Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 67 (2014) 21472-21474

ISSN: 2229-712X

Fuzzy arw-Super Continuous Mappings

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ARTICLE INFO

Article history:

Received: 4 December 2013;

Received in revised form:

24 January 2014;

Accepted: 4 February 2014;

Keywords

Fuzzy semi super open,

Set, Fuzzy g-super closed set,

Fuzzy w-super closed set,

Fuzzy w-super continuous mapping,

Fuzzy g-super continuous mapping,

Fuzzy w-super irresolute mapping.

ABSTRACT

The purpose of present paper to study and introduces the concepts of fuzzy arw- super closed mappings and obtains some of their basic properties and characterizations in fuzzy topology.

Introduction

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha \in \Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by x_β (y)=β for y=x and x(y) =0 for $y \neq x$, $\beta \in [0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x \in X$ such that $A(x) + B(x) > 1$ .A $\le$ B if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1[5]: Let (X,τ) fuzzy topological space and $A \subseteq X$ then

1. Fuzzy Super closure $scl(A)=\{x \in X: cl(U) \cap A \neq \phi\}$
2. Fuzzy Super interior $sint(A) =\{x \in X: cl(U) \le A \neq \phi\}$

Definition 1.2[5]: A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) Fuzzy super closed if $scl(A) \le A$.

(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]: Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]: Let A and B are two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy super closed.

Remark 1.3[5]: The intersection of two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy super closed.

Definition 1.3[7,9,10]: A fuzzy set A of a fuzzy topological space (X,τ) is said to be:

a. Fuzzy semi super open if $A \le cl(int(A))$.

b. Fuzzy g-super closed if $cl(A) \le O$ whenever $A \le O$ and O is fuzzy super open set.

c. Fuzzy g-super open if 1-A is fuzzy g-super closed.

d. Fuzzy w-super closed if $cl(A) \le O$ whenever $A \le O$ and O is a fuzzy semi super open set.

e. Fuzzy w-super open if 1-A is fuzzy w-super closed.

Remark 1.4: Every fuzzy super closed set is fuzzy w-super closed and every fuzzy w-super closed set is fuzzy g-super closed but the converse may not be true.

Definition 1.4 [6]: A mapping $f: (X,\tau) \to (Y,\sigma)$ is said to be ;

a. Fuzzy w-super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy super closed in X.

b. Fuzzy g-super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy g-super closed in X..

c. Fuzzy w-super irresolute if the inverse image of every fuzzy w-super closed set of Y is fuzzy w-super closed in X.

Remark 1.5 [6]: Every fuzzy super continuous mapping is fuzzy w-super continuous and every fuzzy w-super continuous mapping is fuzzy g-super continuous mapping but the converse may not be true.

The concepts of fuzzy w-super irresolute and fuzzy super continuous mapping are independent.

Definition 1.5[9,10]: A fuzzy topological space (X,τ) is said to be fuzzy w-$T_{1/2}$ if every fuzzy w-super closed set is fuzzy super closed set in X. .

Fuzzy arw-Super Continuous Mappings

Definition 2.1: A mapping $f: (X,\tau) \to (Y,\sigma)$ is said to be fuzzy rw-super continuous provided that $cl(A) \le f^{-1}(O)$ whenever O is fuzzy semi super open in Y, A is fuzzy rw-super closed in X and $A \le f^{-1}(O)$.

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Theorem 2.1: Every fuzzy super irresolute is fuzzy arw- super continuous.

Proof: Let O be a fuzzy semi super open set of Y,A is a fuzzy w-super closed set of X and (A) $\leq f^{-1}(O)$ .Now f is fuzzy super irresolute $f^{-1}(O)$ is fuzzy semi super open set of X. Since A is fuzzy rw-super closed and $A \leq f^{-1}(O)$ it follows that $cl(A) \leq f^{-1}(O)$.Hence f is fuzzy arw-super continuous.

Remark 2.1: The converse of theorem (2.1) is not true for,

Example2.1: Let X={a,b} ,Y={x,y} and τ={0,1} and σ={0,A,1} be the fuzzy topologies where A(x)=0.3 ,A(y)=0.4.Then the mapping f: $(X,\tau)\to(Y,\sigma)$ be a mapping defined by f(a)= x , f(b)= y, is fuzzy arw-super continuous but not fuzzy w-super irresolute. Now consider the following example.

Example2.2: Let X={a,b} ,Y={x,y} and τ={0,A,1} and σ={0,1} be the fuzzy topologies where A(a)=0.7 ,A(b)=0.5.Then the mapping f: $(X,\tau)\to(Y,\sigma)$ be a mapping is fuzzy super continuous but not fuzzy arw-super continuous.

Remark 2.2: Example (2.1) and (2.2) asserts that the concepts of fuzzy super continuous and fuzzy arw-super continuous mappings arw-super independent.

Theorem 2.2: Let If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy arw-super continuous and fuzzy super closed mappings then the image of every fuzzy rw-super closed set of X is fuzzy rw-super closed in Y.

Proof: Let A be a fuzzy w-super closed set of X and f(A) $\leq$O where O is the fuzzy semi super open set in Y then $A\leq f^{-1}(O)$ and hence f is fuzzy arw- super continuous $cl(A) \leq f^{-1}(O)$ which implies f(cl(A)) $\leq$ O since f is fuzzy super closed we have cl(f(A)) $\leq$ cl(fcl((A)))=f(cl(A)) $\leq$O .Hence f(A) is fuzzy rw-super closed set in Y.

Theorem 2.3: If (X,τ) is fuzzy w- super $T_{1/2}$ then every mapping If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy arw-super continuous.

Proof: Let A be a fuzzy w-super closed set of X and $A \leq f^{-1}(O)$ where O is fuzzy semi super open set in Y. Since X is fuzzy w-super $T_{1/2}$, A is fuzzy super closed in X therefore $cl(A) = A \leq f^{-1}(O)$.Hence f is fuzzy arw- super continuous.

Theorem 2.4: If FSO(X)=FC(X) then a mapping f: $(X,\tau)\to(Y,\sigma)$ is fuzzy arw-super continuous if and only if $f^{-1}(O)$ is fuzzy super closed in X for every fuzzy semi super open set O in Y.

Proof: Necessity: Let f: $(X,\tau) \to (Y,\sigma)$ is fuzzy arw-super continuous by theorem(1.7) every fuzzy set of X is fuzzy rw-super closed (and hence fuzzy rw-super open) Thus for any fuzzy semi super open set O of Y, $f^{-1}(O)$ is fuzzy rw-super closed in X. Since $f^{-1}(O) \leq f^{-1}(O)$ and f is fuzzy arw-super continuous, $cl(f^{-1}(O)) \leq f^{-1}(O)$.Hence $f^{-1}(O)$ is fuzzy super closed in X.

Sufficiency: Let O be a fuzzy semi super open set of Y and A be a fuzzy rw-super closed set of X such that $A \leq f^{-1}(O)$ then $cl(A) \leq cl(f^{-1}(O)) = f^{-1}(O)$, because by assumption $f^{-1}(O)$ is fuzzy super closed in X, hence f is fuzzy arw-super continuous.

Theorem 2.5: If FSO(X) = FC(Y) then a mapping f: $(X,\tau)\to(Y,\sigma)$ is fuzzy arw- super continuous if and only if it is an fuzzy super irresolute.

Proof: Necessity: Let O be a fuzzy semi super open set of Y, then by theorem(2.4) $f^{-1}(O)$ is fuzzy super closed in X and so $f^{-1}(O)$ is fuzzy semi super open in X and hence f is fuzzy is an super irresolute.

Sufficiency: Let A be a fuzzy w-super closed of X and O be a fuzzy semi super open set of Y and $A \leq f^{-1}(O)$.By hypothesis $f^{-1}(O)$ is fuzzy semi super open and thus fuzzy semi super closed, $cl(A) \leq cl(f^{-1}(O)) = f^{-1}(O)$ hence f is fuzzy arw-super continuous.

Theorem 2.6: If f: $(X,\tau) \to (Y,\sigma)$ is fuzzy arw-super continuous and g:$(Y,\sigma)\to(Z,\omega)$ is a fuzzy super irresolute then g of: (Y,σ)) $\to (Z,\omega)$ is fuzzy arw-super continuous.

Proof: Let A be a fuzzy rw-super closed of X and O be a fuzzy semi super open sub set of Z such that $A \leq (gof)^{-1}(O)$. Since g is a fuzzy super irresolute $g^{-1}(O)$ is fuzzy semi super open in Y. Since f is fuzzy arw-super continuous $cl(A) \leq f^{-1}(g^{-1}(O)) = (gof)^{-1}(O)$. Hence (gof) is fuzzy arw-super continuous.

Definition 2.2:A mapping f: $(X,\tau) \to (Y,\sigma)$ is called fuzzy arw-super closed provided that f(A) $\leq$ int(O) whenever A is fuzzy semi super closed in X and O is fuzzy rw-super open in Y and f(A) $\leq$ O.

Theorem 2.7: Every fuzzy rw-continues and fuzzy arw-super closed mappings are fuzzy rw- super irresolute.

Proof: A mapping f: $(X,\tau) \to (Y,\sigma)$ is fuzzy rw-super continuous and fuzzy arw-super closed .Let A is fuzzy rw-super closed in Y and $f^{-1}(A) \leq O$ where O is fuzzy rw-super open in X. Then $1-O \leq f^{-1}(1-A)$ which implies f(1-O) $\leq$ (1-A) .Since f is fuzzy arw-super closed f(1 - O) $\leq$ int(1 - A) = 1 - cl(A).Hence $f^{-1}(cl(A)) \leq O$. Since f is fuzzy rw-super continuous $f^{-1}(cl(A))$ is fuzzy rw-super closed set in X. Therefore $cl(f^{-1}(cl(A))) \leq O$ which implies that $cl(f^{-1}(A) \leq O$ .Hence $f^{-1}(A)$ is fuzzy rw-super closed in X.

Theorem 2.8: A mapping f: $(X,\tau) \to (Y,\sigma)$ is fuzzy super continuous and fuzzy arw-super closed mapping then it is fuzzy rw- super irresolute.

Proof: Follows from theorem (2.7).

Theorem 2.9: If $(Y,\sigma)$is fuzzy w-$T_{1/2}$ then every mapping f:$(X,\tau) \to (Y,\sigma)$ is fuzzy arw-super closed.

Proof: Obvious

Theorem 2.10: If FSO(X) = FC(Y) then a mapping f: $(X,\tau)\to(Y,\sigma)$ is fuzzy arw-super closed if and only if f(O) is super open for every fuzzy semi super closed subset O of X.

Proof: Necessity: Let A mapping f: $(X,\tau) \to (Y,\sigma)$ is fuzzy super continuous and fuzzy arw-super closed mapping by theorem (1.7) ,all fuzzy set of Y are fuzzy rw-super closed and hence all are fuzzy w-super open .Thus for any fuzzy semi super closed subset O of X. f(O) is fuzzy rw-super open in Y. Since f is arw-super closed f(O) $\leq$ int(f(O)).Hence f(O) is fuzzy super open.

Sufficiency: Let O be a fuzzy semi super closed set of X and A be a fuzzy w-super open set of Y and f(O) $\leq$ A. By hypothesis f(O) is fuzzy super open in Y and so $f^{-1}(O)$ = int(f(O)) $\leq$ int(A) hence f is fuzzy arw-super closed.

Definition 2.3:A mapping f:$(X,\tau) \to (Y,\sigma)$ is fuzzy pre semi super closed if the image of every fuzzy semi super closed set of X is fuzzy semi super closed in Y.

Theorem 2.11: If FSO(X) =FC(Y) then a mapping f: $(X,\tau) \to (Y,\sigma)$ is fuzzy arw-super closed if and only if f is fuzzy pre semi rw-super closed .

Proof: Necessity: Let O be a semi super closed set of X then by theorem (2.10) f(O) is fuzzy super open and fuzzy super closed and hence it is fuzzy rw-super closed and f is fuzzy pre semi super closed.

Sufficiency: Let O be a fuzzy semi super closed set of X and A be a fuzzy w-super open set of Y and f(O) $\leq$A. By hypothesis f(O) is fuzzy rw-super closed in Y. Thus f(O) is fuzzy semi .Hence f(O) = int(f(O)) $\leq$ int(A) and f is fuzzy arw-super closed.

Theorem 2.12: Let mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is fuzzy pre rw-super closed and g :$(Y,\sigma) \rightarrow (Z, \omega)$ is fuzzy arw-super closed then (g of) is fuzzy arw-super closed.

Proof: Let O be a fuzzy rw-super closed set of X and A be a fuzzy w-super open set of Z, such that (gof (O))$\leq$ A since f is fuzzy pre rw-super closed f(O) is fuzzy rw-super closed in Y. Therefore gof(O) $\leq$ int(A) because g is fuzzy arw-super closed and hence (gof) is fuzzy arw-super closed.

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