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Dynamic programming method approach for optimizing stock allocation via the step function

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ABSTRACT

Stock control is an important functional branch of any manufacturing/service organization. Stock control has been discussed using the dynamic programming model approach. The distribution of goods to warehouses is a multistage process which the dynamic programming is adapted to providing solution to and also guarantees optimal feasible solution. The stock in warehouses is used as a partition of the firms main store. An equation ensues from the stock record which is shown to be piecewise continuous. The Laplace transform is used to test the existence of the function. The step function that derives from the supply to the warehouses is transformed to a piecewise linear function which can be approximated as continuous function. The step function is then applied on the function to obtain the returns from the respective allocations if the function exists. The allocation with the highest return is the optimum. Some relevant theorem have been stated and proved and illustrative examples included.

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Introduction

Faulty stock allocation has led to huge losses to manufacturing and wholesale firms in the past. In recent times, companies stepped up deliberate effort to minimize the cost incurred from non-availability of their goods in some locations. The conscious effort to ensure that goods reach the consumers has led to warehouses being built/spread at many locations in a region. It is obvious that these activities attract some costs that may affect the amount of profit being made. The stock allocation model seeks to find a stock level that would be maintained in order to optimize returns [9].

Mathematics is a viable tool in attempting to solve this problem of stock allocation. Mathematics model which is a collection of logical and mathematical relationship that represents aspects of the situation is used. In this work a technique for the planning of multistage processes, dynamic programming model is used. This model describes a process in terms of states, decision, transaction and returns. The process begins in some initial state where a decision is made and this decision causes a transition to a new state. The problem is to find the sequence that maximizes the total returns [2]

Goods are stockpiled in different warehouses. The originating warehouse is the main outlet from which the items are sent and stocked in smaller/subsidiary outlet. The subsidiary outlets serve as the subrectangles to the main store.

A rectangle is defined as the Cartesian product of two closed intervals [a, b] and [c, d], $Q = [a,b]\, x\, [c,d] = \{(x,y): x \in [a,b]\ and\ y \in [c.d]$ where φ is a rectangle. A function f defined on a rectangle φ is called step function if a partition p of Q exists such that f is constant on each of the open subrectangles of p. A partition p of $[a,b]$ is a finite set of real numbers $(x_0\, x_1, x_2 - - - x_1)$, such that $a = x_0 \leq x_1 \leq x_2 \leq x_3 \leq - - - \leq x_i = b$ and such that the subintervals therein in p are the intervals $[x_{i-1}, x_1], i = 1, 2, 3, - - - n$. The partition of $[a,b]$ is a finite collection of non $-$ overlapping intervals whose union is $[a,b]$.

In this work, the warehouses are the subrectagles and they have regular replenishment and thus supply is constant. The quantity to each warehouse comes in batches which represents the partitions. The step functions that derive from the supply to the warehouses are transformed to piecewise linear function which can be approximated as continuous function whose domain is a closed interval [a,b]. The Laplace transform is used to check for the existence of the resulting functions. If it exists then the function is solved to obtain an optimal allocation.

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Definitions

Stock is the supply of goods that is available for sale in a warehouse [1].

Dynamic programming model is a method of linear optimization that determines the optimum solution of a multivariable problem by decomposing it into stages, each stage comprises a single variable subproblem. It is a recursive equation that links the different stages of the problem in a manner that guarantees that the optimal feasible solution of each stage is also optimal and feasible for the entire problem [3]. The dynamic programming problem to be solved is stated as; $\max Z = f_1(x_1) + f_2(x_2) + --- + f_n(x_n)$

Subject to $x_1 + x_2 + --- + x_n \leq k$

where k is the number of products allocated and $x_1, x_2 ---- x_n \geq 0$ are integral.

Suppose f is a function with Domain in R^P and range in R^q, f is a step function if it assumes only a finite number of distinct values in R^q each non-zero value being taken on an interval in R^P, then f is a step function. f is defined on a rectangle Q of a partition p on Q exists such that f is constant on each of the open subrectangles of p [4].

Suppose g is a function defined on a compact interval [a, b] of R g is called piecewise linear if there are a finite number of points C_r with

$a = C_0 < C_1 < C_2 < --- < C_n = b$ and corresponding real numbers

$A_r, B_r,\ r = 0, 1, 2, ---, n$ such that x satisfies the relation $C_{r-1} < x < C_r$, the function g has the form

$g(x) = A_r + B_r, r = 0, 1, 2, --- n$. Continuous functions can be approximated uniformly by simple functions which are also continuous. Step functions by nature are not continuous, hence we adopt a simple approach where p = q = 1 to make them piecewise linear functions [5].

Suppose f_p is a given function with domain D contained in R^P and range in R^q. then a function g approximates f uniformly on D within $\varepsilon > 0\ if$

$$IIg(x) - f(x)II \leq \varepsilon \qquad \forall\, x\ in\ D.$$

Laplace transform: Let f be a function defined for $t \geq o$. then the integral $\mathcal{L}(f(x)) = \int_o^\infty e^{-st} f(x)dt$ is said to be the Laplace transform of f provided the integral converges. When $\mathcal{L}(f(x))$ converges, the result is a function of s. The Laplace transform is a method where operations of calculus on function are replaced by operations of algebra on transforms. The Laplace transform has a linearity property which makes it very useful for solving linear differential equation with constant coefficients.

Theorems

Let f be a continuous function whose domain is a compact cell in R^P and whose values belong to R^q. Then f can be uniformly approximated on D by step function. [6]

Proof: Assume $\varepsilon > 0$ be given

But it is true that if f is a continuous function with domain D in R^P and range in R^q and if $N \subseteq D$ is compact, then f is uniformly continuous on N.

$\therefore$ there is a number $\delta(\varepsilon) > 0$ such that if $x < y$ belong to D and $IIx - yII < \delta(\varepsilon)$, then $IIf(x) - f(y)II < \varepsilon$.

Now divide the domain D of f into disjoint cells $I_1, I_2, --- I_n$ such that x, y belong to $I_k, k = 1, 2, --- n$ and define $g(x) = f(x_k)$ for $x\ in\ I_k$ and $g(x) = 0$ for x not in D.

i.e. $$g(x) = \begin{cases} x_k\ if\ xED \\ 0\ if\ x \notin D \end{cases}$$

Then clearly $IIg(x) - f(x_k)\ II < \varepsilon$ for x in D.

So that g approximates f uniformly on D within ε.

Suppose f is a continuous function whose domain is a closed interval $[a, b]$. then f can be uniformly approximately on [a, b] by continuous piecewise linear function.

Proof: The uniform continuity of f is compact on the cell [a, b].

$\therefore$ Given $\varepsilon > 0$, we can partition [a, b] into cells by adding intermediate points $C_r, r = 0, 1, 2, ---, n$ with

$a = C_0 < C_1 < C_2 < - - - < C_n = b$ so that $C_r - C_{r-1} < \delta\,(\varepsilon)$.

now if the points $(C_r, f\,(C_r))$ are connected by line segments and define the resulting continuous piecewise linear function g, Thus clearly, g approximates f uniformly on [a, b] with ε [8] Laplace transform is a linear operation i.e. for any function $f(x)$ and $g(x)$ whose transform exists and any constants a and b, the transform of $af(x) + gb(x)$ exist and

$$\mathcal{L}[af(x) + bg(x)] = a\,\mathcal{L}\,[f(x) + b[g(x)]$$

Proof: By definition $F[s] = \mathcal{L}(t) = \int_o^\infty e^{-st} f(t)dt$ (1)

$\therefore \mathcal{L}[af(t)] = \int_o^\infty ae^{-st} f(t)dt, \quad \mathcal{L}\,[bg(t)] = \int_o^\infty be^{-st} g(t)dt$ (2)

$$\Rightarrow \mathcal{L}[af(t) + bg(x)] = \int_o^\infty ae^{-st} f(t)dt + \int_o^\infty be^{-st} g(t)dt$$

$$= a\int_o^\infty e^{-st} f(t)dt + b\int_o^\infty e^{-st} g(t)dt \qquad (3)$$

From (1) above, (3) becomes

$a\mathcal{L}[\,f(t)] + b\mathcal{L}[g(t)]$

$\therefore \mathcal{L}[af(t) + bg(t)] = a\mathcal{L}[\,f(t)] + b\mathcal{L}[\,g(t)]$.

4.

[10] If $f(t)$ is piecewise continuous on $[o, \infty)$ and of exponential order c for $t > T$,

Then $\mathcal{L}[f(t)]$ exists for $s > c$.

Proof: $\mathcal{L}[\,f(t)] = \int_o^T e^{-st} f(t)dt + \int_T^\infty e^{-st} f(t)dt = I_1 + I_2$.

The integral I_1 exists because it can be written as a sum of integrals over intervals on which $e^{-st} f(t)$ is continuous.

Now $\overline{I_2} \leq \int_T^\infty / \, e^{-st} f(t)dt \leq M\int_T^\infty e^{-st} e^{ct} dt$

$= M\int_T^\infty e^{-(t-c)ti} dt$

$= -M\left[\frac{e^{-(t-c)s}}{s-c}\right]_T^\infty = M\frac{e^{-(S-c)T}}{s-c}$ for s > c

Since $\int_T^\infty Me^{-(s-c)t} dt$ converges, the integral

$\int_T^\infty / e^{-st} f(t)/dt$ converges by the comparison test for improper integrals.

$\Rightarrow$ I_2 exists for $s > c$

$\therefore$ the existence of I_1 and I_2 implies that

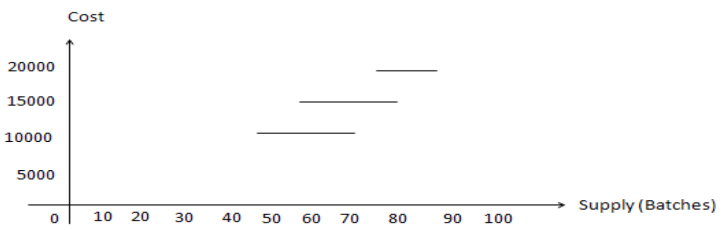
$\mathcal{L}\,[f(t)\,] = \int_0^\infty e^{-st} f(t)dt$ exists for $s > c$.

Illustration Examples

A firm has three warehouses through which it distributes from the depot. The following is the record of batches supplied to the warehouses and their corresponding costs incured. Obtain the optimum allocation of the company.

(a) Warehouse	Cost (units)	supply (batches in 000)
A.	10,000	if $50 \leq t \leq 70$
B.	15, 000	if $75 \leq t \leq 90$
C.	12,000	if $60 \leq t \leq 80$

The graph appears thus:



The function for the supply is $f(t) = f(t_1) + f(t_2) + f(t_3)$. This function is assumed piecewise continuous hence apply the Laplace transform to test existence. i.e.

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)dt$$

$$\int_{50}^{70} 10{,}000e^{-st}\,dt + \int_{75}^{90} 15{,}000e^{-st}dt + \int_{60}^{80} 12{,}000e^{-st}dt$$

$$= \left[\frac{10{,}000e^{-st}}{-s}\right]_{50}^{70} + \left[\frac{15{,}000e^{-st}}{-s}\right]_{75}^{90} + \left[\frac{12{,}000}{-s}\right]_{60}^{80}$$

$$= \frac{1}{-s}\{[10{,}000e^{-st}]_{50}^{70} + [15{,}000e^{-st}]_{75}^{90} + [12{,}000e^{-st}]_{60}^{80}$$

$= \frac{1}{s} e^{56/25}$

Since $s > 0$ $\lim_{s\to\infty} \frac{1}{-s} e^{2.24}$ exists. We can then obtain the respective returns

i.e. $\int_{50}^{70} 10{,}000\,dx = [10{,}000x]_{50}^{70} = 200{,}000$

$$\int_{50}^{70} 15{,}000\,dx = [15{,}000x]_{75}^{90} = 225{,}000$$

and

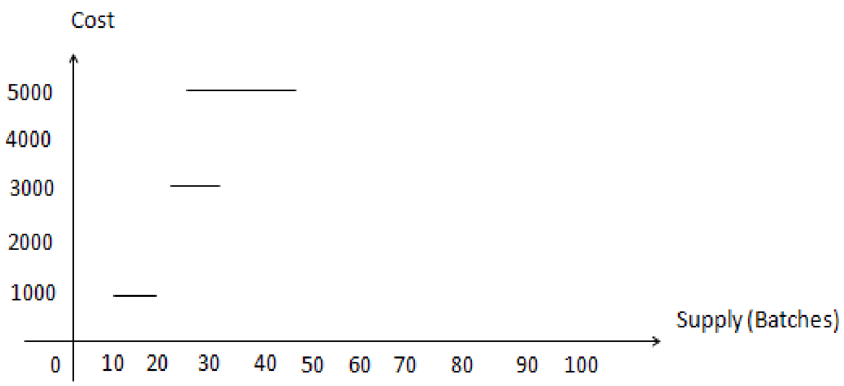
$$\int_{60}^{80} 12{,}000\,dx = [12{,}000x]_{60}^{80} = 240{,}000$$

The returns are 200,000, 225,000 and 240,000 respectively. Warehouse C has the best and hence the optimal allocation is 60,000 to 80,000 batches

(b)

Warehouse	Cost (units)		supply (batches in 000)
A.	1,000	if	$10 \le x \le 20$
B.	3, 000	if	$20 \le x \le 30$
C.	5,000	if	$30 \le x \le 50$

The graph appears thus:



Using Laplace transform to ascertain existence or otherwise gives the following

$$\int_{10}^{20} 1{,}000e^{-st}\,dt + \int_{20}^{30} 3{,}000e^{-st}dt + \int_{30}^{50} 5{,}000e^{-st}dt$$

$$= \left[\frac{1{,}000e^{-st}}{-s}\right]_{10}^{20} + \left[\frac{3{,}000e^{-st}}{-s}\right]_{20}^{30} + \left[\frac{5{,}000}{-s}\right]_{30}^{50}$$

$= \frac{1}{s} e^{\frac{20}{10}} + e^{\frac{30}{20}} + e^{\frac{50}{30}}$

$s > 0,$ $\lim_{s\to\infty} \frac{e^s}{-s}$ exists

Hence

$$\int_{10}^{20} 1{,}000\, dx = [20{,}000 - 10{,}000 \quad = 10{,}000$$

$$\int_{20}^{30} 3{,}000\, dx = 90{,}000 - 60{,}000 \quad = 30{,}000$$

$$\int_{30}^{50} 5{,}000\, dx = 250{,}000 - 150{,}000 = 100{,}000$$

The highest return is 100,000 and hence the optimum allocation is 30,000 to 50,000 batches of warehouse Z.

Summary/Conclusion

Stock control is an important functional branch of any organization and needs proper management [7]. Stock allocation has been x-rayed using the step function as an approximation method. The given stock allocation are represented graphically to show that it is a piecewise linear function. A step function is not continuous but can be approximated using the piecewise continuous linear functions [8]. When a variety allocation of stock to different warehouses are presented, the Laplace transform is applied to ascertain the existence of the function that results from the allocation to the warehouses. If the function exists, then the unit step function is applied to calculate the corresponding return of each channel. The allocation with the highest cost is the optimal.

Some effort has been made to apply some mathematical tools in stock allocation. Stock allocation has been medelled such that the step function can be used to obtain the approximate optimal allocation.

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