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# Remarks on soft $\beta$ locally closed sets

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# ABSTRACT

The aim of this paper is to introduce a class of sets called soft  $\beta$ - locally closed sets. Further soft  $\beta$  - LC continuous map and its irresoluteness are defined in soft topological space along with their characterization.

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#### Keywords

Soft  $\beta$ - locally closed set, Soft  $\beta$  - LC continuous map, Soft  $\beta$  - LC irresolute map.

#### 1. Introduction

In 1999, D.Molodtsov [12] initiated soft set theory which was very convenient and easily applicable to the real life problems that are complicated due to some uncertainty. He gave a detailed analysis of inherent difficulties for dealing with uncertainty and incompleteness of information. In 2003, Maji et al [10] studied the soft set theory and developed several basic notions for this theory. Following which there was a rapid growth in applying soft set theory for solving decision making problems. Thus the applications of soft sets are enormously increasing day by day.

In 2011, Muhammad Shabir and Munazza Naz [13] gave a new dimension to the soft set by developing the soft topology. Many researchers started [4,7,10] to contribute to the development of this new innovation by redesigning the topological settings of set theory to soft set theory.

The notion of  $\beta$ - sets was introduced by Njastad[14] and several researchers have studied  $\beta$ - open sets and their related properties. The idea of locally closed set in a topological space was created by Kuratowski and Sierpienski [9]. Balachandran et al [6] defined  $\beta$ - locally closed sets and studied their properties. The concepts relating soft  $\beta$ - sets was introduced by Arockiarani et al [4], following the pioneer work we have taken the  $\beta$ - locally closed sets to the soft topology and the classes of soft  $\beta$  - LC continuous maps and soft  $\beta$  - LC irresolute maps are discussed along with their properties.

#### 2. Preliminaries

**Definition : 2.1[12]** A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U. **Definition : 2.2[10]** A soft set (F,A) over U is said to be a NULL soft set denoted by  $\mathbf{\Phi}$ , if  $\forall e \in A$ ,  $F(e) = \emptyset$ .

**Definition: 2.3[10]** A soft set (F,A) over U is said to be absolute soft set denoted by  $\tilde{A}$ , if  $\forall e \in A$ , F(e) = U. Clearly  $\Phi^c = \tilde{A}$  and  $\check{A}^c = \Phi$ .

**Definition: 2.4 [10]** The union of two soft sets (F,A) and (G,B) over the common universe U is the soft set (H,C), where  $C = A \cup B$  and  $\forall e \in C$ ,

$$H(e) = \begin{cases} F(e)if \ e \in A - B \\ G(e)if \ e \in B - A \\ F(e) \cup G(e)if \ e \in A \cap B \end{cases}$$

And it is written as  $(F,A) \cup (G,B) = (H,C)$ .

**Definition: 2.5[10]** The intersection (H, C) of two soft sets (F,A) and (G,B) over the common universe U, denoted by (F,A)  $\cap$  (*G*, *B*) is defined as  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$ ,  $\forall e \in C$ .

**Definition: 2.6[10]** Let  $\tau$  be the collection of soft sets over X, then  $\tau$  is said to be a soft topology on X if

i)  $\Phi$  ,  $\widetilde{\pmb{X}}$  belong to  $\tau$ 

ii) the union of any number of soft sets in  $\tau$  belongs to  $\tau$ .

iii) the intersection of any two soft sets in  $\tau$  belongs to  $\tau$ . The triplet (X,  $\tau$ ,E) is called soft topological space over X. The members of  $\tau$  are said to be soft open sets in X.

**Definition: 2.7** A soft set (F,E) of a soft topological space(X,  $\tau$ ,E) is called

(i) soft pre-open set[4] if  $(F,E) \cong int(cl(F,E))$ .

(ii) soft  $\alpha$  -open set[4] if (F,E)  $\cong$  int(cl(int(F,E))).

(iii) soft  $\beta$  -open set[15] if (F,E)  $\cong$  cl(int(cl(F,E))).

**Definition: 2.8[15]** Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be two soft topological spaces. A function f:  $(X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be soft  $\beta$  - continuous if  $f^{-1}((G, E))$  is soft  $\beta$  - open in  $(X, \tau, E)$ , for every soft open set (G, E) of  $(Y, \tau', E)$ .

**Definition: 2.9** A subset S of a topological space(X,  $\tau$ ) is called

(i) pre-closed set[11] if  $cl(int(S)) \subseteq S$ .

(ii)  $\alpha$  -open set[14] if (S)  $\subseteq$  int(cl(int(S))).

(iii)  $\beta$  -open set[6] if (S)  $\subseteq$  cl(int(cl(S))).

(iv) Q-set [8] if cl(int(S)) = int(cl(S)).

(v)  $\beta$  - locally closed [6] if  $S = A \cap B$ , where A is  $\beta$  -open and B is  $\beta$  - closed in X.

**Definition: 2.10 [6]** If the collection of  $\beta$  - open subsets of (X,  $\tau$ ) is closed under finite intersection then the space is called a B space.

**Definition: 2.11 [6]** If  $\beta O(X) =$  then  $(X, \tau)$  is termed as a  $\beta$  space.

3. Soft  $\beta$ - locally closed sets

**Definition: 3.1** A soft set (F,E) of  $(X,\tau,E)$  is called soft  $\beta$ - locally closed (soft  $\beta$  -lc) if (F,E) = (G,E)  $\widetilde{\cap}$  (H,E) where (G,E) is soft  $\beta$ - open and (H,E) is soft  $\beta$ - closed.

**Definition: 3.2** A soft set (F,E) of (X, $\tau$ ,E) is called soft  $\alpha$  -locally closed (soft  $\alpha$  -lc) if (F,E) = (G,E)  $\widetilde{\cap}$  (H,E) where (G,E) is soft  $\alpha$  - open and (H,E) is soft  $\alpha$  - closed.

**Definition: 3.3** Let  $(X,_{\tau},E)$  be a soft topological space and (F,E) be a soft set in X. Then

 $\beta$ int(F,E) =  $\widetilde{U}$  { (K,E) / (K,E) is soft  $\beta$ -open in X and (K,E)  $\cong$  (F,E) }

 $\beta cl(F,E) = \widetilde{\cap} \{ (K,E) / (K,E) \text{ is soft } \beta \text{-closed in } X \text{ and } (F,E) \cong (K,E) \}$ 

**Proposition: 3.4** Every soft  $\beta$ - open (soft  $\beta$ - closed) set is soft  $\beta$ - locally closed.

**Remark: 3.5** Every soft  $\alpha$  -open is soft  $\beta$ -open,  $\alpha$  -LC(X)  $\Rightarrow \beta$ -LC(X) but soft  $\beta$ -locally closed need not be soft  $\alpha$  -locally closed.

Theorem: 3.6 For a soft set (F,E) of a soft topological space the following are equivalent

(i) (F,E) is soft  $\beta$ -locally closed.

(ii)  $(F,E) = (G,E) \widetilde{\cap} \beta cl(F,E)$  for some soft  $\beta$ -open set (G,E).

(iii)  $\beta cl(F,E)$ -(F,E) is soft  $\beta$ -closed.

(iv) (F,E)  $\widetilde{U}$  (X- $\beta$ cl(F,E)) is soft  $\beta$ -open.

(v) (F,E)  $\cong \beta int((F,E) \widetilde{U} (X-\beta cl(F,E))).$ 

#### **Proof:**

(i)  $\rightarrow$  (ii) Let (F,E) = (G,E)  $\cap$  (H,E) where (G,E) is soft  $\beta$ -open and (H,E) is soft  $\beta$ -closed. (F,E)  $\cong$  (H,E), hence  $\beta$ cl(F,E)  $\cong$  (H,E). Thus (F,E) = (G,E)  $\cap$  (H,E)  $\cong$  (G,E)  $\cap$   $\beta$ cl(F,E).

 $(F,E) \cong (G,E) \text{ and } (F,E) \cong \beta cl(F,E) \Rightarrow (F,E) \cong (G,E) \widetilde{\cap} \beta cl(F,E). \text{Hence } (F,E) = (G,E) \widetilde{\cap} \beta cl(F,E).$ 

(ii)  $\rightarrow$  (iii)  $\beta cl(F,E)-(F,E) = \beta cl(F,E) \cap (X-(G,E))$  which is soft  $\beta$ -closed.

(iii)  $\rightarrow$ (iv) (F, E)  $\widetilde{U}$  (X- $\beta$ cl(F,E)) = X - [ $\beta$ cl(F,E)-(F,E)]. Therefore (F,E)  $\widetilde{U}$  (X- $\beta$ cl(F,E)) is soft  $\beta$ -open.

(iv) →(v) Since (F,E)  $\widetilde{U}$  (X- $\beta$ cl(F,E)) is soft  $\beta$  open, (F,E)  $\cong \beta$ int((F,E)  $\widetilde{U}$  (X- $\beta$ cl(F,E))).

 $(v) \rightarrow (i) (F,E) \subseteq \beta int((F,E) \widetilde{U}(X-\beta cl(F,E))) \Rightarrow (F,E) = \beta int((F,E) \widetilde{U}(X-\beta cl(F,E))) \cap \beta cl(F,E).$  Hence (F,E) is soft  $\beta$ -locally closed.

#### Proposition: 3.7

If a soft Q set is soft  $\beta$ -open then it is soft  $\alpha$ -open.

#### **Proof:**

Consider soft 0 set (F,E)which is also soft then (F.E) ĩ cl(int(cl(F,E))) а β-open,  $= cl(cl(int(F,E))) = cl(int(F,E)) = cl(int(int(F,E))) = int(cl(int(F,E))), thus (F,E) is soft \alpha$ -open.

#### **Proposition: 3.8**

Each soft  $\beta$ - open subset which is soft pre-closed is soft semi- open.

**Proof:** Let (F, E) be soft  $\beta$  -open and soft pre-closed. Now  $pcl(F,E) \subseteq cl(int(pcl(F,E)))$ . Since (F,E) is soft pre-closed,  $(F,E) \subseteq cl(int(F,E))$ , thus (F,E) is soft semi-open.

#### **Proposition: 3.9**

If (F,E) is soft  $\beta$ -locally closed in (X, $\tau$ ,E), then there exists soft  $\beta$ -locally closed set (G,E) in X such that (F,E)  $\widetilde{\cap}$  (G,E) =  $\Phi$ .

**Proof:** Let (F,E) be soft  $\beta$ - locally closed, hence (F,E) = (I,E)  $\cap$  (J,E) where (I,E) is soft  $\beta$ - open and (J,E) is soft  $\beta$ - closed in X.

Assume  $(G,E) = [(I,E) - (J,E)] \cap [(J,E) - (I,E)]$ . Now consider,  $(F,E) \cap (G,E) = \{(I,E) \cap (J,E)\} \cap \{[(I,E) - (J,E)] \cap [(J,E) - (I,E)]\} = \{(I,E) \cap (J,E)\} \cap \{[(I,E) \cap (J,E)^c] \cap \{[(J,E) \cap (I,E)^c] = \mathbf{\Phi}.$ 

#### **Proposition: 3.10**

Let (I,E), (J,E)  $\cong$  X be such that (I,E) is soft  $\beta$ -open and (J,E) is soft  $\beta$ -closed in (X, $\tau$ ,E). Then there exists a soft  $\beta$ -open set (G, E) and a soft  $\beta$ -closed set (F,E), such that (I,E)  $\cap$  (J,E)  $\cong$  (F,E),

 $(G, E) \cong (I, E) \widetilde{U} (J, E).$ 

**Proof:** Let  $(F,E) = \beta cl (I,E) \widetilde{\cap} (J,E)$  and  $(G,E) = (I,E) \widetilde{\cup} \beta int(J,E), (I,E) \widetilde{\subseteq} \beta cl (I,E) \Rightarrow (I,E) \widetilde{\cap} (J,E) \widetilde{\subseteq} \beta cl (I,E) \widetilde{\cap} (J,E) \widetilde{\subseteq} (F,E).$ 

 $\beta int((J,E) \cong (J,E) \Rightarrow (G, E) \cong (I, E) \widetilde{\cup} \ \beta int (J,E) \Rightarrow (G, E) \cong (I, E) \widetilde{\cup} \ \beta int (J,E) \Rightarrow (G, E) \cong (I, E) \widetilde{\cup} \ (J,E) \ and \ (I,E) \widetilde{\cap} \ (J,E) \cong (I,E) \ (I$ 

# $(I, E) \Rightarrow (I,E) \widetilde{\cap} (J,E) \widetilde{\subseteq} (G, E)$

Also  $(F,E) \cong (J,E) \cong (I, E) \widetilde{\cup} ((J,E) .$  Thus  $(I,E) \cap (J,E) \cong (F,E), (G,E) \cong (I,E) \widetilde{\cup} (J,E).$ 

**Definition: 3.11** If the collection of all soft  $\beta$ -open subsets of  $(X, \tau, E)$  are closed under finite intersection then the space is called a SAB space.

**Proposition: 3.12** Let  $(X,\tau,E)$  be a SAB space and  $(F,E) \cong X$  be soft  $\beta$ -locally closed. Then (i)  $\beta$ int (F,E) is soft  $\beta$ -locally closed. (ii)  $\beta$ cl (F,E) is contained in a soft  $\beta$ -closed set. (iii) (F,E) is soft  $\beta$ -open if  $\beta$ cl (F,E) is soft  $\beta$ -open.

#### **Proof:**

(i) Let  $(F,E) = (I,E) \cap \beta cl (F,E)$  where (I,E) is soft  $\beta$ -open in X.

Now  $\beta int(F,E) = \beta int((I,E) \cap \beta cl (F,E)) = \beta int(I,E) \cap \beta int(\beta cl (F,E)) = \beta int(I,E) \cap \beta cl(\beta int(F,E)), [3] thus <math>\beta int(F,E)$  is soft  $\beta$ -locally closed.

(ii)  $\beta cl(F,E) = \beta cl[(I,E) \cap \beta cl(F,E)] \cong \beta cl(I,E) \cap \beta cl(F,E)$  which is a soft  $\beta$ -closed set.

(iii)  $\beta int(F,E) = \beta int((I,E) \cap \beta cl (F,E)) = \beta int(I,E) \cap \beta int(\beta cl (F,E)) = (I,E) \cap \beta cl (F,E) = (F,E)$ , since  $\beta cl (F,E)$  is soft  $\beta$  open. Thus (F,E) is soft  $\beta$ - open.

#### **Proposition: 3.13**

If  $(F,E) \cong (G,E) \cong X$  and (G,E) is soft  $\beta$ -locally closed then there exist a soft  $\beta$ - locally closed (I,E) such that  $(F,E) \cong (I,E) \cong (G,E)$ .

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# **Proof:**

As (G,E) is soft  $\beta$ -locally closed, (G,E) = (P,E)  $\widetilde{\cap} \beta cl$  (G,E) where (P,E) is soft  $\beta$ -open.

As  $(F,E) \cong (G,E)$ ,  $(F,E) \cong (P,E)$ , also  $(F,E) \cong \beta cl (F,E)$ .

Thus  $(F,E) \cong (P,E) \cap \beta cl (F,E)=(I,E)$  impling that (I,E) is soft  $\beta$ -locally closed and  $(F,E) \cong (I,E) = (P,E) \cap \beta cl (F,E) \cong (P,E) \cap \beta cl (G,E) \cong (G,E)$ . Thus  $(F,E) \cong (I,E) \cong (G,E)$ .

**Definition: 3.14** A subset (F,E) of  $(X,\tau,E)$  is called soft  $\beta$ -dense if  $\beta$ cl (F,E)=X.

**Definition: 3.15** Let (F,E),  $(G,E) \cong X$ . Then (F,E) and (G,E) are said to be soft  $\beta$ -separated if  $(F,E) \cap \beta cl (G,E) = \Phi$  and  $(G,E) \cap \beta cl (F,E) = \Phi$ .

**Proposition: 3.16** Suppose  $(X,\tau,E)$  is a SAB space and if  $(F,E),(G,E) \in \beta LC(X,\tau,E)$  which are also soft  $\beta$ -separated then  $(F,E) = \widetilde{U}(G,E) \in \beta LC(X,\tau,E)$ .

**Proof:** Since (F,E), (G,E)  $\epsilon \beta LC(X,\tau,E)$ , (F,E) = (I,E)  $\widetilde{\cap} \beta cl(F,E)$  and (G,E) = (J,E)  $\widetilde{\cap} \beta cl(G,E)$  where (I,E) and (J,E) are soft  $\beta$ -open in X.

Consider (U,E) =(I,E)  $\widetilde{\cap}$  (X-\betacl (G,E)) and (V,E) =(I,E)  $\widetilde{\cap}$  (X-\betacl (F,E)) , then

 $(U,E) ~\widetilde{\cap} ~ \beta cl~(F,E) ~=~ ((I,E) ~\widetilde{\cap} ~ (X-\beta cl~(G,E)) ~~) ~\widetilde{\cap} ~ \beta cl~(F,E) ~=~ (F,E) ~\widetilde{\cap} ~ (X-\beta cl~(G,E)) = (F,E)$ 

since (F,E)  $\cong$  (X- $\beta$ cl (G,E)).

 $\label{eq:similarly} \text{ (V,E) } \widetilde{\cap} \ \beta \text{cl} \ (G,E) \ = (G,E) \ \text{and} \ (U,E) \ \widetilde{\cap} \ \beta \text{cl} \ (G,E) \ = \Phi \ , \ (V,E) \ \widetilde{\cap} \ \beta \text{cl} \ (F,E) = \Phi.$ 

Since X is a SAB space (U,E) and (V,E) are soft  $\beta$ -open in X.

 $((U,E) \ \widetilde{U} \ (V,E) \ ) \ \widetilde{\cap} \ \beta cl \ ((F,E) \ \widetilde{U} \ (G,E)) = ((U,E) \ \widetilde{U} \ (V,E) \ ) \ \widetilde{\cap} \ (\beta cl \ (F,E) \ \widetilde{U} \ \beta cl \ (G,E))$ 

 $= [(U,E) \cap \beta cl (F,E)] \widetilde{U} [(U,E) \cap \beta cl (G,E)] \widetilde{U} [(V,E) \cap \beta cl (F,E)] \widetilde{U} [(V,E) \cap \beta cl (G,E)] = (F,E) \widetilde{U} (G,E).As ((U,E) \widetilde{U} (V,E)) is soft \beta- open. (F,E) \widetilde{U} (G,E) \epsilon \beta LC (X,\tau,E).$ 

# **Proposition: 3.17**

The intersection of a soft  $\beta$ -locally closed set and a soft  $\alpha$ -locally closed set is a soft  $\beta$ -locally closed set.

**Proof:** In view of the fact that intersection of a soft  $\alpha$ -open set and a soft  $\beta$ -open set is soft  $\beta$ -open set and intersection of a soft  $\alpha$ -closed set and a soft  $\beta$ -closed set is soft  $\beta$  - closed set the result is derivable.

# Example: 3.18

Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2, e_3\}$ . Let the soft sets (F,E) and (G,E) be defined as

 $(F,E) = \{(e_1,\{X\}), (e_2, \{h_2, h_3\}), (e_3,\{h_1,h_2\})\} ; (G,E) = \{(e_1, \pmb{\phi}), (e_2, \{h_1\}), (e_3,\{h_3\})\} \text{ and } (F,E) = \{(e_1,\{X\}), (e_3,\{h_2, h_3\}), (e_3,\{h_1,h_2\})\}$ 

 $\boldsymbol{\tau} = \{ \boldsymbol{\tilde{X}}, \boldsymbol{\Phi}, (F,E), (G,E) \}$  be a soft topology defined using the universal set X along with the parameter set E. Here (F,E), (G,E) are both soft  $\beta$ -open and soft  $\beta$ -closed. Hence (F,E), (G,E) are soft  $\beta$ -locally closed sets.

# Example: 3.19

Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2, e_3\}$ . Let F,G,H,I be the mappings from E to P(X) defined by,

 $(F,E) = \{(e_1,\{X\}), (e_2, \{h_2, h_3\}), (e_3,\{h_1,h_2\})\} \ ; \ (G,E) = \{(e_1, \pmb{\phi}), (e_2, \{h_1\}), (e_3,\{h_3\})\}$ 

 $(H,E) = \{(e_1, \{h_2\}), (e_2, \{h_1, h_3\}), (e_3, \{h_3\})\} ; (I,E) = \{(e_1, \{h_2\}), (e_2, \{h_3\}), (e_3, \boldsymbol{\phi})\}$ 

Let  $\boldsymbol{\tau} = \{ \boldsymbol{\tilde{X}}, \boldsymbol{\Phi}, (F,E), (G,E), (H,E), (I,E) \}$ ,  $\beta O(X,E) = \{ \boldsymbol{\tilde{X}}, \boldsymbol{\Phi}, (F,E), (G,E), (H,E), (I,E) \}$  here the soft  $\beta$ -open sets are closed under finite intersection, hence this space is a SAB space.

# 4. Soft $\beta$ -LC – continuous maps and soft $\beta$ - LC irresolute maps

The soft  $\beta$  -LC – Continuous Maps along with Soft  $\beta$  - LC Irresolute Maps are introduced in this section and a characterization study is carried out.

**Definition: 4.1** A map  $\varphi_{\psi}$ : (X,<sub> $\tau$ </sub>,E) $\rightarrow$ (Y, $\sigma$ ,K) is called soft  $\beta$ -LC – continuous if the inverse image of every soft open set in Y is soft  $\beta$ -locally closed in X.

**Definition: 4.2** A map  $\varphi_{\psi}$ : (X, $\tau$ ,E) $\rightarrow$ (Y, $\sigma$ ,K) is called soft  $\beta$  -LC – irresolute if the inverse image of every soft  $\beta$ -locally closed set

in Y is soft  $\beta$ -locally closed in X.

Remark: 4.3 The following results follow from the definition

(i) Every soft  $\beta$  -LC – irresolute function is soft  $\beta$  -LC – continuous.

(ii) Every soft  $\beta$ - continuous function is soft  $\beta$ -LC – continuous.

However the converses are not true in general.

**Definition: 4.4** If the set of all  $\beta$ - open sets of X are in  $\tau$ , then  $(X,\tau,E)$  is defined as a S $\beta$  space.

**Proposition: 4.5** A soft  $\beta$ -continuous function defined into a S $\beta$ - space is soft  $\beta$ -LC – irresolute.

**Proof:** Let  $\varphi_{\psi}$ :  $(X,\tau,E) \rightarrow (Y,\sigma,K)$  be soft  $\beta$ - continuous and Y be a S $\beta$ - space. Let (F,E) be a soft  $\beta$ -locally closed set in Y. Then  $(F,E) = (P,E) \cap (Q,E)$  where (P,E) is soft  $\beta$  - open and (Q,E) is soft  $\beta$  - closed set. Since  $\varphi_{\psi}$  is soft  $\beta$ - continuous,  $\varphi_{\psi}^{-1}$  (P,E) is soft  $\beta$  - open and  $\varphi_{\psi}^{-1}$  (Q,E) is soft  $\beta$  - closed in X. That is  $\varphi_{\psi}^{-1}$  (F,E) =  $\varphi_{\psi}^{-1}$  (P,E)  $\cap \varphi_{\psi}^{-1}$  (Q,E) is soft  $\beta$ -locally closed in X.

**Proposition:** 4.6 Let  $\varphi_{\psi}$ : (X, $\tau$ ,E) $\rightarrow$ (Y, $\sigma$ ,K) be soft  $\beta$ - LC continuous and (F,E) be soft  $\beta$  - closed in X. Then the restriction  $\varphi_{\psi} \setminus (F,E) : (F,E) \rightarrow Y$  is soft  $\beta$ - LC continuous.

**Proof:** Let (V,E) be soft open in Y, by the assumption  $\varphi_{\psi}^{-1}$  (V,E) is soft  $\beta$ -locally closed in X.

Hence  $\varphi_{ib}^{-1}$  (V,E) can be written as a intersection of a soft  $\beta$  - open and soft  $\beta$  - closed set in X.

Let us now consider  $[\boldsymbol{\varphi}_{\psi} \setminus (\boldsymbol{F}, \boldsymbol{E})]^{-1} = [(P,E) \cap (Q,E)] \cap (F,E) = [(P,E) \cap (F,E)] \cap [(Q,E) \cap (F,E)]$  where  $[(P,E) \cap (F,E)]$  is soft  $\beta$  - open and  $[(Q,E) \cap (F,E)]$  is soft  $\beta$  - closed in X. Thus  $[\boldsymbol{\varphi}_{\psi} \setminus (\boldsymbol{F}, \boldsymbol{E})]^{-1}$  is soft  $\beta$ -locally closed in X. Hence  $\boldsymbol{\varphi}_{\psi} \setminus (F,E)$  is soft  $\beta$ - LC continuous.

**Theorem 4.7** Let  $\varphi_{\psi}$ : (X, $\tau$ ,E) $\rightarrow$ (Y, $\sigma$ ,K) be soft  $\beta$ - continuous and soft open map then (i) the inverse image of every soft pre-open set in Y is soft  $\beta$ -open in X. (ii) the inverse image of every soft semi-open set in Y is soft  $\beta$ -open in X.

**Theorem: 4.8** Let  $\varphi_{\psi}$ :  $(X,\tau,E) \rightarrow (Y,\sigma,K)$  be soft  $\beta$ - continuous and soft open map then the inverse image of every soft  $\alpha$ -locally closed set in Y is  $\beta$ - locally closed set in X.

**Proof:** Let (F,E) be a soft  $\alpha$  -locally closed set in Y. Then (F,E) = (P,E)  $\cap$  (Q,E) where (P,E) is soft  $\alpha$  - open and (Q,E) is soft  $\alpha$  - closed set in Y. Then (P,E) is soft semi-open and (Q,E) is soft semi - closed set in Y. By theorem 4.7,  $\varphi_{\psi}^{-1}$  (P,E) is soft  $\beta$  - open and  $\varphi_{\psi}^{-1}$  (Q,E) is soft  $\beta$  - closed in X. And so  $\varphi_{\psi}^{-1}$  (F,E) =  $\varphi_{\psi}^{-1}$  (P,E)  $\cap \varphi_{\psi}^{-1}$  (Q,E) is soft  $\beta$ -locally closed in X.

**Theorem: 4.9** Let  $\varphi_{\psi}$ : (X, $\tau$ ,E) $\rightarrow$ (Y, $\sigma$ ,K) and  $\theta_{\eta}$ : (Y, $\sigma$ ,K) ) $\rightarrow$ (Z, $\mu$ ,R) be two functions, then

(i) If  $\theta_{\eta}$  is soft  $\beta$ -LC continuous and  $\varphi_{\psi}$  is soft  $\beta$ -LC continuous then  $\theta_{\eta} o \varphi_{\psi}$  is soft  $\beta$ -LC continuous.

(ii) If  $\theta_{\eta}$  is soft  $\beta$ -LC irresolute and  $\varphi_{\psi}$  is soft  $\beta$ -LC irresolute then  $\theta_{\eta} \sigma \varphi_{\psi}$  is soft  $\beta$ -LC irresolute.

(iii) If  $\theta_{\eta}$  is soft  $\beta$ -LC continuous and  $\varphi_{\psi}$  is soft  $\beta$ -LC irresolute then  $\theta_{\eta} o \varphi_{\psi}$  is soft  $\beta$ -LC continuous.

**Proof:** It is obvious from the definition.

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