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Reliability and sensitivity analysis of a two unit warm standby system with low efficiency unit

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ABSTRACT

In the present paper the system considered consists of two subsystems A and B. Subsystem A consists of identical operating and warm standby units. While subsystem B has two dissimilar units: main unit and a unit in cold standby. Main unit of subsystem B is assumed to be more efficient than the standby unit so when the main unit fails the system goes to the state of low efficiency. Main unit is connected to cold standby unit with a switching over device. Further, whenever there is a failure in one of the units of A and in the main unit of subsystem B, the system goes to critical state where system has to stop functioning to avoid the further failures. Also we have considered that the company providing repair facility has appointed a repairman. The repairman repairs the system in case of minor failures but when the system fails completely he has to take it to the nearest service station of the company for repair. By applying Supplementary variable technique, Laplace transformations and copula methodology transition state probabilities, asymptotic behaviour, reliability, availability, M.T.T.F., cost effectiveness and sensitivity of the system have been determined. Particular cases corresponding to the situations when the standby unit of subsystem A is in cold standby and in hot standby have also been considered. At last some numerical examples have been taken to illustrate the model.

Introduction

It is well known that the reliability of a system can be improved by using standby units in the system. We can find a number of applications of these types of systems in various industrial setups. In general there are three types of standbys: (i) Cold standby in which the standby unit is only called upon when the primary or operating unit fails. Here the unit in the state of standby does not fail; (ii) Warm standby in which the standby unit runs in the background of operating unit. In this case the unit can fail in the state of standby but its failure rate is less than that of operating unit. As the standby unit comes in operating state its failure rate is equal to operating unit; (iii) Hot standby in which the standby unit run with the operating unit and the unit in the state of standby can fail with the same failure rate as operating unit. In real world there are various systems having standby units such as electric generator in a power plant, inverter in houses, batteries in various electronic setups, wheel assembly in a truck etc. In these systems when operating unit fails the load is transferred to the standby with the help of switching over device in case of cold standby or automatically in case of hot and warm standbys. In many situations the assumption that the standby unit is capable to bear the total load as efficiently as the primary unit is not seems to be realistic. Many researchers [1, 3, 8, 9] have considered the system with standby units but they did not take into account the case of low efficiency of the standby units. Also, when the standby units are not as efficient as the primary unit the chances of failure of the system may be greater than the normal situation. Further, we can find many cases in which prolong running of the system in the standby brings the system to a risky state where it is advisable to stop the functioning of the system to prevent the system from major damages. For instance in case of a two-engine aeroplane if one engine fails the pilot has a provision of emergency landing so as to minimize the risk of life.

Keeping all these facts in view in the present paper we have considered a system which consists of two subsystems A and B in series configuration. Subsystem A has two homogeneous units: Operating and warm standby. The failure rates of operating and standby units are λ_1 and λ_2 respectively. When standby becomes functional by virtue of failure of operating unit of A its failure rate is assumed to be λ_1. Subsystem B is a heterogeneous system having two units: Main and a unit in cold standby. Here it is assumed that

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the main unit is more efficient than the standby unit. Whenever the main unit fails, load transfers to the standby unit with the help of a switching over device and the system goes to the state of low efficiency. Further, whenever one unit of Subsystem A and the main unit of B fail, system goes to critical state and is stopped deliberately to avoid the further risks of failure. The company providing the repair facility has appointed a person (repairman) to look after the system. The repairman repairs the system whenever there is a minor failure in the system. But if the system fails completely due to major failures, policy of the company is to take it to the nearest service station of the company for repair. Therefore, in this situation repairing of the system cannot be started immediately, hence it has to wait for the repair. At the service station better facilities and expert repairmen are available where the system can be repaired effectively in a short period of time. Failure rates of the system are assumed to be constant whereas repairs follow general distributions. It is also assumed that from state S_0 to S_3 and from S_1 to S_5 there are two different types of failures. This is a realistic assumption since the warm standby unit can also fail in state of standby with a failure rate less than that of operating unit. The joint probability distribution of failure rates has been analysed by using copula methodology [5, 7]. Transition state probabilities, asymptotic behaviour, various reliability measures such as reliability, availability, M.T.T.F., cost analysis and sensitivity analysis of the system have been obtained by using Supplementary variable technique, Laplace transformation and copula. Transition state probabilities when the second unit of subsystem A is a cold standby or a hot standby with the operating unit have also been examined and a comparison on the basis of the reliability obtained in these three cases has also been made. Numerical examples have been provided to illustrate the model at last.

Assumptions

(1) Initially the system is in perfectly operating state.

(2) In subsystem A one unit is in warm standby with the operating unit. Both the units are similar.

(3) In subsystem B one unit is in cold standby with the main unit. The standby unit is not as efficient as the main unit so when the main unit of subsystem B fails the system goes to the state of low efficiency.

(4) The system S_5 in which one unit of subsystem A and the main unit of subsystem B have failed is a critical state. In this state the functioning of system stopped deliberately with the emergency failure rate λ_E.

(5) The switchover device is perfect and switchover is instantaneous.

(6) The main unit of subsystem B has given priority over other units for repair.

(7) Subsystem A can repair only when both of its units have failed.

(8) When the system is in complete failure state the repairman provided by company carries it to the nearest service station of company due to which the system has to wait for some time.

(9) After repair the system is as good as new.

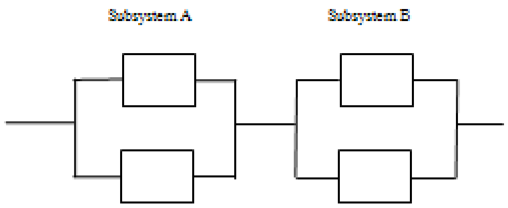
(10) The joint probability distribution of failure rates is given by Gumbel-Hougaard family of copula.

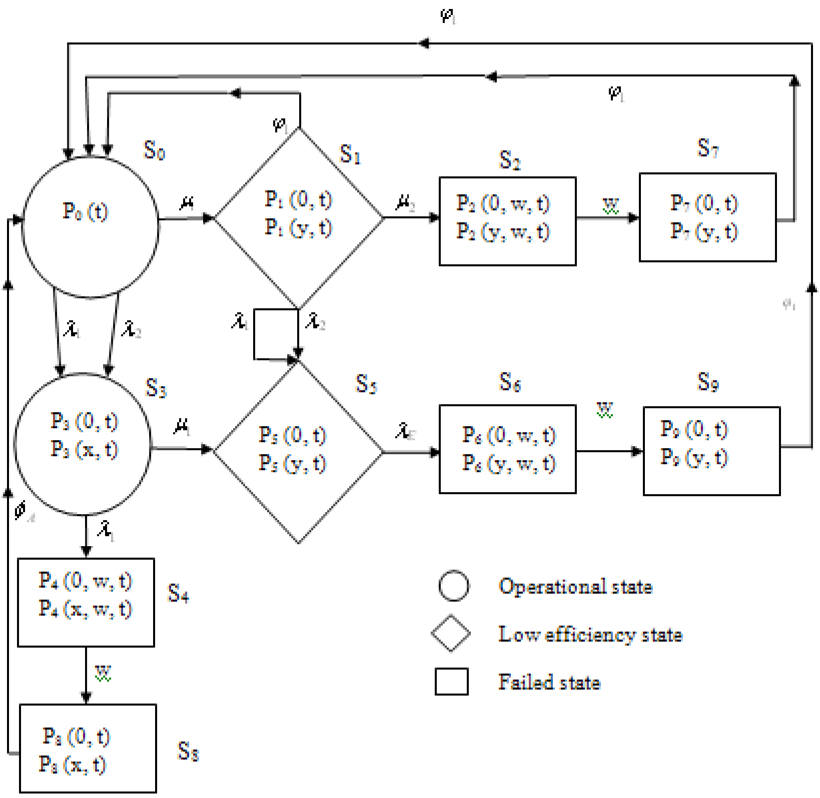
State Specification

States	Subsystem A: Number of good units	Subsystem B: Number of good units	System state
S_0	2	2	G
S_1	2	1	L
S_2	2	0	F_w
S_3	1	2	G
S_4	0	2	F_w
S_5	1	1	C
S_6	1	1	F_w
S_7	2	0	F_R
S_8	0	2	F_R
S_9	1	1	F_R

Table 1: State specification

G: Good state, L: Low efficiency state, C: Critical state, F_w: Failed under waiting, F_R: Failed under repair





Notations

λ_1 : Failure rate of operating unit of A.

λ_2 : Failure rate of standby unit of A.

μ_1 : Failure rate of main unit of B.

μ_2 : Failure rate of standby unit of B.

λ_E : Rate of emergency failure in the system.

$\phi_i(r)$: Repair rate of main unit and standby unit of subsystem B. If i = 1/2 then r = y/z.

$\phi_A(x)$: Repair rate of subsystem A.

x, y, z: Elapsed repair time for both the units of subsystem A, the main unit of subsystem B and the standby unit of B.

P_i (t): Probability that the system is in S_i state at instant t for i = 1 to i = 9.

$\overline{P}_i(s)$: Laplace transform of P_i (t).

P_4(x, t): Probability density function that at time t the system is in failed state S_4 and the system is under repair, elapsed repair time is x.

E_p (t): Expected profit during the interval (0, t].

K_1, K_2: Revenue per unit time and service cost per unit time respectively.

$S_\eta(x)$: $\eta(x)\exp(-\int_0^x \eta(x)dx)$

$\bar{S}_\eta(x)$: Laplace transform of $S_\eta(x) = \int_0^\infty \eta(x)\exp(-sx - \int_0^x \eta(x)dx$

If $u_1 = \phi_P(y)$, $u_2 = \psi_P(y)$ then the expression for the joint probability according to Gumbel-Hougaard family of copula is given as

$C_\theta(u_1, u_2) = \exp[-\{(-\log\phi_P(y))^\theta + (-\log\psi_P(y))^\theta\}^{1/\theta}]$

5. Formulation of Mathematical Model

By elementary probability and continuity arguments, one can obtain the following set of integro-differential equations.

$$\left[\frac{d}{dt} + \mu_1 + \exp[-\{(-\log\lambda_1)^\theta + (-\log\lambda_2)^\theta\}^{1/\theta}]\right]P_0(t) = \int_0^\infty \phi_1(y)P_1(y,\, t)dy$$

$$+\int_0^\infty \phi_1(y)P_7(y,\, t)dy + \int_0^\infty \phi_A(x)P_8(x,\, t)dy + \int_0^\infty \phi_1(y)P_9(y, t)dy \quad \ldots(1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2 + \phi_1(y) + \exp[-\{(-\log\lambda_1)^\theta + (-\log\lambda_2)^\theta\}^{1/\theta}]\right]P_1(y,t) = 0 \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + w\right]P_2(y,w,t) = 0 \quad \ldots(3)$$

$$\left[\frac{d}{dt} + \mu_1 + \lambda_1\right]P_3(t) = \exp[-\{(-\log\lambda_1)^\theta + (-\log\lambda_2)^\theta\}^{1/\theta}]P_0(t) \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + w\right]P_4(x,w,t) = 0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_E\right]P_5(y,t) = 0 \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + w\right]P_6(y,w,t) = 0 \quad \ldots(7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \phi_1(y)\right]P_7(y,t) = 0 \quad \ldots(8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi_A(x)\right]P_8(x,t) = 0 \quad \ldots(9)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \phi_1(y)\right]P_9(y,t) = 0 \quad \ldots(10)$$

Boundary conditions:

$$P_1(0,t) = \mu_1 P_0(t) \quad \ldots(11)$$

$$P_2(0,w,t) = \mu_2 P_1(y,t) \quad \ldots(12)$$

$$P_3(t) = 0 \quad \ldots(13)$$

$$P_4(0,w,t) = \lambda_1 P_3(t) \quad \ldots(14)$$

$$P_5(0,t) = \mu_1 P_3(t) + \exp[-\{(-\log\lambda_1)^\theta + (-\log\lambda_2)^\theta\}^{1/\theta}]P_1(y,t) \quad \ldots(15)$$

$$P_6(0,w,t) = \lambda_E P_5(y,t) \quad \ldots(16)$$

$P_7(0,t) = wP_2(y,w,t)$ …(17)

$P_8(0,t) = wP_4(x,w,t)$ …(18)

$P_9(0,t) = wP_6(y,w,t)$ …(19)

Initial condition:

$P_0(t) = 1$ at t = 0 and all other probabilities are zero initially. …(20)

Solution of the model

Taking Laplace transformation of equations (1-19) and using (20), we get

$$\left[s+\mu_1+\exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}]\right]\overline{P}_0(s) = 1+\int_0^\infty \phi_1(y)\overline{P}_1(y,\ s)dy$$
$$+\int_0^\infty \phi_1(y)\overline{P}_7(y,\ s)dy+\int_0^\infty \phi_A(x)\overline{P}_8(x,\ s)dy+\int_0^\infty \phi_1(y)\overline{P}_9(y,s)dy \quad \text{…(21)}$$

$$\left[s+\frac{\partial}{\partial y}+\mu_2+\phi_1(y)+\exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}]\right]\overline{P}_1(y,s) = 0 \quad \text{…(22)}$$

$$\left[s+\frac{\partial}{\partial y}+w\right]\overline{P}_2(y,w,s) = 0 \quad \text{…(23)}$$

$$\left[s+\mu_1+\lambda_1\right]\overline{P}_3(s) = \exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}]\overline{P}_0(s) \quad \text{…(24)}$$

$$\left[s+\frac{\partial}{\partial x}+w\right]\overline{P}_4(x,w,s) = 0 \quad \text{…(25)}$$

$$\left[s+\frac{\partial}{\partial y}+\lambda_E\right]\overline{P}_5(y,s) = 0 \quad \text{…(26)}$$

$$\left[s+\frac{\partial}{\partial y}+w\right]\overline{P}_6(y,w,s) = 0 \quad \text{…(27)}$$

$$\left[s+\frac{\partial}{\partial y}+\phi_1(y)\right]\overline{P}_7(y,s) = 0 \quad \text{…(28)}$$

$$\left[s+\frac{\partial}{\partial x}+\phi_A(x)\right]\overline{P}_8(x,s) = 0 \quad \text{…(29)}$$

$$\left[s+\frac{\partial}{\partial y}+\phi_1(y)\right]\overline{P}_9(y,s) = 0 \quad \text{…(30)}$$

Boundary conditions:

$\overline{P}_1(0,s) = \mu_1\overline{P}_0(s)$ …(31)

$\overline{P}_2(0,w,s) = \mu_2\overline{P}_1(y,s)$ …(32)

$\overline{P}_3(s) = 0$ …(33)

$\overline{P}_4(0,w,s) = \lambda_1\overline{P}_3(s)$ …(34)

$\overline{P}_5(0,s) = \mu_1\overline{P}_3(s)+\exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}]\overline{P}_1(y,s)$ …(35)

$\overline{P}_6(0,w,s) = \lambda_E\overline{P}_5(y,s)$ …(36)

$\overline{P}_7(0,s) = w\overline{P}_2(y,w,s)$ …(37)

$$\overline{P}_8(0,s) = w\overline{P}_4(x,w,s) \quad \ldots(38)$$

$$\overline{P}_9(0,s) = w\overline{P}_6(y,w,s) \quad \ldots(39)$$

Solving equations (21-30) and using equations (31-39), we get the following transition state probabilities

$$\overline{P}_0(s) = 1/D(s) \quad \ldots(40)$$

$$\overline{P}_1(s) = \mu_1\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(s+\lambda+\mu_2)}{s+\lambda+\mu_2}\right] \quad \ldots(41)$$

$$\overline{P}_2(s) = \mu_1\mu_2\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(2s+\lambda+w+\mu_2)}{2s+\lambda+w+\mu_2}\right] \quad \ldots(42)$$

$$\overline{P}_3(s) = \lambda\overline{P}_0(s)\left[\frac{1}{s+\lambda_1+\mu_1}\right] \quad \ldots(43)$$

$$\overline{P}_4(s) = \lambda_1\overline{P}_3(s)\left[\frac{1}{s+w}\right] \quad \ldots(44)$$

$$\overline{P}_5(s) = \mu_1\overline{P}_3(s)\left[\frac{1}{s+\lambda_E}\right] + \mu_1\lambda\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(2s+\lambda+\lambda_E+\mu_2)}{2s+\lambda+\lambda_E+\mu_2}\right] \quad \ldots(45)$$

$$\overline{P}_6(s) = \mu_1\lambda_E\overline{P}_3(s)\left[\frac{1}{2s+\lambda_E+w}\right] + \mu_1\lambda\lambda_E\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(3s+\lambda+w+\lambda_E+\mu_2)}{3s+\lambda+w+\lambda_E+\mu_2}\right] \quad \ldots(46)$$

$$\overline{P}_7(s) = \mu_1 w\mu_2\overline{P}_0(s)\left[\frac{1-\overline{S}_{2\phi_1}(3s+\lambda+w+\mu_2)}{2(3s+\lambda+w+\mu_2)}\right] \quad \ldots(47)$$

$$\overline{P}_8(s) = w\lambda_1\overline{P}_3(s)\left[\frac{1-\overline{S}_{\phi_A}(2s+w)}{2s+w}\right] \quad \ldots(48)$$

$$\overline{P}_9(s) = \mu_1\lambda_E w\overline{P}_3(s)\left[\frac{1-\overline{S}_{\phi_1}(3s+w+\lambda_E)}{3s+w+\lambda_E}\right] + w\mu_1\lambda\lambda_E\overline{P}_0(s)\times$$
$$\times\left[\frac{1-\overline{S}_{2\phi_1}(4s+\lambda+w+\lambda_E+\mu_2)}{2(4s+\lambda+w+\lambda_E+\mu_2)}\right] \quad \ldots(49)$$

where

$$D(s) = s+\mu_1+\lambda-\mu_1\overline{S}_{\phi_1}(s+\mu_2+\lambda)-\frac{w}{2}\mu_1\mu_2\overline{S}_{2\phi_1}(3s+w+\lambda+\mu_2)-\frac{w\lambda\lambda_1}{s+\mu_1+\lambda_1}\times$$
$$\times\overline{S}_{\phi_A}(2s+w)-\frac{w\lambda_E\mu_1\lambda}{s+\mu_1+\lambda_1}\overline{S}_{\phi_1}(3s+w+\lambda_E)-\frac{w}{2}\lambda\lambda_E\mu_1\overline{S}_{2\phi_1}(4s+\lambda+\mu_2+w+\lambda_E) \quad \ldots(50)$$

$$\lambda = \exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}] \quad \ldots(51)$$

Also up and down state probabilities of the system are given by

$$\overline{P}_{\mathrm{up}}(s) = \overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_3(s)+\overline{P}_5(s)$$

$$=\left[1+\mu_1\left\{\frac{1-\overline{S}_{\phi_1}(s+\lambda+\mu_2)}{s+\lambda+\mu_2}\right\}+\frac{\lambda}{s+\lambda_1+\mu_1}+\frac{\mu_1\lambda}{(s+\lambda_E)(s+\mu_1+\lambda_1)}\right.$$
$$\left.+\mu_1\lambda\left\{\frac{1-\overline{S}_{\phi_1}(2s+\lambda+\lambda_E+\mu_2)}{2s+\lambda+\lambda_E+\mu_2}\right\}\right]\overline{P}_0(s) \quad \ldots(52)$$

$$\overline{P}_{\mathrm{down}}(s) = \overline{P}_2(s)+\overline{P}_4(s)+\overline{P}_6(s)+\overline{P}_7(s)+\overline{P}_8(s)+\overline{P}_9(s)$$

$$= \Bigg[\mu_1\mu_2 \left\{ \frac{1-\bar{S}_{\phi_1}(2s+\lambda+w+\mu_2)}{2s+\lambda+w+\mu_2} \right\} + \frac{\lambda_1\lambda}{(s+w)(s+\mu_1+\lambda_1)}$$

$$+ \frac{\mu_1\lambda_E\lambda}{(2s+\lambda_E+w)(s+\lambda_1+\mu_1)} + \mu_1\lambda\lambda_E \left\{ \frac{1-\bar{S}_{\phi_1}(3s+\lambda+w+\lambda_E+\mu_2)}{3s+\lambda+w+\lambda_E+\mu_2} \right\}$$

$$+ \mu_1 w\mu_2 \left\{ \frac{1-\bar{S}_{2\phi_1}(3s+\lambda+w+\mu_2)}{2(3s+\lambda+w+\mu_2)} \right\} + w\lambda_1\lambda \left\{ \frac{1-\bar{S}_{\phi_A}(2s+w)}{(2s+w)(s+\lambda_1+\mu_1)} \right\}$$

$$+ \mu_1\lambda_E w\lambda \left\{ \frac{1-\bar{S}_{\phi_1}(3s+w+\lambda_E)}{(3s+w+\lambda_E)(s+\mu_1+\lambda_1)} \right\} + w\mu_1\lambda\lambda_E \times$$

$$\times \left\{ \frac{1-\bar{S}_{2\phi_1}(4s+\lambda+w+\lambda_E+\mu_2)}{2(4s+\lambda+w+\lambda_E+\mu_2)} \right\} \Bigg] \bar{P}_0(s) \qquad \ldots(53)$$

Also it is noticeable that

$$\bar{P}_{\text{up}}(s) + \bar{P}_{\text{down}}(s) = 1/s \qquad \ldots(54)$$

Asymptotic behaviour

Using Able's lemma

$$\lim_{s\to 0} \{s\bar{F}(s)\} = \lim_{t\to\infty} F(t) = F(\text{say})$$

in equations (52) and (53), one can obtain the following time independent probabilities

$$P_{\text{up}} = \Bigg[1 + \mu_1 \left\{ \frac{1-\bar{S}_{\phi_1}(\lambda+\mu_2)}{\lambda+\mu_2} \right\} + \frac{\lambda}{\lambda_1+\mu_1} + \frac{\mu_1\lambda}{\lambda_E(\mu_1+\lambda_1)}$$

$$+ \mu_1\lambda \left\{ \frac{1-\bar{S}_{\phi_1}(\lambda+\lambda_E+\mu_2)}{\lambda+\lambda_E+\mu_2} \right\} \Bigg] \frac{1}{D(0)} \qquad \ldots(55)$$

$$P_{\text{down}} = \Bigg[\mu_1\mu_2 \left\{ \frac{1-\bar{S}_{\phi_1}(\lambda+w+\mu_2)}{\lambda+w+\mu_2} \right\} + \frac{\lambda_1\lambda}{w(\mu_1+\lambda_1)} + \frac{\mu_1\lambda_E\lambda}{(\lambda_E+w)(\lambda_1+\mu_1)}$$

$$+ \mu_1\lambda\lambda_E \left\{ \frac{1-\bar{S}_{\phi_1}(\lambda+w+\lambda_E+\mu_2)}{\lambda+w+\lambda_E+\mu_2} \right\} + \mu_1 w\mu_2 \left\{ \frac{1-\bar{S}_{2\phi_1}(\lambda+w+\mu_2)}{2(\lambda+w+\mu_2)} \right\}$$

$$+ w\lambda_1\lambda \left\{ \frac{1-\bar{S}_{\phi_A}(w)}{w(\lambda_1+\mu_1)} \right\} + \mu_1\lambda_E w\lambda \left\{ \frac{1-\bar{S}_{\phi_1}(w+\lambda_E)}{(w+\lambda_E)(\mu_1+\lambda_1)} \right\} + w\mu_1\lambda\lambda_E \times$$

$$\times \left\{ \frac{1-\bar{S}_{2\phi_1}(\lambda+w+\lambda_E+\mu_2)}{2(\lambda+w+\lambda_E+\mu_2)} \right\} \Bigg] \frac{1}{D(0)} \qquad \ldots(56)$$

where

$$D(0) = \lim_{s\to 0} D(s)$$

$$\lambda = \exp[-\{(-\log\lambda_1)^\theta + (-\log\lambda_2)^\theta\}^{1/\theta}]$$

Particular cases

(i) When repair follows exponential distribution.

In this case the results can be derived by putting

$$\bar{S}_{\phi_1}(s) = \frac{\phi_1(y)}{s+\phi_1(y)}, \bar{S}_{\phi_A}(s) = \frac{\phi_A(x)}{s+\phi_A(x)} \qquad \ldots(57)$$

in equations (52) and (53), we get

$$\bar{P}_{\text{up}}(s)=\left[1+\frac{\mu_1}{s+\lambda+\mu_2+\phi_1(y)}+\frac{\lambda}{s+\lambda_1+\mu_1}+\frac{\mu_1\lambda}{(s+\lambda_E)(s+\mu_1+\lambda_1)}\right.$$
$$\left.+\frac{\lambda\mu_1}{2s+\lambda+\lambda_E+\mu_2+\phi_1(y)}\right]\frac{1}{D_1(s)} \qquad \ldots(58)$$

$$\bar{P}_{\text{down}}(s)=\left[\frac{\mu_1\mu_2}{2s+\lambda+w+\mu_2+\phi_1(y)}+\frac{\lambda_1\lambda}{(s+w)(s+\mu_1+\lambda_1)}\right.$$
$$+\frac{\mu_1\lambda_E\lambda}{(2s+\lambda_E+w)(s+\lambda_1+\mu_1)}+\frac{\mu_1\lambda\lambda_E}{3s+\lambda+w+\lambda_E+\mu_2+\phi_1(y)}$$
$$+\frac{\mu_1 w\mu_2}{2(3s+\lambda+w+\mu_2+2\phi_1(y))}+\frac{w\lambda_1\lambda}{(2s+w+\phi_A(x))(s+\lambda_1+\mu_1)}$$
$$\left.+\frac{\mu_1\lambda_E w\lambda}{(3s+w+\lambda_E+\phi_1(y))(s+\mu_1+\lambda_1)}+\frac{w\mu_1\lambda\lambda_E}{2(4s+\lambda+w+\lambda_E+\mu_2+2\phi_1(y))}\right]\frac{1}{D_1(s)} \qquad \ldots(59)$$

where

$$D_1(s)=s+\mu_1+\lambda-\frac{\mu_1\phi_1(y)}{s+\mu_2+\lambda+\phi_1(y)}-\frac{w\mu_1\mu_2\phi_1(y)}{3s+w+\lambda+\mu_2+2\phi_1(y)}-\frac{w\lambda\lambda_1}{s+\mu_1+\lambda_1}\times$$
$$\times\frac{\phi_A(x)}{2s+w+\phi_A(x)}-\frac{w\lambda_E\mu_1\lambda}{s+\mu_1+\lambda_1}\frac{\phi_1(y)}{3s+w+\lambda_E+\phi_1(y)}-\frac{w\lambda\lambda_E\mu_1\phi_1(y)}{4s+\lambda+\mu_2+w+\lambda_E+2\phi_1(y)} \qquad \ldots(60)$$

$$\lambda=\exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}] \qquad \ldots(61)$$

(ii) When waiting time is zero.

In this case up and down state probabilities of the system can be obtain by putting w = 0 in equations, we get

$$\bar{P}_{\text{up}}(s)=\left[1+\mu_1\left\{\frac{1-\bar{S}_{\phi_1}(s+\lambda+\mu_2)}{s+\lambda+\mu_2}\right\}+\frac{\lambda}{s+\lambda_1+\mu_1}+\frac{\mu_1\lambda}{(s+\lambda_E)(s+\mu_1+\lambda_1)}\right.$$
$$\left.+\mu_1\lambda\left\{\frac{1-\bar{S}_{\phi_1}(2s+\lambda+\lambda_E+\mu_2)}{2s+\lambda+\lambda_E+\mu_2}\right\}\right]\frac{1}{D_2(s)} \qquad \ldots(62)$$

$$\bar{P}_{\text{down}}(s)=\left[\mu_1\mu_2\left\{\frac{1-\bar{S}_{\phi_1}(2s+\lambda+\mu_2)}{2s+\lambda+\mu_2}\right\}+\frac{\lambda_1\lambda}{s(s+\mu_1+\lambda_1)}\right.$$
$$\left.+\frac{\mu_1\lambda_E\lambda}{(2s+\lambda_E)(s+\lambda_1+\mu_1)}+\mu_1\lambda\lambda_E\left\{\frac{1-\bar{S}_{\phi_1}(3s+\lambda+\lambda_E+\mu_2)}{3s+\lambda+\lambda_E+\mu_2}\right\}\right]\frac{1}{D_2(s)} \qquad \ldots(63)$$

where

$$D_2(s)=s+\mu_1+\lambda-\mu_1\bar{S}_{\phi_1}(s+\mu_2+\lambda) \qquad \ldots(64)$$

$$\lambda=\exp[-\{(-\log\lambda_1)^\theta+(-\log\lambda_2)^\theta\}^{1/\theta}] \qquad \ldots(65)$$

(iii) When the standby unit in subsystem A is in cold standby.

This can be derived by putting $\lambda_2 = 0$ or $\lambda = \lambda_1 = \lambda_A$ in equations (40-49). The Laplace transformations of various transition state probabilities are as follows:

$$\bar{P}_0(s)=1/D_3(s) \qquad \ldots(66)$$

$$\overline{P}_1(s) = \mu_1 \overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(s+\lambda_A)}{s+\lambda_A+\mu_2}\right] \quad \text{...(67)}$$

$$\overline{P}_2(s) = \mu_1\mu_2 \overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(2s+\lambda_A+w+\mu_2)}{2s+\lambda_A+w+\mu_2}\right] \quad \text{...(68)}$$

$$\overline{P}_3(s) = \lambda_A \overline{P}_0(s)\left[\frac{1}{s+\lambda_A+\mu_1}\right] \quad \text{...(69)}$$

$$\overline{P}_4(s) = \lambda_A \overline{P}_3(s)\left[\frac{1}{s+w}\right] \quad \text{...(70)}$$

$$\overline{P}_5(s) = \mu_1 \overline{P}_3(s)\left[\frac{1}{s+\lambda_E}\right] + \mu_1\lambda_A \overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(2s+\lambda_A+\lambda_E+\mu_2)}{2s+\lambda_A+\lambda_E+\mu_2}\right] \quad \text{...(71)}$$

$$\overline{P}_6(s) = \mu_1\lambda_E \overline{P}_3(s)\left[\frac{1}{2s+\lambda_E+w}\right] + \mu_1\lambda_A\lambda_E \overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(3s+\lambda_A+w+\lambda_E+\mu_2)}{3s+\lambda_A+w+\lambda_E+\mu_2}\right] \quad \text{...(72)}$$

$$\overline{P}_7(s) = \mu_1 w\mu_2 \overline{P}_0(s)\left[\frac{1-\overline{S}_{2\phi_1}(3s+\lambda_A+w+\mu_2)}{2(3s+\lambda_A+w+\mu_2)}\right] \quad \text{...(73)}$$

$$\overline{P}_8(s) = w\lambda_A \overline{P}_3(s)\left[\frac{1-\overline{S}_{\phi_A}(2s+w)}{2s+w}\right] \quad \text{...(74)}$$

$$\overline{P}_9(s) = \mu_1\lambda_E w\overline{P}_3(s)\left[\frac{1-\overline{S}_{\phi_1}(3s+w+\lambda_E)}{3s+w+\lambda_E}\right] + w\mu_1\lambda_A\lambda_E \overline{P}_0(s)\times\left[\frac{1-\overline{S}_{2\phi_1}(4s+\lambda_A+w+\lambda_E+\mu_2)}{2(4s+\lambda_A+w+\lambda_E+\mu_2)}\right] \quad \text{...(75)}$$

where

$$\begin{aligned} D_3(s) = {} & s+\mu_1+\lambda_A-\mu_1\overline{S}_{\phi_1}(s+\mu_2+\lambda_A)-\frac{w}{2}\mu_1\mu_2\overline{S}_{2\phi_1}(3s+w+\lambda_A+\mu_2) \\ & -\frac{w\lambda_A^2}{s+\mu_1+\lambda_A}\times\overline{S}_{\phi_A}(2s+w)-\frac{w\lambda_E\mu_1\lambda_A}{s+\mu_1+\lambda_A}\overline{S}_{\phi_1}(3s+w+\lambda_E) \\ & -\frac{w}{2}\lambda_A\lambda_E\mu_1\overline{S}_{2\phi_1}(4s+\lambda_A+\mu_2+w+\lambda_E) \end{aligned} \quad \text{...(76)}$$

Also up and down state probabilities of the system are given by

$$\begin{aligned} \overline{P}_{\text{up}}(s) = {} & \left[1+\mu_1\left\{\frac{1-\overline{S}_{\phi_1}(s+\lambda_A+\mu_2)}{s+\lambda_A+\mu_2}\right\}+\frac{\lambda_A}{s+\lambda_A+\mu_1}+\frac{\mu_1\lambda_A}{(s+\lambda_E)(s+\mu_1+\lambda_A)}\right. \\ & \left.+\mu_1\lambda_A\left\{\frac{1-\overline{S}_{\phi_1}(2s+\lambda_A+\lambda_E+\mu_2)}{2s+\lambda_A+\lambda_E+\mu_2}\right\}\right]\overline{P}_0(s) \end{aligned} \quad \text{...(77)}$$

$$\begin{aligned} \overline{P}_{\text{down}}(s) = {} & \left[\mu_1\mu_2\left\{\frac{1-\overline{S}_{\phi_1}(2s+\lambda_A+w+\mu_2)}{2s+\lambda_A+w+\mu_2}\right\}+\frac{\lambda_A{}^2}{(s+w)(s+\mu_1+\lambda_A)}\right. \\ & +\frac{\mu_1\lambda_E\lambda_A}{(2s+\lambda_E+w)(s+\lambda_A+\mu_1)}+\mu_1\lambda_A\lambda_E\left\{\frac{1-\overline{S}_{\phi_1}(3s+\lambda_A+w+\lambda_E+\mu_2)}{3s+\lambda_A+w+\lambda_E+\mu_2}\right\} \\ & +\mu_1 w\mu_2\left\{\frac{1-\overline{S}_{2\phi_1}(3s+\lambda_A+w+\mu_2)}{2(3s+\lambda_A+w+\mu_2)}\right\}+w\lambda_A{}^2\left\{\frac{1-\overline{S}_{\phi_A}(2s+w)}{(2s+w)(s+\lambda_A+\mu_1)}\right\} \end{aligned}$$

$$+\mu_1\lambda_E w\lambda_A\left\{\frac{1-\overline{S}_{\phi_1}(3s+w+\lambda_E)}{(3s+w+\lambda_E)(s+\mu_1+\lambda_A)}\right\}+w\mu_1\lambda_A\lambda_E\times$$

$$\times\left\{\frac{1-\overline{S}_{2\phi_1}(4s+\lambda_A+w+\lambda_E+\mu_2)}{2(4s+\lambda_A+w+\lambda_E+\mu_2)}\right\}\Bigg]\overline{P}_0(s) \qquad \ldots(78)$$

(iv) When standby unit in subsystem A is in hot standby with the operating unit.

In this case by putting $\lambda_2=\lambda_1=\lambda_A$ and $\lambda=2\lambda_A$ in equations (40-49), one can obtain following transition state probabilities.

$$\overline{P}_0(s)=1/D_4(s) \qquad \ldots(79)$$

$$\overline{P}_1(s)=\mu_1\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(s+2\lambda_A+\mu_2)}{s+2\lambda_A+\mu_2}\right] \qquad \ldots(80)$$

$$\overline{P}_2(s)=\mu_1\mu_2\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(2s+2\lambda_A+w+\mu_2)}{2s+2\lambda_A+w+\mu_2}\right] \qquad \ldots(81)$$

$$\overline{P}_3(s)=2\lambda_A\overline{P}_0(s)\left[\frac{1}{s+\lambda_A+\mu_1}\right] \qquad \ldots(82)$$

$$\overline{P}_4(s)=\lambda_A\overline{P}_3(s)\left[\frac{1}{s+w}\right] \qquad \ldots(83)$$

$$\overline{P}_5(s)=\mu_1\overline{P}_3(s)\left[\frac{1}{s+\lambda_E}\right]+2\mu_1\lambda_A\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(2s+2\lambda_A+\lambda_E+\mu_2)}{2s+2\lambda_A+\lambda_E+\mu_2}\right] \qquad \ldots(84)$$

$$\overline{P}_6(s)=\mu_1\lambda_E\overline{P}_3(s)\left[\frac{1}{2s+\lambda_E+w}\right]+\mu_1 2\lambda_A\lambda_E\overline{P}_0(s)\left[\frac{1-\overline{S}_{\phi_1}(3s+2\lambda_A+w+\lambda_E+\mu_2)}{3s+2\lambda_A+w+\lambda_E+\mu_2}\right] \qquad \ldots(85)$$

$$\overline{P}_7(s)=\mu_1 w\mu_2\overline{P}_0(s)\left[\frac{1-\overline{S}_{2\phi_1}(3s+2\lambda_A+w+\mu_2)}{2(3s+2\lambda_A+w+\mu_2)}\right] \qquad \ldots(86)$$

$$\overline{P}_8(s)=w\lambda_A\overline{P}_3(s)\left[\frac{1-\overline{S}_{\phi_A}(2s+w)}{2s+w}\right] \qquad \ldots(87)$$

$$\overline{P}_9(s)=\mu_1\lambda_E w\overline{P}_3(s)\left[\frac{1-\overline{S}_{\phi_1}(3s+w+\lambda_E)}{3s+w+\lambda_E}\right]+w\mu_1 2\lambda_A\lambda_E\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{2\phi_1}(4s+2\lambda_A+w+\lambda_E+\mu_2)}{2(4s+2\lambda_A+w+\lambda_E+\mu_2)}\right] \qquad \ldots(88)$$

where

$$D_4(s)=s+\mu_1+2\lambda_A-\mu_1\overline{S}_{\phi_1}(s+\mu_2+2\lambda_A)-\frac{w}{2}\mu_1\mu_2\overline{S}_{2\phi_1}(3s+w+2\lambda_A+\mu_2)$$

$$-\frac{2w\lambda_A^2}{s+\mu_1+\lambda_A}\times\overline{S}_{\phi_A}(2s+w)-\frac{2w\lambda_E\mu_1\lambda_A}{s+\mu_1+\lambda_A}\overline{S}_{\phi_1}(3s+w+\lambda_E)$$

$$-w\lambda_A\lambda_E\mu_1\overline{S}_{2\phi_1}(4s+2\lambda_A+\mu_2+w+\lambda_E) \qquad \ldots(89)$$

Also the up and down state probabilities of the system are given by

$$\overline{P}_{\mathrm{up}}(s)=\Bigg[1+\mu_1\left\{\frac{1-\overline{S}_{\phi_1}(s+2\lambda_A+\mu_2)}{s+2\lambda_A+\mu_2}\right\}+\frac{2\lambda_A}{s+\lambda_A+\mu_1}+\frac{2\mu_1\lambda_A}{(s+\lambda_E)(s+\mu_1+\lambda_A)}$$

$$+2\mu_1\lambda_A\left\{\frac{1-\overline{S}_{\phi_1}(2s+2\lambda_A+\lambda_E+\mu_2)}{2s+2\lambda_A+\lambda_E+\mu_2}\right\}\Bigg]\overline{P}_0(s) \qquad \ldots(90)$$

$$\bar{P}_{\text{down}}(s)=\Bigg[\mu_1\mu_2\left\{\frac{1-\bar{S}_{\phi_1}(2s+2\lambda_A+w+\mu_2)}{2s+2\lambda_A+w+\mu_2}\right\}+\frac{2\lambda_A^{\ 2}}{(s+w)(s+\mu_1+\lambda_A)}$$

$$+\frac{2\mu_1\lambda_E\lambda_A}{(2s+\lambda_E+w)(s+\lambda_A+\mu_1)}+2\mu_1\lambda_A\lambda_E\left\{\frac{1-\bar{S}_{\phi_1}(3s+2\lambda_A+w+\lambda_E+\mu_2)}{3s+2\lambda_A+w+\lambda_E+\mu_2}\right\}$$

$$+\mu_1 w\mu_2\left\{\frac{1-\bar{S}_{2\phi_1}(3s+2\lambda_A+w+\mu_2)}{2(3s+2\lambda_A+w+\mu_2)}\right\}+2w\lambda_A^{\ 2}\left\{\frac{1-\bar{S}_{\phi_A}(2s+w)}{(2s+w)(s+\lambda_A+\mu_1)}\right\}$$

$$+2\mu_1\lambda_E w\lambda_A\left\{\frac{1-\bar{S}_{\phi_1}(3s+w+\lambda_E)}{(3s+w+\lambda_E)(s+\mu_1+\lambda_A)}\right\}+2w\mu_1\lambda_A\lambda_E\times$$

$$\times\left\{\frac{1-\bar{S}_{2\phi_1}(4s+2\lambda_A+w+\lambda_E+\mu_2)}{2(4s+2\lambda_A+w+\lambda_E+\mu_2)}\right\}\Bigg]\bar{P}_0(s) \qquad \ldots(91)$$

Numerical computation

(1) Availability analysis

Let us take $\lambda_1 = 0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$, $\Phi_1 = \Phi_2 = \Phi_A = 1$, $\theta = 1$ and $x = y = z = 1$. Also let the repair follows exponential distribution i. e. equation (57) holds, then putting all these values in equation (52), taking inverse Laplace transformation, we get

$$P_{up}(t)= -0.1301054620\, e^{(-0.5000000000\, t)} - 0.003113014003\, e^{(-0.9100000000\, t)} -0.05846608969\, e^{(-1.577110005\, t)} -0.1942010441\, e^{(-0.8601482323\, t)}\cos(0.007394121216\, t) -0.2512567850\, e^{(-0.8601482323\, t)}\sin(0.007394121216\, t) + 0.4613082557\, e^{(-0.7862665523\, t)} + 0.1663598755\, e^{(-0.7285677930\, t)}+0.002024434936\, e^{(-0.6042513821\, t)}+1.17130413\, e^{(-0.1268411365\, t)} \qquad \ldots(92)$$

Now setting t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, in above equation (92), one can obtain Table 2 and correspondingly Fig. 2 which represents the variation of availability with respect to time.

(2) Reliability Analysis

Let the failure rates be $\lambda_1 = 0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, emergency failure rate be $\lambda_E = 0.5$, waiting rate $w = 0.3$, repair rates be $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\theta = 1$ and $x = y = z = 1$. Also let the repair follows exponential distribution. Now by putting all these values in equations (52), (77) and (90) taking inverse Laplace transform, using (57) and varying time from t = 0 to t = 10, one can obtain Table 3 and Fig. 3 which demonstrate the manner in which reliability varies as time passes when the unit of subsystem A is in warm standby, cold standby or in hot standby.

(3) M.T.T.F. Analysis

We know that M.T.T.F. = $\lim_{s\to 0} \bar{P}_{\text{up}}(s)$

Also suppose that repair follows exponential distribution then using equation (57) and

(a) Setting $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$, $x = y = z =1$, $\theta = 1$ and varying λ_1 as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, one can obtain Table 4 which demonstrates variation of M.T.T.F. with respect to λ_1.

(b) Let us take $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\lambda_1 = 0.6$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$, $x = y = z =1$, $\theta = 1$ then by varying λ_2 as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 Table 5 can be obtained which shows how M.T.T.F. varies as the value of λ_2 increases.

(c) Fixing $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\lambda_1=0.6$, $\lambda_2 = 0.2$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$, $x = y = z =1$, $\theta = 1$ and varying μ_1 as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, one can obtain Table 6 which shows variation of M.T.T.F. with respect to μ_1.

(d) Putting $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\lambda_1=0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\lambda_E = 0.5$, $w = 0.3$, $x = y = z =1$, $\theta = 1$ and varying μ_2 from 0.10 to 0.90 one can obtain Table 7 which represents the manner in which M.T.T.F. varies with respect to μ_2.

Variations of M.T.T.F with respect to λ_1, λ_2, μ_1 and μ_2 in the cases (a), (b), (c) and (d) have been shown by the Figs. 4, 5, 6 and 7 respectively.

(4) Cost Analysis

Letting $\lambda_1 = 0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, w = 0.3, $\Phi_1 = \Phi_2 = \Phi_A = 1$, $\theta = 1$ and x = y = z = 1. Furthermore, if the repair follows exponential distribution then using equations (57), we can obtain equation (92). If the service facility is always available, then expected profit during the interval (0, t] is given by

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

where K_1 and K_2 are the revenue and service cost per unit time respectively, then

$$\begin{aligned} E_P(t) = K_1[&0.2602109240\, e^{(-0.5000000000\, t)} + 0.003420894509\, e^{(-0.9100000000\, t)} + 0.3707166241\, e^{(-1.577110005\, t)} \\ &+ 0.2282704474\, e^{(-0.8601482323\, t)} \cos(0.007394121216\, t) + 0.2901464146\, e^{(-0.8601482323\, t)} \sin(0.007394121216\, t) \\ &- 0.05867072106\, e^{(-0.7862665523\, t)} - 0.2283382234\, e^{(-0.7285677930\, t)} - 0.003350319082\, e^{(-0.6042513821\, t)} - 9.234941008\, e^{(-0.1268411365\, t)} \\ &+ 8.996326343] - K_2 t \end{aligned} \qquad \text{.....(93)}$$

Keeping $K_1 = 1$ and varying K_2 at 0.1, 0.2, 0.3, 0.4, 0.5 in equation (93), one can obtain Table 8 which is depicted by Fig. 8.

(5) Sensitivity Analysis

We have performed sensitivity analysis of system reliability along with the change in specific values of system parameters. For this it is assumed that $\Phi_1 = \Phi_2 = \Phi_A = 0$, $\theta = 1$ and x = y = z = 1. Putting all these values in equation (52), we get

$$R(s) = \left[1 + \frac{\mu_1}{s+\lambda+\mu_2} + \frac{\lambda}{s+\lambda_1+\mu_1} + \frac{\mu_1\lambda}{(s+\lambda_E)(s+\mu_1+\lambda_1)} + \frac{\mu_1\lambda}{2s+\lambda+\lambda_E+\mu_2}\right]\frac{1}{s+\mu_1+\lambda} \qquad \text{...(94)}$$

Sensitivity of any reliability characteristic with respect to some specific parameter concludes how that particular reliability characteristic of the system changes with the change in the value of that specific parameter. In the present work we have done sensitivity analysis of reliability of the system with the change in the values of μ_1 and λ_E in two different cases as given below.

(a) Let us find $\partial R(s)/\partial \mu_1$ i.e. differentiate equation (94) with respect to μ_1, take its inverse Laplace Transform, put $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\mu_2 = 0.2$, $\lambda_E = 0.2$ and then varying the time as t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 for μ_1 = 0.1, 0.3 and 0.5, we get Table 9 and Fig. 9.

(b) Calculating $\partial R(s)/\partial \lambda_E$ by using equation (94), taking its inverse Laplace Transform and putting $\lambda_1 = 0.2$, $\lambda_2 = 0.2$, $\mu_2 = 0.2$, $\mu_1 = 0.2$. Now varying the time as t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 for λ_E = 0.1, 0.3 and 0.5, one can get Table 10 and Fig. 10.

Table 2: Time vs. Availability

Time	P_{up}
0	1.0000000000
1	0.9590418057
2	0.8715461826
3	0.7791496106
4	0.6921953409
5	0.6131108306
6	0.5421330976
7	0.4788411880
8	0.4226140077
9	0.3727850563
10	0.3287014135

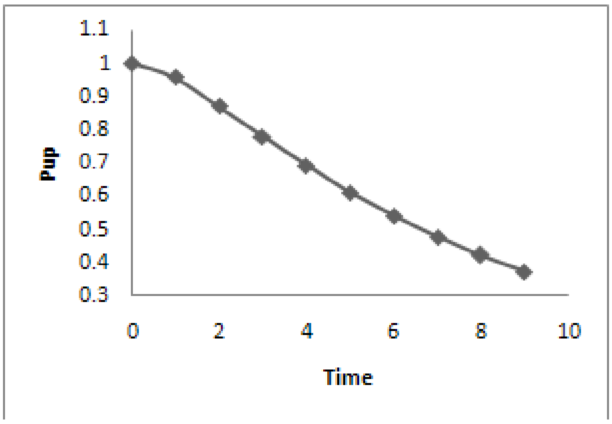


Fig. 2: Time vs. Availability

Table 3: Time vs. Reliability for warm, cold and hot standby

Time	P_{up}		
	When second unit of subsystem A is in		
	Warm standby	Cold standby	Hot standby
0	1.0000000000	1.00000000000	1.00000000000
1	0.9487251480	0.8623605488	0.7838561437
2	0.8226149560	0.5911884120	0.4595508769
3	0.6785373788	0.3704252246	0.2569312398
4	0.5424140387	0.2224483544	0.1446395654
5	0.4244550259	0.1308681494	0.0829366105
6	0.3271103372	0.07631141945	0.0483763152
7	0.2492350393	0.04439843895	0.0285830145
8	0.1882474642	0.02586978019	0.0170392734
9	0.1412130185	0.01512608618	0.0102182145
10	0.1053535797	0.00888284790	0.0061517085

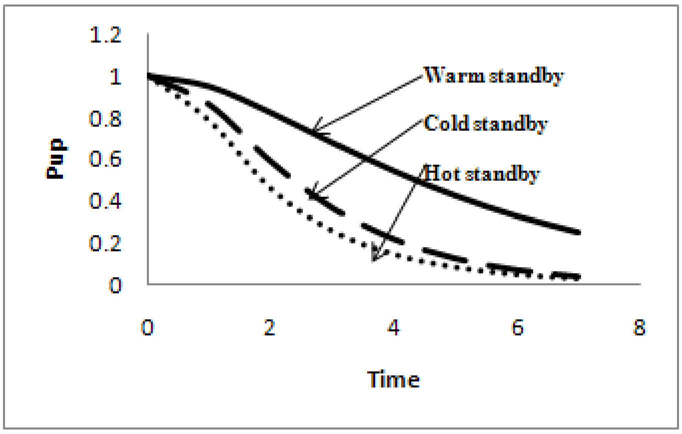


Fig. 3: Time vs. Reliability for warm, cold and hot standby

Table 4: λ_1 vs. M.T.T.F

λ_1	MTTF
.10	7.662405306
.20	7.041812399
.30	6.493139899
.40	6.013302485
.50	5.593750000
.60	5.225561363
.70	4.900802139
.80	4.612770782
.90	4.355911085

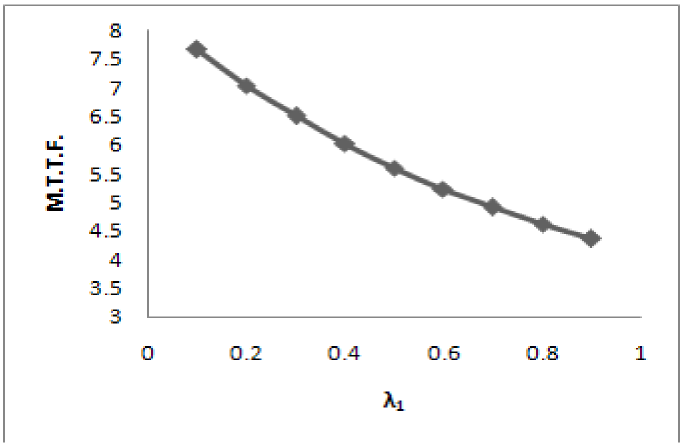


Fig. 4: λ_1 vs. M.T.T.F.

Table 5: λ_2 vs. M.T.T.F

λ_2	MTTF
.10	6.344991754
.20	5.225561363
.30	4.522577751
.40	4.046445543
.50	3.705555556
.60	3.450910725
.70	3.254218937
.80	3.098132952
.90	2.971488770

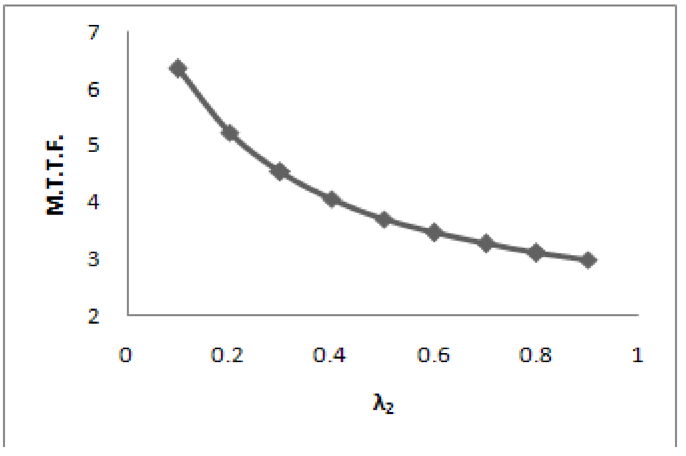


Fig. 5: λ_2 vs. MTTF

Table 6: μ_1 vs. M.T.T.F.

μ_1	MTTF
.10	6.967492872
.20	5.825838415
.30	5.225561363
.40	4.854878049
.50	4.602987984
.60	4.420562330
.70	4.282289616
.80	4.173837297
.90	4.086477760

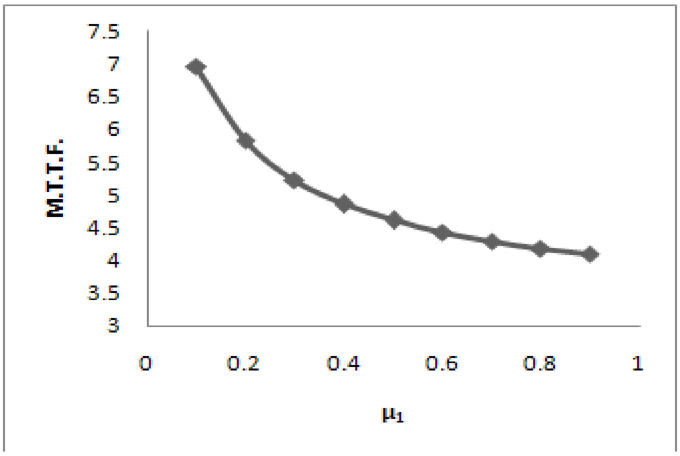


Fig. 6: μ_1 vs. M.T.T.F

Table 7: μ_2 vs. M.T.T.F

μ_2	MTTF
.10	6.254689756
.20	5.225561363
.30	4.682736863
.40	4.346548876
.50	4.117493234
.60	3.951209992
.70	3.824904095
.80	3.725648577
.90	3.645559977

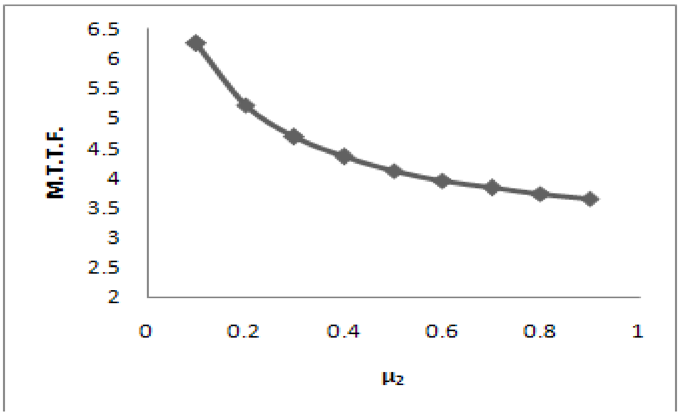


Fig. 7: μ_2 vs. M.T.T.F

Table 8: Time vs. expected profit

Time	$E_P(t)$				
	K_2=0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5
0	0	0	0	0	0
1	0.88710374	0.78710374	0.68710374	0.58710374	0.48710374
2	1.70371498	1.50371498	1.30371498	1.10371498	0.90371498
3	2.42883645	2.12883645	1.82883645	1.52883645	1.22883645
4	3.06389938	2.66389938	2.26389938	1.86389938	1.46389938
5	3.61587061	3.11587061	2.61587061	2.11587061	1.61587061
6	4.09282952	3.49282952	2.89282952	2.29282952	1.69282952
7	4.50270065	3.80270065	3.10270065	2.40270065	1.70270065
8	4.85286721	4.05286721	3.25286721	2.45286721	1.65286721
9	5.15006122	4.25006122	3.35006122	2.45006122	1.55006122
10	5.40035199	4.40035199	3.40035199	2.40035199	1.40035199

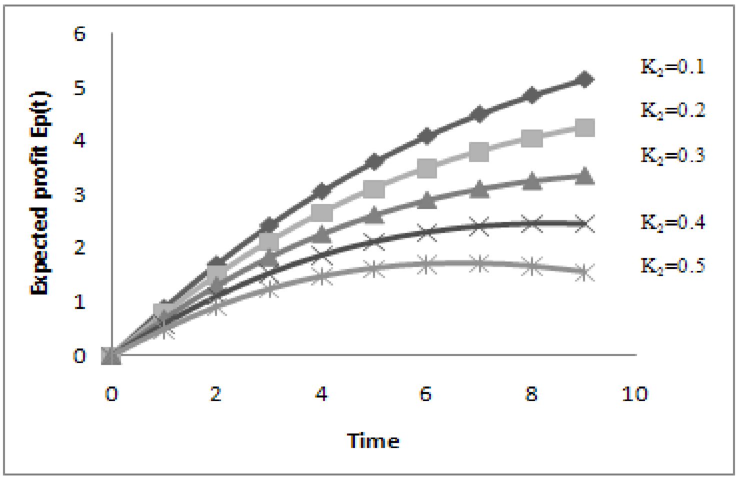


Fig. 8: Time vs. expected profit

Table 9: Sensitivity analysis of the system reliability with respect to μ_1

Time	Value of $\partial R(s)/\partial \mu_1$		
	$\mu_1 = 0.1$	$\mu_1 = 0.3$	$\mu_1 = 0.5$
0	0	0	0
10	-2.207363154	-0.594832647	-0.203725633
20	-1.722542140	-0.128186963	-0.024671693
30	-0.779167373	-0.016897352	-0.002562110
40	-0.286178687	-0.001933326	-0.000271869
50	-0.094478242	-0.000213954	-0.000029950
60	-0.029274170	-0.000023925	-0.000003429
70	-0.008702018	-0.000002751	-0.000000407
80	-0.002512753	-0.000000326	-0.000000049
90	-0.000710245	-0.000000040	-0.000000006

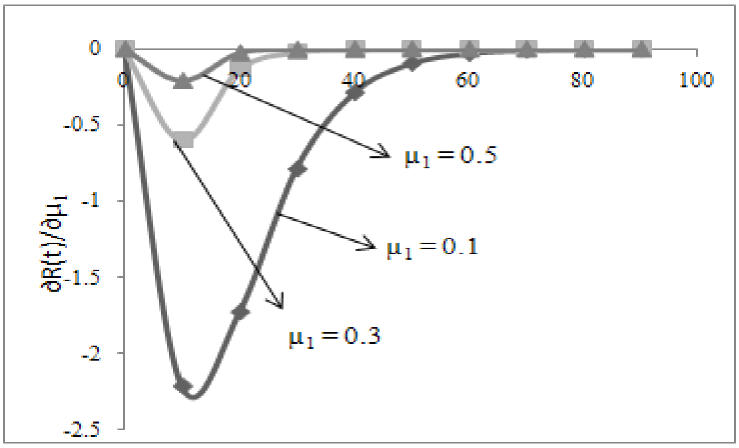
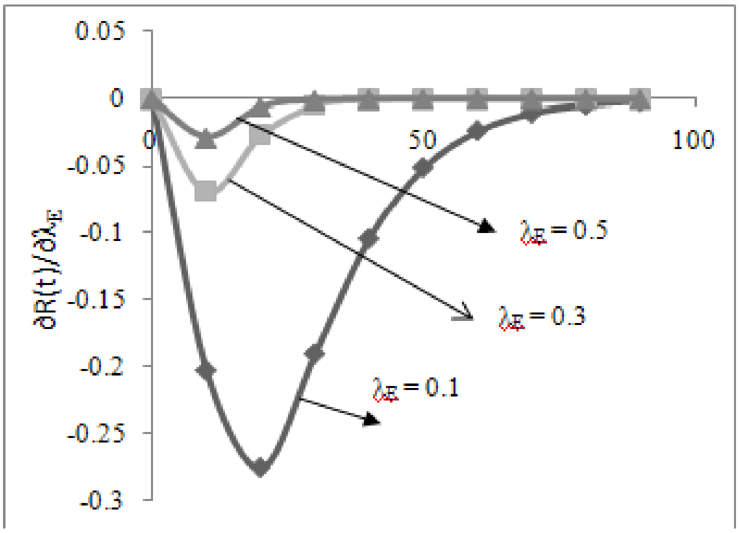


Fig. 9: Sensitivity analysis of the system reliability with respect to μ_1

Table 10: Sensitivity analysis of the system reliability with respect to λ_E

Time	Value of $\partial R(s)/\partial \lambda_E$		
	$\lambda_E = 0.1$	$\lambda_E = 0.3$	$\lambda_E = 0.5$
0	0	0	0
10	-0.202538570	-0.069460274	-0.0289654462
20	-0.275121845	-0.026720690	-0.0055961010
30	-0.190092374	-0.004682627	-0.0006652043
40	-0.104018154	-0.000608585	-0.0000573077
50	-0.050951388	-0.000068250	-0.0000052534
60	-0.023432531	-0.000007052	-0.0000004778
70	-0.103497566	-0.000000694	-0.0000000433
80	-0.004444750	-0.000000066	-0.0000000039
90	-0.001869828	-0.000000006	-0.0000000003

Fig. 10: Sensitivity analysis of the system reliability with respect to λ_E



Conclusions

The following conclusions may be drawn on the basis of study conducted in the present paper.

(1) When $\lambda_1 = 0.6$, $\lambda_2 = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\lambda_E = 0.5$, $w = 0.3$ the availability of the system decreases as the time increases. This variation of availability with respect to time is depicted in Fig. 2.

(2) Table 3 is corresponding to the reliabilities obtained for the cases when the second unit of subsystem A is in warm standby, cold standby and in hot standby. One can easily conclude by observing Fig. 3 that reliability in each case decreases as the time increases but the system has highest reliability in case when second unit of subsystem A is in warm standby.

(2) By critically examine Figs. 4, 5, 6 and 7 one can see that the M.T.T.F. of the system decreases with the increment in the values of λ_1, λ_2, μ_1 and μ_2. M.T.T.F. is found to be highest with respect to of λ_1. Also in case of μ_1 and μ_2 the decrement is more rapid in comparison to the cases of λ_1 and λ_2. The value of M.T.T.F. varies from 7.662-4.355, 6.344-2.971, 6.967-4.086 and from 6.254-3.645 with respect to λ_1, λ_2, μ_1 and μ_2 respectively for considered parameters.

(3) Keeping revenue cost per unit time at 1 and varying service cost from 0.1 to 0.5, one can obtain Fig. 8. It is very clear from Fig. 8 that increasing service cost implies decrement in profit. Here highest and lowest values of expected profit are obtained to be 5.40 and 0.4871 respectively for considered values.

(4) Tables 9 and 10 are corresponding to the sensitivity analysis of the system reliability with respect to change in μ_1 and λ_E respectively. This behaviour of sensitivity has been shown in Figs. 9 and 10. One can observe that sensitivity of the system reliability decreases with the increase in the value of μ_1 and λ_E. Also one can analyze that the system reliability is more sensitive in case of μ_1 than λ_E.

References

(1) El-Damcese, M. A., Analysis of warm standby systems subject to common-cause failures with time varying failure and repair rates, *Applied Mathematical Sciences.* 3 (18) (2009) 853-860.

(2) Goel, L. R. and Gupta, P., Stochastic analysis of a two unit parallel system with partial and catastrophic failures and preventive maintenance, *Microelectron. Reliab*., 24 (3) (1984) 881-883.

(3) Goel, L. R. and Gupta, R. (1985), "Profit analysis of a cold standby system with two repair distributions", *Microelectron. Reliab*., Vol. 25, No. 3, pp. 467-472.

(4) Kumar, A. and Agarwal, M., A review of standby systems. *IEEE Transactions on Reliability*, 29 (1980) 290-294.

(5) Nelson R. B. (2006), *An Introduction to Copulas*, 2nd edn. Newyork, Springer.

(6) Ram, Mangey and Singh, S. B. (2008), Availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family of copula in repair, *International Journal of Reliability, Quality & Safety Engineering*, Vol. 15 (4), pp. 341-365.

(7) Sen, P. K., Copulas: concepts and novel applications, *International journal of statistics*, LXI (3), 323-353.

(8) Srinivasan, S. K. and Subramanian, R., Reliability analysis of a three unit warm standby redundant system with repair, *Ann Oper Res*, 143 (2006), 227-235.

(9) Wang, K. H., Ke J. B. and Lee W. C., Reliability and sensitivity analysis of a repairable system with warm standbys and R unreliable service stations, *Int J Adv Manuf Technol*, 31(2007), 1223-1232.