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Notes on anti s-fuzzy subfields of a field

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti S-fuzzy subfield of a field.

Introduction

After the introduction of fuzzy sets by L.A.Zadeh[15], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[4, 5]. In this paper, we introduce the some theorems in anti S-fuzzy subfield of a field.

Preliminaries:

Definition: Let X be a non-empty set. A **fuzzy subset A** of X is a function $A : X \to [0, 1]$.

Definition: A S-norm is a binary operation S: $[0, 1]\times[0, 1] \to [0, 1]$ satisfying the following requirements;

(i) $S(0, x) = x$, $S(1, x) = 1$ (boundary condition)

(ii) $S(x, y) = S (y, x)$ (commutativity)

(iii) $S(x, S (y, z))=S (S(x, y), z)$(associativity)

(iv) if $x \le y$ and $w \le z$, then $S(x, w)\le S (y, z)$(monotonicity).

Definition: Let (F, +, ·) be a field. A fuzzy subset A of F is said to be an **anti S-fuzzy subfield** (anti fuzzy subfield with respect to S-norm) of F if the following conditions are satisfied:

(i) $A(x+y) \le S (A(x), A(y))$, for all x and y in F,

(ii) $A(- x) \le A(x)$, for all x in F,

(iii) $A(xy) \le S (A(x), A(y))$, for all x and y in F,

(iv) $A(x^{-1}) \le A(x)$, for all $x \ne 0$ in F, where 0 is the additive identity of F.

Definition: Let (F, +, ·) and (F^{I}, +, ·) be any two fields. Let $f : F \to F^{I}$ be any function and A be an anti S-fuzzy subfield in F, V be an anti S-fuzzy subfield in f(F) = F^{I}, defined by $V(y) = \inf_{x\in f^{-1}(y)} A(x)$, for all x in F and y in F^{I}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition: Let A and B be any two fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by A×B, is defined as A×B = { ⟨ (x, y), A×B(x, y) ⟩ / for all x in G and y in H }, where A×B(x, y) = max {A(x), B(y)}, for all x in G and y in H.

Definition: Let A be a fuzzy subset in a set S, the **anti-strongest fuzzy relation** on S, that is a fuzzy relation on A is V= {⟨(x,y), V(x,y) ⟩ / x and y in S } given by V(x, y) = max {A(x), A(y) }, for all x and y in S.

Definition: Let A be an anti S-fuzzy subfield of a field (F, +, ·) and a in F. Then the pseudo anti S-fuzzy coset $(aA)^p$ is defined by $((aA)^p)(x) = p(a)A(x)$, for every x in F and for some p in P.

Properties:

Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then $A(-x) = A(x)$, for all x in F and $A(x^{-1}) = A(x)$, for all $x \ne 0$ in F and $A(x) \ge A(0)$, for all x in F and $A(x) \ge A(1)$, for all x in F, where 0 and 1 are identity elements in F.

Proof: For x in F and 0, 1 are identity elements in F. Now, $A(x) = A(-(-x)) \le A(-x) \le A(x)$. Therefore, $A(-x) = A(x)$, for all x in F. And, $A(x) = A((x^{-1})^{-1}) \le A(x^{-1}) \le A(x)$. Therefore, $A(x^{-1}) = A(x)$, for all $x \ne 0$ in F. And, $A(0) = A(x-x) \le S(A(x), A(-x)) = A(x)$. Therefore, $A(0) \le A(x)$, for all x in F. And, $A(1)=A(xx^{-1}) \le S(A(x), A(x^{-1}))=A(x)$. Therefore, $A(1)\le A(x)$, for all $x \ne 0$ in F.

Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then

(i) $A(x-y) = A(0)$ gives $A(x) = A(y)$, for all x and y in F,

(ii) $A(xy^{-1}) = A(1)$ gives $A(x) = A(y)$, for all x and $y \ne 0$ in F, where 0 and 1are identity elements in F.

Proof: Let x and y in F and 0, 1 are identity elements in F. (i) Now, $A(x) = A(x-y+y) \le S(A(x-y), A(y)) = S(A(0), A(y)) = A(y) = A(x-(x-y)) \le S(A(x-y), A(x)) = S (A(0), A(x)) = A(x)$. Therefore, $A(x) = A(y)$, for all x and y in F. (ii) Now, $A(x) = A(xy^{-1}y) \le S (A(xy^{-1}), A(y)) = S (A(1), A(y)) = A(y) = A((xy^{-1})^{-1}x) \le S (A(xy^{-1}), A(x)) = S(A(1), A(x)) = A(x)$. Therefore, $A(x) = A(y)$, for all x and $y \ne 0$ in F.

Theorem: Let A be a Fuzzy subset of a field (F, +, ·). If $A(e) = A(e^{I}) = 0$, $A(x-y) \le S (A(x), A(y))$, for all x and y in F and $A(xy^{-1}) \le S (A(x), A(y))$, for all x and $y \ne e$ in F, then A is an anti S-fuzzy subfield of F, where e and e^{I} are identity elements of F.

Proof: Let e and e^{1} be identity elements of F and x and y in F. Now $A(-x) = A(e-x) \le S(A(e), A(x)) = S(0, A(x)) = A(x)$. Therefore, $A(-x) \le A(x)$, for all x in F. And $A(x^{-1}) = A(e^{I}x^{-1}) \le S(A(e^{I}), A(x)) = S(0, A(x)) = A(x)$. Therefore, $A(x^{-1}) \le A(x)$, for all $x \ne e$ in F. And $A(x+y) = A(x-(-y)) \le S(A(x), A(-y)) \le S (A(x), A(y))$. Therefore, $A(x+y) \le S (A(x), A(y))$, for all x and y in F. And $A(xy) = A(x(y^{-1})^{-1}) \le S (A(x), A(y^{-1})) \le S$

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(A(x), A(y)). Therefore, A(xy) ≤ S(A(x), A(y)), for all x and y ≠ e in F. Hence A is an anti S-fuzzy subfield of F.

Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then H = { x / x∈F: A(x) = 0 } is either empty or is a subfield of F

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then A(x–y) ≤ S (A(x), A(–y)) = S (A(x), A(y)) = S (0, 0) = 0. Therefore, A(x–y) = 0, for all x and y in F. We get x–y in H. And, $A(xy^{-1})$ ≤ S (A(x), $A(y^{-1})$) = S(A(x), A(y)) = S (0, 0) = 0. Therefore, $A(xy^{-1})$ = 0, for all x and y ≠ 0 in F. We get xy^{-1} in H. Therefore, H is a subfield of F. Hence H is either empty or is a subfield of F.

Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then H ={ x∈F: A(x)= A(e) = A(e^{I}) } is either empty or is a subfield of F, where e and e^{I} are identity elements of F.

Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then A(–x) = A(x) = A(e), for all x in F and $A(x^{-1})$ = A(x) = A(e^{I}), for all x ≠ e in F, by Theorem 2.1. Therefore, A(–x)=A(e), for all x in F and $A(x^{-1})$=A(e^{I}), for all x ≠ e in F. Hence –x , x^{-1} in H. Now, A(x–y) ≤ S (A(x), A(–y)) ≤ S(A(x), A(y)) = S (A(e), A(e)) = A(e). Therefore, A(x–y) ≤ A(e)----------------(1). And, A(e) = A((x–y)–(x–y)) ≤ S (A(x–y), A(–(x–y))) ≤ S(A(x–y), A(x–y)) = A(x–y). Therefore, A(e) ≤ A(x–y) ---------------------(2). From (1) and (2), we get A(e) = A(x–y), for all x and y in F. Now, $A(xy^{-1})$ ≤ S (A(x), $A(y^{-1})$) ≤ S (A(x), A(y)) = S (A(e^{I}), A(e^{I})) = A(e^{I}).

Therefore, $A(xy^{-1})$ ≤ A(e^{I}) ----------------(3). And, A(e^{I}) = A($(xy^{-1})(xy^{-1})^{-1}$) ≤ S ($A(xy^{-1})$, $A((xy^{-1})^{-1})$) ≤ S($A(xy^{-1})$, $A(xy^{-1})$) = $A(xy^{-1})$. Therefore, A(e^{I}) ≤ A(xy^{-1})------------(4). From (3) and (4), we get A(e^{I}) = $A(xy^{-1})$, for all x and y ≠ e in F. Hence A(e) = A(x–y), A(e^{I}) = $A(xy^{-1})$. We get x–y, xy^{-1} in H. Hence H is either empty or is a subfield of F.

Theorem: Let A be an anti S-fuzzy subfield of a field (F, +, ·). Then (i) if A(x–y) = 0, then A(x) = A(y), for x and y in F (ii) if $A(xy^{-1})$ = 0, then A(x) = A(y), for all x and y ≠ e in F, where e and e^{I} are identity elements of F.

Proof: Let x and y in F. Now, A(x) = A(x–y+y) ≤ S(A(x–y), A(y)) = S(0, A(y)) = A(y) = A(–y) = A(–x+x–y) ≤ S (A(–x), A(x–y)) = S (A(–x), 0) = A(–x)=A(x). Therefore, A(x) =A(y), for all x and y in F. And, A(x) = $A(xy^{-1}y)$ ≤ S ($A(xy^{-1})$, A(y)) = S (0, A(y)) = A(y) = $A(y^{-1})$ = A $(x^{-1}xy^{-1})$ ≤ S ($A(x^{-1})$, $A(xy^{-1})$) = S ($A(x^{-1})$, 0) = A(x^{-1}) = A(x). Therefore, A(x) = A(y), for all x ≠ e and y≠ e in F.

Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then (i) if A(x–y) =1, then either A(x) = 1or A(y) = 1, for x and y in F,

(ii) if $A(xy^{-1})$ =1, then either A(x) = 1or A(y) = 1, for all x and y≠ e in F.

Proof: Let x and y in F. By the definition A(x–y) ≤ S (A(x), A(y)), which implies that 1≤ S(A(x), A(y)). Therefore, either A(x) =1or A(y) = 1, for all x and y in F. And by the definition $A(xy^{-1})$ ≤ S(A(x), A(y)), which implies that 1≤ S(A(x), A(y)). Therefore, either A(x) =1or A(y) =1, for all x and y≠ e in F.

Theorem: Let (F, +, ·) be a field. If A is an anti S-fuzzy subfield of F, then A(x+y) = S (A(x), A(y)), for all x and y in F and A(xy) = S(A(x), A(y)), for all x ≠ 0 and y in F with A(x) ≠ A(y).

Proof: Let x and y belongs to F. Assume that A(x) < A(y). Now, A(y) = A(–x+x+y) ≤ S(A(–x), A(x+y)) ≤ S(A(x), A(x+y)) = A(x+y) ≤ S(A(x), A(y)) = A(y). Therefore, A(x+y) = A(y) = S (A(x), A(y)), for all x and y in F. And, A(y) = $A(x^{-1}xy)$ ≤ S($A(x^{-1})$, A(xy)) ≤ S(A(x), A(xy)) = A(xy) ≤ S(A(x), A(y)) = A(y). Therefore, A(xy) = A(y) = S (A(x), A(y)), for all x ≠ 0 and y in F.

Theorem: If A and B are any two anti S-fuzzy subfields of a field (F, +, ·), then A∪B is an anti S-fuzzy subfield of F.

Proof: Let x and y belongs to F and A = {⟨x, A(x)⟩ /x∈F } and B = {⟨ x, B(x)⟩ / x∈F }. Let C = A∪B and C = {⟨ x, C(x) ⟩ / x∈F }, C(x) = max {A(x), B(x) }. (i) C(x–y) = max { A(x–y), B(x–y) } ≤ max { S(A(x), A(y)), S(B(x), B(y)) }= S (max {A(x), B(x) }, max {A(y), B(y)}) = S(C(x),C(y)). Therefore, C(x–y) ≤ S (C(x), C(y)), for all x and y in F. (ii) $C(xy^{-1})$ = max {$A(xy^{-1})$, $B(xy^{-1})$ } ≤ max { S(A(x), A(y)), S (B(x), B(y)) } = S (max { A(x), B(x) }, max { A(y), B(y) }) = S (C(x), C(y)). Therefore, $C(xy^{-1})$ ≤ S (C(x), C(y)), for all x and y ≠ 0 in F. Hence A∪B is an anti S-fuzzy subfield of a field F.

Theorem: The union of a family of anti S-fuzzy subfields of a field (F, +, ·) is an anti S-fuzzy subfield of F.

Proof: Let $\{ A_i \}_{i\in I}$ be a family of anti S-fuzzy subfields of a field F and $A = \bigcup_{i\in I} A_i$. Then for x and y belongs to F, we have (i) $A(x-y) = \sup_{i\in I} A_i(x-y) \le \sup_{i\in I} S(A_i(x), A_i(y)) \le S(\sup_{i\in I}(A_i(x)), \sup_{i\in I}(A_i(y))) = S(A(x), A(y))$. Therefore, A(x–y) ≤ S(A(x), A(y)), for all x and y in F. (ii) $A(xy^{-1}) = \sup_{i\in I} A_i(xy^{-1}) \le \sup_{i\in I} S(A_i(x), A_i(y)) \le S(\sup_{i\in I}(A_i(x)), \sup_{i\in I}(A_i(y))) = S(A(x), A(y))$. Therefore, $A(xy^{-1})$ ≤ S (A(x), A(y)), for all x and y ≠ 0 in F. Hence the union of a family of anti S-fuzzy subfields of a field F is an anti S-fuzzy subfield of F.

Theorem: Let A be an anti S-fuzzy subfield of a field (F, +, ·). If A(x) > A(y), for some x and y in F, then A(x+y) = A(x) = A(y+x), for all x and y in F and A(xy) = A(x) = A(yx), for all x and y≠ 0 in F.

Proof: Let A be an anti S-fuzzy subfield of a field F. Also we have A(x) > A(y), for some x and y in F, Now, A(x+y) ≤ S (A(x), A(y)) = A(x); and A(x) = A(x+y–y) ≤ S(A(x+y), A(–y)) ≤ S(A(x+y), A(y)) = A(x+y). Therefore, A(x+y) = A(x) , for all x and y in F. Hence A(x+y) = A(x) = A(y+x), for all x and y in F. Now, A(xy) ≤ S (A(x), A(y)) = A(x); and A(x) = $A(xyy^{-1})$ ≤ S(A(xy), $A(y^{-1})$)) ≤ S (A(xy), A(y)) = A(xy). Therefore, A(xy) = A(x), for all x and y≠ 0 in F. Hence A(xy) = A(x) = A(yx), for all x and y≠ 0 in F.

Theorem: Let A be an anti S-fuzzy subfield of a field (F, +, ·). If A(x) < A(y), for some x and y in F, then A(x+y) = A(y) = A(y+x), for all x and y in F and A(xy) = A(y) = A(yx), for all x and y≠ 0 in F.

Proof: It is trivial.

Theorem: Let A be an anti S-fuzzy subfield of a field (F, +, ·) such that Im A= {α }, where α in L. If A = B∩C, where B and C are anti S-fuzzy subfields of F, then either B ⊆ C or C ⊆ B.

Proof: It is trivial.

Theorem: If A and B are anti S-fuzzy subfields of the fields G and H, respectively, then the anti-product A×B is an anti S-fuzzy subfield of G×H. **Proof:** Let A and B be anti S-fuzzy subfields of the fields G and H respectively. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in G×H. Now, A×B [(x_1, y_1) – (x_2, y_2)] = A×B (x_1– x_2, y_1– y_2) = max (A(x_1– x_2), B(y_1– y_2)) ≤ max (S (A(x_1), A(x_2)), S (

$B(y_1), B(y_2)$)) = S (max ($A(x_1), B(y_1)$) , max ($A(x_2), B(y_2)$)) = S($A\times B(x_1, y_1), A\times B(x_2, y_2)$).

Therefore, $A\times B[(x_1, y_1) - (x_2, y_2)] \leq S(A\times B(x_1, y_1), A\times B(x_2, y_2))$, for all x_1 and x_2 in G and y_1 and y_2 in H. And, $A\times B[(x_1, y_1)(x_2, y_2)^{-1}] = A\times B(x_1x_2^{-1}, y_1y_2^{-1}) = \max(A(x_1x_2^{-1}), B(y_1y_2^{-1})) \leq \max(S(A(x_1), A(x_2)), S(B(y_1), B(y_2))) = S(\max(A(x_1), B(y_1)), \max(A(x_2), B(y_2))) = S(A\times B(x_1, y_1), A\times B(x_2, y_2))$.

Therefore, $A\times B[(x_1, y_1)(x_2, y_2)^{-1}] \leq S(A\times B(x_1, y_1), A\times B(x_2, y_2))$, for all x_1 and $x_2 \neq 0$ in G and y_1 and $y_2 \neq 0^{l}$ in H. Hence anti-product A×B is an anti S-fuzzy subfield of G×H.

Theorem: Let A and B be fuzzy subsets of the fields G and H, respectively. Suppose that 0, 1and $0^{l}, 1^{l}$ are the identity elements of G and H, respectively. If the anti-product A×B is an anti S-fuzzy subfield of G×H, then at least one of the following two statements must hold.

(i) $B(0^{l}) \leq A(x)$, for all x in G and $B(1^{l}) \leq A(x)$, for all $x \neq 0$ in G,

(ii) $A(0) \leq B(y)$, for all y in H and $A(1) \leq B(y)$, for all $y \neq 0^{l}$ in H.

Proof: Let the anti-product A×B be an anti S-fuzzy subfield of G×H. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in G and b in H such that $A(a) < B(0^{l})$, $A(a) < B(1^{l})$ and $B(b) < A(0)$, $B(b) < A(1)$. We have, $A\times B(a, b) = \max(A(a), B(b)) < \max(A(0), B(0^{l})) = A\times B(0, 0^{l})$. And, $A\times B(a, b) = \max(A(a), B(b)) < \max(A(1), B(1^{l})) = A\times B(1, 1^{l})$. Thus anti-product A×B is not an anti S-fuzzy subfield of G×H. Hence either $B(0^{l}) \leq A(x)$, for all x in G and $B(1^{l}) \leq A(x)$, for all $x \neq 0$ in G or $A(0) \leq B(y)$, for all y in H and $A(1) \leq B(y)$, for all $y \neq 0^{l}$ in H.

Theorem: Let A and B be fuzzy subsets of the fields G and H, respectively and the anti-product A×B is an anti S-fuzzy subfield of G×H. Then the following are true:

(i) if $A(x) \geq B(0^{l})$, $A(x) \geq B(1^{l})$, then A is an anti S-fuzzy subfield of G.

(ii) if $B(x) \geq A(0)$, $B(x) \geq A(1)$, then B is an anti S-fuzzy subfield of H.

(iii) either A is an anti S-fuzzy subfield of G or B is an anti S-fuzzy subfield of H, where 0, 1and $0^{l}, 1^{l}$ are the identity elements of G and H, respectively.

Proof: Let the anti-product A×B be an anti S-fuzzy subfield of G×H and x, y in G. Then $(x, 0^{l})$, $(x, 1^{l})$ and $(y, 0^{l})$, $(y, 1^{l})$ are in G×H. Now, using the property $A(x) \geq B(0^{l})$, $A(x) \geq B(1^{l})$, for all x in G, we get, $A(x-y) = \max(A(x-y), B(0^{l}+0^{l})) = A\times B((x-y), (0^{l}+0^{l})) = A\times B[(x, 0^{l})+(-y, 0^{l})] \leq S(A\times B(x, 0^{l}), A\times B(-y, 0^{l})) = S(\max(A(x), B(0^{l})), \max(A(-y), B(0^{l}))) = S(A(x), A(-y)) \leq S(A(x), A(y))$. Therefore, $A(x-y) \leq S(A(x), A(y))$, for all x and y in G. And, $A(xy^{-1}) = \max(A(xy^{-1}), B(1^{l}1^{l})) = A\times B((xy^{-1}), (1^{l}1^{l})) = A\times B[(x, 1^{l})(y^{-1}, 1^{l})] \leq S(A\times B(x, 1^{l}), A\times B(y^{-1}, 1^{l})) = S(\max(A(x), B(1^{l})), \max(A(y^{-1}), B(1^{l}))) = S(A(x), A(y^{-1})) \leq S(A(x), A(y))$. Therefore, $A(xy^{-1}) \leq S(A(x), A(y))$, for all x and $y \neq 0$ in G. Hence A is an anti S-fuzzy subfield of G. Thus (i) is proved. Now, using the property $B(x) \geq A(0)$, for all x in H and $B(x) \geq A(1)$, for all $x \neq 0^{l}$ in H, we get, $B(x-y) = \max(B(x-y), A(0+0)) = A\times B((0+0), (x-y)) = A\times B[(0, x)+(0, -y)] \leq S(A\times B(0, x), A\times B(0, -y)) = S(\max(A(0), B(x)), \max(A(0), B(-y))) = S(B(x), B(-y)) \leq S(B(x), B(y))$. Therefore, $B(x-y) \leq S(B(x), B(y))$, for all x and y in H. And, $B(xy^{-1}) = \max(B(xy^{-1}), A(1.1)) = A\times B((1.1), (xy^{-1})) = A\times B[(1, x)(1, y^{-1})] \leq S(A\times B(1, x), A\times B(1, y^{-1})) = S(\max(A(1), B(x)), \max(A(1), B(y^{-1}))) = S(B(x), B(y^{-1})) \leq S(B(x), B(y))$. Therefore, $B(xy^{-1}) \leq S(B(x), B(y))$, for all x and $y \neq 0^{l}$ in H.

Hence B is an anti S-fuzzy subfield of H. Thus (ii) is proved. And (iii) is clear.

Theorem: Let A be a Fuzzy subset of a field (F, +, .) and V be the anti-strongest S-fuzzy relation of F. Then A is an anti S-fuzzy subfield of F if and only if V is an anti S-fuzzy subfield of F×F.

Proof: Suppose that A is an anti S-fuzzy subfield of F. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in F×F. We have, $V(x-y) = V[(x_1, x_2) - (y_1, y_2)] = V(x_1 - y_1, x_2 - y_2) = \max(A(x_1-y_1), A(x_2-y_2)) \leq \max(S(A(x_1), A(y_1)), S(A(x_2), A(y_2))) = S(\max(A(x_1), A(x_2)), \max(A(y_1), A(y_2))) = S(V(x_1, x_2), V(y_1, y_2)) = S(V(x), V(y))$. Therefore, $V(x-y) \leq S(V(x), V(y))$, for all x and y in F×F. And $V(xy^{-1}) = V[(x_1, x_2)(y_1, y_2)^{-1}] = V(x_1y_1^{-1}, x_2y_2^{-1}) = \max(A(x_1y_1^{-1}), A(x_2y_2^{-1})) \leq \max(S(A(x_1), A(y_1)), S(A(x_2), A(y_2))) = S(\max(A(x_1), A(x_2)), \max(A(y_1), A(y_2))) = S(V(x_1, x_2), V(y_1, y_2)) = S(V(x), V(y))$. Therefore, $V(xy^{-1}) \leq S(V(x), V(y))$, for all x and $y \neq (0, 0)$ in F×F. This proves that V is an anti S-fuzzy subfield of F×F. Conversely, assume that V is an anti S-fuzzy subfield of F×F, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in F×F, we have $\max\{A(x_1-y_1), A(x_2-y_2)\} = V(x_1 - y_1, x_2 - y_2) = V[(x_1, x_2) - (y_1, y_2)] = V(x - y) \leq S(V(x), V(y)) = S(V(x_1, x_2), V(y_1, y_2)) = S(\max(A(x_1), A(x_2)), \max(A(y_1), A(y_2)))$. If we put $x_2 = y_2 = 0$, we get, $A(x_1 - y_1) \leq S(A(x_1), A(y_1))$, for all x_1 and y_1 in F. And $\max\{A(x_1y_1^{-1}), A(x_2y_2^{-1})\} = V(x_1y_1^{-1}, x_2y_2^{-1}) = V[(x_1, x_2)(y_1, y_2)^{-1}] = V(xy^{-1}) \leq S(V(x), V(y)) = S(V(x_1, x_2), V(y_1, y_2)) = S(\max(A(x_1), A(x_2)), \max(A(y_1), A(y_2)))$. If we put $x_2 = y_2 = 1$, We get, $A(x_1y_1^{-1}) \leq S(A(x_1), A(y_1))$, for all x_1 and $y_1 \neq 0$ in F. Hence A is an anti S-fuzzy subfield of F.

Theorem: Let (F, +, ·) and (F^{l}, +, ·) be any two fields. The homomorphic image of an anti S-fuzzy subfield of F is an anti S-fuzzy subfield of F^{l}.

Proof: Let (F, +, ·) and (F^{l}, +, ·) be any two fields and $f : F \rightarrow F^{l}$ be a homomorphism. That is f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V=f(A), where A is an anti S-fuzzy subfield of F. We have to prove that V is an anti S-fuzzy subfield of F^{l}. Now, for f(x) and f(y) in F^{l}, we have $V(f(x)-f(y)) = V(f(x-y)) \leq A(x-y) \leq S(A(x), A(y))$, which implies that $V(f(x)-f(y)) \leq S(V(f(x)), V(f(y)))$, for all f(x) and f(y) in F^{l}. And $V(f(x)(f(y))^{-1}) = V(f(xy^{-1})) \leq A(xy^{-1}) \leq S(A(x), A(y))$, which implies that $V(f(x)(f(y))^{-1}) \leq S(V(f(x)), V(f(y)))$, for all f(x) and $f(y) \neq 0^{l}$ in F^{l}. Hence V is an anti S-fuzzy subfield of a field F^{l}.

Theorem: Let (F, +, ·) and (F^{l}, +, ·) be any two fields. The homomorphic pre-image of an anti S-fuzzy subfield of F^{l} is an anti S-fuzzy subfield of F.

Proof: Let (F, +, ·) and (F^{l}, +, ·) be any two fields and $f : F \rightarrow F^{l}$ be a homomorphism. That is f(x+y) = f(x)+f(y), for all x and y in F and f(xy) = f(x)f(y), for all x and y in F. Let V=f(A), where V is an anti S-fuzzy subfield of F^{l}. We have to prove that A is an anti S-fuzzy subfield of F. Let x and y in F. Then, $A(x-y) = V(f(x-y)) = V(f(x)-f(y)) \leq S(V(f(x)), V(f(y))) = S(A(x), A(y))$, which implies that $A(x-y) \leq S(A(x), A(y))$, for all x and y in F. And, $A(xy^{-1}) = V(f(xy^{-1})) = V(f(x)f(y^{-1})) = V(f(x)(f(y))^{-1}) \leq S(V(f(x)), V(f(y))) = S(A(x), A(y))$, which implies that $A(xy^{-1}) \leq S(A(x), A(y))$, for all x and $y \neq 0$ in F. Hence A is an anti S-fuzzy subfield of a field F.

In the following Theorem ◦ is the composition operation of functions :

Theorem: Let A be an anti S-fuzzy subfield of a field H and f is an isomorphism from a field F onto H. Then A◦f is an anti S-fuzzy subfield of F.

Proof: Let x and y in F and A be an anti S-fuzzy subfield of a field H. Then we have, $(A\circ f)(x-y) = A(f(x-y)) = A(f(x)+f(-y)) = A(f(x)-f(y)) \le S(A(f(x)), A(f(y))) \le S((A\circ f)(x), (A\circ f)(y))$, which implies that $(A\circ f)(x-y) \le S((A\circ f)(x), (A\circ f)(y))$, for all x and y in F. And, $(A\circ f)(xy^{-1}) = A(f(xy^{-1})) = A(f(x)f(y^{-1})) = A(f(x)(f(y))^{-1}) \le S(A(f(x)), A(f(y))) \le S((A\circ f)(x), (A\circ f)(y))$, which implies that $(A\circ f)(xy^{-1}) \le S((A\circ f)(x), (A\circ f)(y))$, for all x and $y \neq 0$ in F. Therefore $(A\circ f)$ is an anti S-fuzzy subfield of a field F.

Theorem: If A is an anti S-fuzzy subfield of a field (F, +, .), then the pseudo anti S-fuzzy coset $(aA)^p$ is an anti S-fuzzy subfield of a field F, for every $a \in F$.

Proof : Let A be an anti S-fuzzy subfield of a field (F, +, .). For every x and y in F, we have, $((aA)^p)(x-y) = p(a)A(x-y) \le p(a) S(A(x), A(y)) = S(p(a)A(x), p(a)A(y)) = S(((aA)^p)(x), ((aA)^p)(y))$. Therefore, $((aA)^p)(x-y) \le S(((aA)^p)(x), ((aA)^p)(y))$, for all x and y in F. And for every x and $y \neq 0$ in F, $((aA)^p)(xy^{-1}) = p(a)A(xy^{-1}) \le p(a) S(A(x), A(y)) = S(p(a)A(x), p(a)A(y)) = S(((aA)^p)(x), ((aA)^p)(y))$. Therefore, $((aA)^p)(xy^{-1}) \le S(((aA)^p)(x), ((aA)^p)(y))$, for all x and $y \neq 0$ in F. Hence $(aA)^p$ is an anti S-fuzzy subfield of a field F.

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