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Efficient Ratio-Cum Product Estimator using Stratified Ranked Set Sampling

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ABSTRACT

This paper proposed a modified ratio-cum-product estimators of finite population mean using information on coefficient of variation and co-efficient of kurtosis of auxiliary variable in Stratified Ranked Set Sampling (SRSS). It has been shown that this method is highly beneficial to the estimation based on Stratified Simple Random Sampling (SSRS). The bias and mean squared error of the proposed estimators are derived. Theoretically, it is shown that these suggested estimators are more efficient than the estimators in Stratified simple random sampling. The results have been illustrated by numerical examples.

Introduction

The literature on Ranked set sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ranked set sampling (RSS) was first suggested by McIntyre (1952) and Stratified Ranked Set Sampling was introduced by Samawi (1996) to increase the efficiency of estimator of population mean. The performance of the combined and the separate ratio estimates using the stratified ranked set sample (SRSS) was given by Samawi and Siam (2003). Here we shall improve ratio cum product estimators given by Singh et al.(2005) and Tailor et al. (2011), respectively by using SRSS based on auxiliary variable.

The combined ratio and product estimator of population mean $\bar{Y}$ in stratified random sampling is defined by

$$\bar{y}_{R,SSRS} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \quad (1.1)$$

$$\bar{y}_{P,SSRS} = \bar{y}_{st}\left(\frac{\bar{x}_{st}}{\bar{X}}\right) \quad (1.2)$$

where $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$ are the unbiased estimators of population mean $\bar{Y}$ and $\bar{X}$ respectively.

When the population coefficient of variation C_x is known, Motivated by Sisodia and Dwivedi (1981), Kadilar and Cingi (2003) suggested a modified ratio estimator for $\bar{Y}$ in stratified random sampling as

$$\bar{y}_{stSD} = \bar{y}_{st}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{x_h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + C_{x_h}\right)} \quad (1.3)$$

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Motivated by Singh and Kakran (1993), Kadilar and Cingi (2003) developed ratio-type estimator for $\overline{Y}$ as

$$\overline{y}_{stSK} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h \left(\overline{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\overline{x}_h + \beta_{2h}(x)\right)} \tag{1.4}$$

Estimators based on Upadhyaya and Singh (1999), using both coefficient of variation and kurtosis in stratified random sampling, Kadilar and Cingi (2003) considered the following ratio and product estimators respectively

$$\overline{y}_{stUS1} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h \left(\overline{X}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\overline{x}_h C_{x_h} + \beta_{2h}(x)\right)} \tag{1.5}$$

$$\overline{y}_{stUS2} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h \left(\overline{x}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\overline{X}_h C_{x_h} + \beta_{2h}(x)\right)} \tag{1.6}$$

To estimate $\overline{Y}$, Singh et al.(2005) suggested the combined ratio- product estimator as

$$\hat{\overline{Y}}_{RP,SSRS} = \overline{y}_{st} \left[\alpha \left(\frac{\overline{X}}{\overline{x}_{st}} \right) + (1-\alpha) \left(\frac{\overline{x}_{st}}{\overline{X}} \right) \right] \tag{1.7}$$

where α is a real constant to be determined such that the mean squared error (MSE) is minimum.

Utilizing the information on co-efficient of variation and co-efficient of kurtosis of the auxiliary variable x, Tailor et al. (2011) proposed the modified ratio-cum-product estimator of population mean under SSRS is given by

$$\hat{\overline{Y}}_{M,SSRS} = \overline{y}_{st} \left[\alpha \left\{ \frac{\sum_{h=1}^{L} W_h \left(\overline{X}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\overline{x}_h C_{x_h} + \beta_{2h}(x)\right)} \right\} + (1-\alpha) \left\{ \frac{\sum_{h=1}^{L} W_h \left(\overline{x}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\overline{X}_h C_{x_h} + \beta_{2h}(x)\right)} \right\} \right] \tag{1.8}$$

Where α is a suitably chosen scalar. It is to be noted that for α =1 and α =0, $\hat{\overline{Y}}_{M,SSRS}$ reduces to the estimators $\overline{y}_{stUS1}$ and $\overline{y}_{stUS2}$ respectively.

To the first degree of approximation the mean squared error (MSE) of the estimators $\overline{y}_{R,SSRS}$, $\overline{y}_{P,SSRS}$, $\overline{y}_{stSD}$, $\overline{y}_{stSK}$, $\overline{y}_{stUS1}$, $\overline{y}_{stUS2}$, $\hat{\overline{Y}}_{RP,SSRS}$ and $\hat{\overline{Y}}_{M,SSRS}$ respectively are

$$MSE(\overline{y}_{SSRS}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left(S_{y_h}^2 + R^2 S_{x_h}^2 - 2RS_{x_h y_h} \right) \tag{1.9}$$

$$MSE(\overline{y}_{PSRS}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left(S_{y_h}^2 + R^2 S_{x_h}^2 + 2RS_{x_h y_h} \right) \tag{1.10}$$

$$MSE(\overline{y}_{stSD}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left(S_{y_h}^2 + R^2 \lambda_1^2 S_{x_h}^2 - 2R\lambda_1 S_{x_h y_h} \right) \tag{1.11}$$

$$MSE(\bar{y}_{stSK}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2 + R^2\lambda_2^2 S_{x_h}^2 - 2R\lambda_2 S_{x_h y_h}\right) \quad (1.12)$$

$$MSE(\bar{y}_{stUS1}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2 + R^2\gamma_1^2 S_{x_h}^2 - 2R\gamma_1 S_{x_h y_h}\right) \quad (1.13)$$

$$MSE(\bar{y}_{stUS2}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2 + R^2\gamma_2^2 S_{x_h}^2 + 2R\gamma_2 S_{x_h y_h}\right) \quad (1.14)$$

$$MSE(\hat{Y}_{RP,SSRS}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2 + 2R(1-2\alpha)S_{x_h y_h} + R^2(1-2\alpha)^2 S_{x_h}^2\right] \quad (1.15)$$

and
$$MSE(\hat{Y}_{RP,SSRS}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2 + 2Rt_{us}(1-2\alpha)S_{x_h y_h} + R^2 t_{us}^2(1-2\alpha)^2 S_{x_h}^2\right] \quad (1.16)$$

where
$$\lambda_1 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h(\bar{X}_h + C_{x_h})}, \quad \lambda_2 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h(\bar{X}_h + \beta_{2h}(x))}, \quad \gamma_1 = \gamma_2 = t_{us} = \frac{\sum_{h=1}^{L} W_h \bar{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h(\bar{X}_h C_{x_h} + \beta_{2h}(x))},$$

$$S_{y_h}^2 = \frac{\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)^2}{N_h - 1}, \quad S_{x_h}^2 = \frac{\sum_{i=1}^{N_h}(X_{hi} - \bar{X}_h)^2}{N_h - 1} \text{ and } S_{y_h x_h} = \frac{\sum_{i=1}^{N_h}(Y_{hi} - \bar{Y}_h)(X_{hi} - \bar{X}_h)}{N_h - 1}.$$

Stratified ranked set sample

In Ranked set sampling (RSS), r independent random sets, each of size r and each unit in the set being selected with equal probability and without replacement , are selected from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest rank is chosen from the r^{th} set. This cycle may be repeated m times, so $mr\ (= n)$ units have been measured during this process.

In Stratified ranked set sampling, for the h^{th} stratum of the population, first choose r_h independent samples each of size r_h $h = 1, 2, \ldots, L$. Rank each sample, and use RSS scheme to obtain L independent RSS samples of size r_h, one from each stratum. Let $r_1 + r_2 + \ldots\ldots + r_L = r$. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until $n = mr$ *elements* have been obtained. A modification of the above procedure is suggested here to be used for the estimation of the ratio using stratified ranked set sample. For the h^{th} stratum, first choose r_h independent samples each of size r_h of independent bivariate elements from the h^{th} subpopulation, $h = 1, 2, \ldots, L$. Rank each sample with respect to one of the variables say Y or X. Then use the RSS sampling scheme to obtain L independent RSS samples of size r_h one from each stratum. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until $n = mr$ bivariate elements have been obtained. We will use the following notation for the stratified ranked set sample when the ranking is on the variable X. For the k^{th} cycle and the h^{th} stratum, the SRSS is denoted by

$$\left\{(Y_{h[1]k}, X_{h(1)k})(Y_{h[2]k}, X_{h(2)k}), \ldots\ldots\ldots (Y_{h[r_h]k}, X_{h(r_h)k}) : k = 1,2,\ldots m; h = 1,2,\ldots L\right\},$$ where $Y_{h[i]k}$ is the i^{th}

Judgment ordering in the i^{th} set for the study variable and $X_{h(i)k}$ is the i^{th} order statistic in the i^{th} set for the auxiliary variable.

The combined ratio and product estimator of population mean $\bar{Y}$ given by Samawi and Siam (2003) and Bouza (2008) using stratified ranked set sampling are respectively, defined as

$$\bar{y}_{R,SRSS} = \bar{y}_{[SRSS]}\left(\frac{\bar{X}}{\bar{x}_{(SRSS)}}\right) \tag{2.1}$$

$$\bar{y}_{P,SRSS} = \bar{y}_{[SRSS]}\left(\frac{\bar{x}_{(SRSS)}}{\bar{X}}\right) \tag{2.2}$$

where $\bar{y}_{[SRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[r_h]}$ and $\bar{x}_{(SRSS)} = \sum_{h=1}^{L} W_h \bar{x}_{h(r_h)}$.

To the first degree of approximation the mean squared error (MSE) of the estimators $\bar{y}_{R,SRSS}$ and $\bar{y}_{P,SRSS}$ are respectively given by

$$MSE(\bar{y}_{R,SRSS}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2 S_{x_h}^2 - 2RS_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - D_{x_h(i)}\right)^2\right\}\right] \tag{2.3}$$

$$MSE(\bar{y}_{P,SRSS}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2 S_{x_h}^2 + 2RS_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} + D_{x_h(i)}\right)^2\right\}\right] \tag{2.4}$$

where $n_h = mr_h$, $D_{y_h[i]}^2 = \frac{\tau_{y_h[i]}^2}{\bar{Y}^2}$, $D_{x_h(i)}^2 = \frac{\tau_{x_h(i)}^2}{\bar{X}^2}$ and $D_{x_h(i)y_h[i]} = \frac{\tau_{x_h(i)y_h[i]}}{\bar{Y}\bar{X}}$. Here we would also like to remind that $\tau_{x_h(i)} = \mu_{x_h(i)} - \bar{X}_h$, $\tau_{y_h[i]} = \mu_{y_i[i]} - \bar{Y}_h$ and $\tau_{x_h(i)y_h[i]} = (\mu_{x_h(i)} - \bar{X}_h)(\mu_{y_h[i]} - \bar{Y}_h)$ where $\mu_{x_h(i)} = E[x_{h(i)}]$, $\mu_{y_h(i)} = E[y_{h(i)}]$, $\bar{X}_h$ and $\bar{Y}_h$ are the means of the h^{th} stratum for variables X and Y respectively.

Proposed Estimators based on SRSS

Motivated by Kadilar and Cingi (2003), we suggest ratio-type estimator for $\bar{Y}$ using stratified ranked set sampling, when the population coefficient of variation of auxiliary variable C_x is known as follows-

$$\bar{y}_{MM1,str} = \bar{y}_{[SRSS]}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{x_h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(r_h)} + C_{x_h}\right)} \tag{3. 1}$$

where $\bar{y}_{[SRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[r_h]}$ and $\bar{x}_{(SRSS)} = \sum_{h=1}^{L} W_h \bar{x}_{h(r_h)}$.

To obtain bias and MSE of $\bar{y}_{strMM1}$, we put $\bar{y}_{[SRSS]} = \bar{Y}(1+\delta_0)$ and $\bar{x}_{(SRSS)} = \bar{X}(1+\delta_1)$ so that $E(\delta_0) = E(\delta_1) = 0$.

Now

$$V(\delta_0) = E(\delta_0^2) = \frac{V(\bar{y}_{[SRSS]})}{\bar{Y}^2}$$

$$= \sum_{h=1}^{L} W_h^2 \frac{1}{mr_h}\frac{1}{\bar{Y}^2}\left[S_{y_h}^2 - \frac{m}{n_h}\sum_{i=1}^{r_h}\tau_{y_h[i]}^2\right] = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_h[i]}^2\right]$$

Similarly, $E(\delta_1^2) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right]$

and $E(\delta_0,\delta_1) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{x_hy_h}}{\overline{X}\,\overline{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right]$

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\delta_0|$ and $|\delta_1|$ as small so that the terms involving δ_0 and or δ_1 in a degree greater than two will be negligible.

Bias and MSE of the estimator $\bar{y}_{strMM1}$ to the first degree of approximation are respectively given by

$B(\bar{y}_{strMM1}) = E(\bar{y}_{strMM1}) - \overline{Y}$

Here $\bar{y}_{strMM1} = \overline{Y}(1+\delta_0)(1+\lambda_1\delta_1)^{-1}$, where $\lambda_1 = \dfrac{\sum_{h=1}^{L}W_h\overline{X}_h}{\sum_{h=1}^{L}W_h(\overline{X}_h + C_{x_h})}$.

Now $E(\bar{y}_{strMM1}) = \overline{Y}\left[1+\lambda_1^2E(\delta_1^2) - \lambda_1E(\delta_0\delta_1)\right]$, because $E(\delta_0) = E(\delta_1) = 0$

(Using Taylor series expansion, where $O(\delta_1)$ with power more than 2 are neglected for large power of δ_1.)

$$\Rightarrow B(\bar{y}_{strMM1}) = \overline{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{\lambda_1^2S_{x_h}^2}{\overline{X}^2} - \frac{\lambda_1S_{x_hy_h}}{\overline{X}\,\overline{Y}}\right\} - \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\lambda_1^2\sum_{i=1}^{r_h}D_{x_h(i)}^2 - \lambda_1\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right\}\right] \quad (3.2)$$

Now $MSE(\bar{y}_{strMM1}) = E(\bar{y}_{strMM1} - \overline{Y})^2$

$$= \overline{Y}^2E\left[\delta_0^2 + \lambda_1^2\delta_1^2 - 2\lambda_1\delta_0\delta_1\right]$$

$$= \overline{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\overline{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\} + \lambda_1^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\overline{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right\} - 2\lambda_1\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_hy_h}}{\overline{X}\,\overline{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right\}\right]$$

$$MSE(\bar{y}_{strMM1}) = \overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{\frac{S_{y_h}^2}{\overline{Y}^2} + \frac{\lambda_1^2S_{x_h}^2}{\overline{X}^2} - 2\lambda_1\frac{S_{x_hy_h}}{\overline{X}\,\overline{Y}}\right\} - \frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \lambda_1D_{x_h(i)}\right)^2\right]$$

$$MSE(\bar{y}_{strMM1}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2\lambda_1^2S_{x_h}^2 - 2R\lambda_1S_{x_hy_h}\right\} - \overline{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \lambda_1D_{x_h(i)}\right)^2\right\}\right] \quad (3.3)$$

Adapting the estimators in (1.3), given by Kadilar and Cingi (2003), we suggest the new ratio estimator in stratified ranked set sampling is as follows

$$\bar{y}_{strMM2} = \bar{y}_{[SRSS]}\frac{\sum_{h=1}^{L}W_h\left(\overline{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L}W_h\left(\bar{x}_{h(r_h)} + \beta_{2h}(x)\right)} \quad (3.4)$$

The Bias and MSE of $\bar{y}_{strMM2}$ can be found as follows-

$B(\bar{y}_{strMM2}) = E(\bar{y}_{strMM2}) - \overline{Y}$

Here $\bar{y}_{strMM2} = \bar{Y}(1+\delta_0)(1+\lambda_2\delta_1)^{-1}$, where $\lambda_2 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h(\bar{X}_h + \beta_{2h}(x))}$.

$$Now \quad E(\bar{y}_{strMM2}) = \bar{Y}\left[1+\lambda_2^2 E(\delta_1^2) - \lambda_2 E(\delta_0\delta_1)\right], \quad \text{because } E(\delta_0)=E(\delta_1)=0$$

$$\Rightarrow B(\bar{y}_{strMM2}) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{\lambda_2^2 S_{x_h}^2}{\bar{X}^2} - \frac{\lambda_2 S_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\lambda_2^2\sum_{i=1}^{r_h} D_{x_h(i)}^2 - \lambda_2\sum_{i=1}^{r_h} D_{x_h(i)y_h[i]}\right)\right\}\right] \qquad (3.5)$$

Now $MSE(\bar{y}_{strMM2}) = E(\bar{y}_{strMM2} - \bar{Y})^2$

$$= \bar{Y}^2 E\left[\delta_0^2 + \lambda_2^2\delta_1^2 - 2\lambda_2\delta_0\delta_1\right]$$

$$= \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_h[i]}^2\right\} + \lambda_2^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right\} - 2\lambda_2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)y_h[i]}\right\}\right]$$

$$MSE(\bar{y}_{strMM2}) = \bar{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{\frac{S_{y_h}^2}{\bar{Y}^2} + \frac{\lambda_2^2 S_{x_h}^2}{\bar{X}^2} - 2\lambda_2\frac{S_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \lambda_2 D_{x_h(i)}\right)^2\right]$$

$$MSE(\bar{y}_{strMM2}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2\lambda_2^2 S_{x_h}^2 - 2R\lambda_2 S_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \lambda_2 D_{x_h(i)}\right)^2\right\}\right] \qquad (3.6)$$

Motivated by Kadilar and Cingi (2003), we suggest the ratio –type estimators based on Upadhyaya and Singh (1999) considered both coefficients of variation and Kurtosis in stratified ranked set sampling as

$$\bar{y}_{strMM3} = \bar{y}_{[SRSS]}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(r_h)} C_{x_h} + \beta_{2h}(x)\right)} \qquad (3.7)$$

$$\bar{y}_{strMM4} = \bar{y}_{[SRSS]}\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h(r_h) C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)} \qquad (3.8)$$

The Bias and MSE of $\bar{y}_{strMM3}$ can be found as follows-

$$B(\bar{y}_{strMM3}) = E(\bar{y}_{strMM3}) - \bar{Y}$$

Here $\bar{y}_{strMM3} = \bar{Y}(1+\delta_0)(1+\gamma_1\delta_1)^{-1}$, where $\gamma_1 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h(\bar{X}_h C_{x_h} + \beta_{2h}(x))}$.

$$Now \quad E(\bar{y}_{strMM3}) = \bar{Y}\left[1+\gamma_1^2 E(\delta_1^2) - \gamma_1 E(\delta_0\delta_1)\right], \quad \text{because } E(\delta_0)=E(\delta_1)=0$$

$$\Rightarrow B(\bar{y}_{strMM3}) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{\gamma_1^2 S_{x_h}^2}{\bar{X}^2} - \frac{\gamma_1 S_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\gamma_1^2\sum_{i=1}^{r_h} D_{x_h(i)}^2 - \gamma_1\sum_{i=1}^{r_h} D_{x_h(i)y_h[i]}\right)\right\}\right] \qquad (3.9)$$

Now $MSE(\bar{y}_{strMM3}) = E(\bar{y}_{strMM3} - \bar{Y})^2$

$$= \bar{Y}^2 E\left[\delta_0^2 + \gamma_1^2\delta_1^2 - 2\gamma_1\delta_0\delta_1\right]$$

$$= \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\} + \gamma_1^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right\} - 2\gamma_1\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_hy_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right\}\right]$$

$$MSE(\bar{y}_{strMM3}) = \bar{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{\frac{S_{y_h}^2}{\bar{Y}^2} + \frac{\gamma_1^2 S_{x_h}^2}{\bar{X}^2} - 2\gamma_1\frac{S_{x_hy_h}}{\bar{X}\bar{Y}}\right\} - \frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \gamma_1 D_{x_h(i)}\right)^2\right]$$

$$MSE(\bar{y}_{strMM3}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2\gamma_1^2 S_{x_h}^2 - 2R\gamma_1 S_{x_hy_h}\right\} - \bar{Y}\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \gamma_1 D_{x_h(i)}\right)^2\right\}^2\right] \quad (3.10)$$

The Bias and MSE of $\bar{y}_{strMM4}$ can be found as follows-

$$B(\bar{y}_{strMM4}) = E(\bar{y}_{strMM4}) - \bar{Y}$$

Here $\bar{y}_{strMM4} = \bar{Y}(1+\delta_0)(1+\gamma_2\delta_1)$, where

$$\gamma_2 = \frac{\sum_{h=1}^{L}W_h \bar{X}_h C_{x_h}}{\sum_{h=1}^{L}W_h(\bar{X}_h C_{x_h} + \beta_{2h}(x))}.$$

Now $E(\bar{y}_{strMM4}) = \bar{Y}\left[1 + \gamma_2 E(\delta_0\delta_1)\right]$, because $E(\delta_0) = E(\delta_1) = 0$

$$\Rightarrow B(\bar{y}_{strMM4}) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\gamma_2\left\{\frac{S_{x_hy_h}}{\bar{X}\bar{Y}} - \sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right\}\right] \quad (3.11)$$

Now $MSE(\bar{y}_{strMM4}) = E(\bar{y}_{strMM4} - \bar{Y})^2$

$$= \bar{Y}^2 E\left[\delta_0^2 + \gamma_2^2\delta_1^2 + 2\gamma_2\delta_0\delta_1\right]$$

$$= \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\} + \gamma_2^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right\} + 2\gamma_2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_hy_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right\}\right]$$

$$MSE(\bar{y}_{strMM4}) = \bar{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{\frac{S_{y_h}^2}{\bar{Y}^2} + \frac{\gamma_2^2 S_{x_h}^2}{\bar{X}^2} + 2\gamma_2\frac{S_{x_hy_h}}{\bar{X}\bar{Y}}\right\} - \frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} + \gamma_2 D_{x_h(i)}\right)^2\right]$$

$$MSE(\bar{y}_{strMM4}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2\gamma_2^2 S_{x_h}^2 - 2R\gamma_2 S_{x_hy_h}\right\} - \bar{Y}\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} + \gamma_2 D_{x_h(i)}\right)^2\right\}^2\right] \quad (3.12)$$

Ratio-Cum-Product estimators in Stratified Ranked Set Sampling

Adapting the estimators in (1.7), given by Singh et al. (2005) and utilizing the information on co-efficient of variation C_x and co-efficient of kurtosis $\beta_2(x)$ of the auxiliary variable, we propose modified ratio-cum-product estimator of population mean using Stratified ranked set sampling as

$$\hat{Y}_{MM,SRSS} = \bar{y}_{[SRSS]}\left[\alpha\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(r_h)} C_{x_h} + \beta_{2h}(x)\right)}\right\} + (1-\alpha)\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(r_h)} C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)}\right\}\right] \quad (4.1)$$

Where α is a suitably chosen scalar. It is to be noted that for α =1 and α =0, $\hat{Y}_{MM,SRSS}$ reduces to the estimators $\bar{y}_{strMM3}$ and $\bar{y}_{strMM4}$ respectively.

Bias and MSE of the estimator $\hat{Y}_{MM,SRSS}$ to the first degree of approximation are respectively given by

$$B(\hat{Y}_{MM,SRSS}) = E(\hat{Y}_{MM,SRSS}) - \bar{Y}$$

Expressing the estimator $\hat{Y}_{MM,SRSS}$ in terms of δ_0 and δ_1, we get

$$\hat{Y}_{MM,SRSS} = \bar{Y}(1+\delta_0)\left[\alpha(1+t_{us}\delta_1)^{-1} + (1-\alpha)(1+t_{us}\delta_1)\right],$$ where $$t_{us} = \frac{\sum_{h=1}^{L} W_h \bar{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h(\bar{X}_h C_{x_h} + \beta_{2h}(x))}$$

$$= \bar{Y}(1+\delta_0)\left[\alpha(1 - t_{us}\delta_1 + t_{us}^2\delta_1^2) + (1-\alpha)(1+t_{us}\delta_1)\right]$$

Taking expectation of both sides, we get

$$E(\hat{Y}_{MM,SRSS}) = \bar{Y}\left[1 + t_{us}E(\delta_0\delta_1) + \alpha t_{us}^2 E(\delta_1^2) - 2\alpha t_{us}E(\delta_0\delta_1)\right]$$

[using $E(\delta_0) = E(\delta_1) = 0$].

Now $$B(\hat{Y}_{MM,SRSS}) = \bar{Y}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\alpha t_{us}^2\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right\} + (1-2\alpha)t_{us}\left\{\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)y_h[i]}\right\}\right] \quad (4.2)$$

and

$$MSE(\hat{Y}_{MM,SRSS}) = E(\hat{Y}_{MM,SRSS} - \bar{Y})^2$$

$$= \bar{Y}^2 E\left[\delta_0 + (1-2\alpha)t_{us}\delta_1\right]^2$$

$$= \bar{Y}^2 E\left[\delta_0^2 + (1-2\alpha)^2 t_{us}^2\delta_1^2 + 2\delta_0\delta_1(1-2\alpha)t_{us}\right]$$

$$= \bar{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{\frac{S_{y_h}^2}{\bar{Y}^2} + (1-2\alpha)^2 t_{us}^2\frac{S_{x_h}^2}{\bar{X}^2} + 2(1-2\alpha)t_{us}\frac{S_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} + (1-2\alpha)t_{us}D_{x_h(i)}\right)^2\right]$$

$$\Rightarrow MSE(\hat{Y}_{MM,SRSS}) =$$

$$\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + (1-2\alpha)^2 R^2 t_{us}^2 S_{x_h}^2 + 2(1-2\alpha)t_{us}RS_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} + (1-2\alpha)t_{us}D_{x_h(i)}\right)^2\right\}\right] \quad (4.3)$$

Optimality of α

The optimum values of α to minimize the MSE's of $\hat{Y}_{MM,SRSS}$ can easily be found as follows

$$\frac{\partial MSE(\overset{\Lambda}{Y}_{MM,SRSS})}{\partial \alpha}=0$$

$$\Rightarrow \alpha=\frac{1}{2}\left[1+\frac{\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h y_h}}{\overline{X}\,\overline{Y}}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right\}}{t_{us}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\overline{X}^2}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right\}}\right]$$

Efficiency Comparison

On comparing (1.11), (1.12), (1.13), (1.14), and (1.16) with (3.3), (3.6), (3.10), (3.12) and (4.3) respectively, we obtained

1) $MSE(\overline{y}_{stSD})-MSE(\overline{y}_{strMM1})=A_1\geq 0$, where $A_1=\overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\lambda_1 D_{x_h(i)}\right)^2$

$\Rightarrow MSE(\overline{y}_{stSD})\geq MSE(\overline{y}_{strMM1})$

2) $MSE(\overline{y}_{stSK})-MSE(\overline{y}_{strMM2})=A_2\geq 0$, where $A_2=\overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\lambda_2 D_{x_h(i)}\right)^2$

$\Rightarrow MSE(\overline{y}_{stSK})\geq MSE(\overline{y}_{strMM2})$

3) $MSE(\overline{y}_{stUS1})-MSE(\overline{y}_{strMM3})=A_3\geq 0$, where $A_3=\overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\gamma_1 D_{x_h(i)}\right)^2$

$\Rightarrow MSE(\overline{y}_{stUS1})\geq MSE(\overline{y}_{strMM3})$

4) $MSE(\overline{y}_{stUS2})-MSE(\overline{y}_{strMM4})=A_4\geq 0$, where $A_4=\overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}+\gamma_2 D_{x_h(i)}\right)^2$

$\Rightarrow MSE(\overline{y}_{stUS2})\geq MSE(\overline{y}_{strMM4})$

5) $MSE(\overset{\Lambda}{Y}_{M,SSRS})-MSE(\overset{\Lambda}{Y}_{MM,SRSS})=A_5\geq 0$,

where $A_5=\overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}+(1-2\alpha)t_{us}D_{x_h(i)}\right)^2$

$$\Rightarrow MSE(\overset{\Lambda}{Y}_{M,SSRS})\geq MSE(\overset{\Lambda}{Y}_{MM,SRSS})$$

It is easily seen that the MSE of the suggested estimators given in (3.1), (3.4), (3.7), (3.8) and (4.1) are always smaller than the estimator given in (1.3),(1.4), (1.5), (1.6) and (1.8) respectively, because A_1, A_2 A_3 A_4 and A_5 all are non-negative values. As a result, show that the various proposed ratio type, product type and ratio-cum-product estimators $\overline{y}_{strMM1}$, $\overline{y}_{strMM2}$, $\overline{y}_{strMM3}$, $\overline{y}_{strMM4}$ and $\overset{\Lambda}{Y}_{MM,SRSS}$ for the population mean using SRSS are more efficient than the corresponding usual estimators of stratified sampling.

Numerical Example

To compare efficiencies of various estimators of our study, here we take a Stratified population with 3 strata of sizes 12,30 & 17 respectively of page 1119(Appendix) from the book entitled "Advanced Sampling Theory with Applications", Vol.2 , by Sarjinder

Singh published from Kluwer Academic Publishers. The example considers the data of Tobacco for Area and Production in specified countries during 1998, where y is production (study variable) in metric tons and x is area (auxiliary variable) in hectares.

For the above population, the parameters are summarized as below:

For total population, $N = 59$, $\overline{Y} = 76485.42$, $\overline{X} = 26942.29$.

Stratum-1	Stratum-2	Stratum-3
$N_1 = 12$	$N_2 = 30$	$N_3 = 17$
$n_1 = 9$	$n_2 = 15$	$n_3 = 12$
$W_1 = 0.2034$	$W_2 = 0.5085$	$W_3 = 0.2881$
$\overline{X}_1 = 5987.83$	$\overline{X}_2 = 11682.73$	$\overline{X}_3 = 68662.29$
$\overline{Y}_1 = 11788$	$\overline{Y}_2 = 16862.27$	$\overline{Y}_3 = 227371.53$
$S^2_{x_1} = 27842810.5$	$S^2_{x_2} = 760238523$	$S^2_{x_3} = 12187889050$
$S^2_{y_1} = 153854583$	$S^2_{y_2} = 2049296094$	$S^2_{y_3} = 372428238550$
$S_{y_1x_1} = 62846173.1$	$S_{y_2x_2} = 1190767859$	$S_{y_3x_3} = 27342963562$
$C_{x_1} = 0.8812$	$C_{x_2} = 2.3601$	$C_{x_3} = 1.6079$
$\beta_{21}(x) = 1.8733$	$\beta_{22}(x) = 10.7527$	$\beta_{21}(x) = 8.935$
$R_1 = 1.97$	$R_2 = 1.44$	$R_3 = 3.31$

We took ranked set samples of sizes $r_1 = 3$, $r_2 = 5$ & $r_3 = 4$ from stratum 1st , 2nd and 3rd respectively. Further each ranked set sample from each stratum were repeated with number of cycles $m = 3$. So that sample sizes of stratified ranked set samples are equivalent to stratified simple random samples with $n_h (= mr_h)$ for the h^{th} stratum, $h = 1$, 2 ,3.

The estimated relative efficiencies of various proposed Stratified ranked set estimators in comparison with corresponding Stratified SRS estimators are as shown in the next table:

Variances of various Stratified SRS estimators	$\overline{y}_{stSD}$	$\overline{y}_{stSK}$	$\overline{y}_{stUS1}$	$\overline{y}_{stUS2}$	$\hat{Y}_{M,SSRS}$
	2245878377	2245739510	2245816148	4863538453	2158910787
Variances of corresponding proposed Stratified ranked set sampling estimators	$\overline{y}_{strMM1}$	$\overline{y}_{strMM2}$	$\overline{y}_{strMM3}$	$\overline{y}_{strMM4}$	$\hat{Y}_{MM,SRSS}$
	1938091889	1938094290	1938095047	3822937792	1605469167
Relative Efficiencies in %	115.8809	115.8736	115.8775	127.2199	134.4723

In the table above, we see that the proposed Stratified ranked set estimators are more efficient than corresponding Stratified SRS estimators.

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