSNR(SSIM)	PSNR(SSIM)
$\sigma = 10$	33.7(0.9243)	35.6(0.9012)	34.1(0.9356)
$\sigma = 20$	29.7(0.8605)	32.5(0.8471)	30.1(0.8902)
$\sigma = 30$	27.6(0.8066)	30.4(0.8185)	27.8(0.8558)

Comparing with the existing system [3] we have got higher PSNR and better image quality.

Conclusion

This paper proposed a spatially adaptive image denoising scheme by using SVD before principal component analysis (PCA).In this method iterated one more time to improve the denoising performance and preserve the image fine structures while smoothing noise. It presents a competitive denoising solution compared with two stage image denoising using covariance base LPG-PCA.

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