# Fixed point theorems in intuitionistic fuzzy metric spaces by using occasionally weakly compatible maps <br> M.Jeyaraman and R.Balamurugan <br> Department of Mathematics, Raja Duraisingam Govt. Arts College, Sivagangai- 630 561, Tamil Nadu, India. 

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#### Abstract

The purpose of this paper is to obtain common fixed point theorem in intuitionistic fuzzy metric spaces while proving this result we utilize the idea of occasionally weakly compatible maps due to AI Thagafi and N.Shahzad [14]. In this paper we have generalized the result of Kamel Wadhwa and Huriom Dubey [8] and occasionally weakly compatible maps by using intuitionistic fuzzy metric spaces.


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## 1. Introduction

Park [9] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy sets introduced by Zadeh [16] while Atanassov [2] introduced the concept of intuitionistic fuzzy sets. Turkoglu et. al [15], introduced the concept of compatible maps on intuitionistic fuzzy metric spaces. Turkoglu et al. [15] gave generalization of Jungck's common fixed point theorem [7] to intuitionistic fuzzy metric spaces. However, the study of common fixed points of non-compatible maps is also very interesting and this condition has further been weakened by introducing the notion of weakly compatible mappings by Jungck and Rhoades [7]. Sadati and Park [11], Y.J.Cho et al. [4] studied the concept of intuitionistic fuzzy metric spaces and its applications. In 2008 AlThagafi and N. Shahzad [14] introduced the notion of occasionally weakly compatible mappings which is more general than the concept of weakly compatible maps.
In this paper, with the help of occasionally weakly compatible mappings, we prove common fixed point theorem in intuitionistic fuzzy metric space. We extend generalized and improved the corresponding results given by many authors earlier given in intuitionistic fuzzy metric spaces.

## 2. Preliminaries

Definition 1.1: [11] A binary operation $*:[0,1] \times[0,1] \rightarrow[0,1]$ is a continuous t-norm if it satisfies the following conditions:
$(1) *$ is associative and commutative,
(2) $*$ is continuous,
(3) $a^{*} 1=$ a for all $a \in[0,1]$,
(4) $\mathrm{a} * \mathrm{~b} \leq \mathrm{c} * \mathrm{~d}$ whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for each $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in[0,1]$.

Definition 1.2: [11] A binary operation $\rangle:[0,1] \times[0,1] \rightarrow[0,1]$ is a continuous $t$-conorm if it satisfies the following conditions:
(1) $\diamond$ is associative and commutative,
(2) $\diamond$ is continuous,
(3) $\mathrm{a} \diamond 0=\mathrm{a}$ for all $\mathrm{a} \in[0,1]$,
(4) $\mathrm{a} \diamond \mathrm{b} \leq \mathrm{c} \diamond \mathrm{d}$ whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for each $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in[0,1]$.

Definition 1.3: [1] A 5-tuple (X, M, N, *, $\rangle$ ) is called a intuitionistic fuzzy metric space if $X$ is an arbitrary (non-empty) set, * is a continuous t-norm, $\rangle$ is a continuous t-conorm and $\mathrm{M}, \mathrm{N}$ are fuzzy sets on $X^{2} \times(0, \infty)$, satisfying the following conditions:
(IFM-1) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})+\mathrm{N}(\mathrm{x}, \mathrm{y}, \mathrm{t}) \leq 1$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{t}>0$;
(IFM-2) $M(x, y, 0)=0$, for all $x, y \in X$;
(IFM-3) $M(x, y, t)=1$ for all $x, y \in X$ and $t>0$ if and only if $x=y$;
(IFM-4) $M(x, y, t)=M(y, x, t)$ for all $x, y \in X$ and $t>0$
(IFM-5) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t}) * \mathrm{M}(\mathrm{y}, \mathrm{z}, \mathrm{s}) \leq \mathrm{M}(\mathrm{x}, \mathrm{z}, \mathrm{t}+\mathrm{s})$ for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{X}$ and $\mathrm{s}, \mathrm{t}>0$;
(IFM-6) for all $x, y \in X, M(x, y,):.[0, \infty) \rightarrow[0,1]$ is left continuous;
(IFM-7) $\lim _{t \rightarrow \infty} M(\mathrm{x}, \mathrm{y}, \mathrm{t})=1$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{t}>0$;
(IFM-8) $N(x, y, 0)=1$, for all $x, y \in X$;
(IFM-9) $N(x, y, t)=0$ for all $x, y \in X$ and $t>0$ if and only if $x=y$;
$($ IFM-10 $) N(x, y, t)=N(y, x, t)$ for all $x, y \in X$ and $t>0$;
(IFM-11) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t+s)$ for all $x, y, z \in X$ and $s, t>0$;
(IFM-12) for all $x, y \in X, N(x, y,):.[0, \infty) \rightarrow[0,1]$ is right continuous;
(IFM-13) $\lim _{t \rightarrow \infty} N(\mathrm{x}, \mathrm{y}, \mathrm{t})=0$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$.
Then $(M, N)$ is called an intuitionistic fuzzy metric in $X$. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between $x$ and $y$ with respect to $t$, respectively.
Remark 1.4: Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form ( $X, M, 1-M, *, \diamond$ ) such that $t$ norm *and t -conorm $\diamond$ are associated $x \diamond y=1-((1-x) *(1-y))$ for any $x, y \in X$.
Example 1.5:. Let ( $\mathrm{X}, \mathrm{d}$ ) be a metric space. Define t -norm $\mathrm{a} * \mathrm{~b}=\min \{\mathrm{a}, \mathrm{b}\}$ and the t -conorm $\mathrm{a} \diamond \mathrm{b}=\min \{1, \mathrm{a}+\mathrm{b}\}$ and for all $\mathrm{a}, \mathrm{b} \in[0,1]$ and let $M_{d}$ and $N_{d}$ be fuzzy sets on $\mathrm{X}^{2} \times[0, \infty)$, define as follows $M_{d}(\mathrm{x}, \mathrm{y}, \mathrm{t})=\frac{t}{t+d(x, y)}$ and $N_{d}(\mathrm{x}, \mathrm{y}, \mathrm{t})=\frac{d(x, y)}{t+d(x, y)}$

Then ( $\mathrm{X}, \mathrm{M}, \mathrm{N}, *, \diamond$ ) is an intuitionistic fuzzy metric spaces. We call $\left(M_{d}, N_{d}\right)$ intuitionistic fuzzy metric induced by a metric ' d ' the standard intuitionistic fuzzy metric.
Remark 1.6: In intuitionistic fuzzy metric space $(X, M, N, *, \diamond), M(x, y,$.$) is non-decreasing and N(x, y,$.$) is non-increasing for all$ $x, y \in X$.
Definition 1.7: Let $(X, M, N, *, \diamond)$ be a intuitionistic fuzzy metric space. Consider I : $X \rightarrow X$ and
$T: X \rightarrow C B(X)$. A point $z \in X$ is called a coincidence point of $I$ and $T$ if and only if $I z \in T z$. We denote by $C B(X)$ the set of all nonempty bounded and closed subsets of $X$.
Definition 1.8: A pair of self mappings (A, S) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. $A x=S x$ for some $x$ in $X$, then $A S x=S A x$.
It is easy to see that two compatible maps are weakly compatible but converse is not true.
Definition 1.9: Two self mappings $A$ and $S$ of intuitionistic fuzzy metric space ( $X, M, N, *, \diamond$ ) are said to be occasionally weakly compatible (OWC) iff there is a point $x$ in $X$ which is coincidence point of $A$ and $S$ at which $A$ and $S$ commute.
Example 1.10: Define $A, S: R \rightarrow R$ by $A x=x$ and $S x=x^{2}$ for all $x \in R$, for $x=0,1$ then $\operatorname{AS}(0)=S A(0)$, and $\operatorname{AS}(1)=S A(1)$ for $\mathrm{x}=0,1$. Thus A and S are OWC maps and weakly compatible.
Example 1.11: Define $A, S: R \rightarrow R$ by $A x=4 x$ and $S x=x^{3}$ for all $x \in R$. Then $A x=S x$ for $x=0,2$ but $\operatorname{AS}(0)=\operatorname{SA}(0)$, and AS $(2) \neq \mathrm{SA}(2)$. Thus A and S are OWC maps but not weakly compatible

## 2. Preliminaries

Lemma 2.1: Let $\left(X, M, N,{ }^{*}, \diamond\right)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t>0$ and if for a number $k \in(0,1), M(x$, $y, k t) \geq M(x, y, t)$ and $N(x, y, k t) \leq N(x, y, t)$ then $x=y$.

Lemma 2.2: Let $X$ be a set, $A$ and $B$ be OWC self maps of $X$. If $A$ and $B$ have a unique point of coincidence $w=A x=B x$, then $w$ is the unique common fixed point of $A$ and $B$.

## 3. Main Results :

Theorem 3.1 : Let $\left(\mathrm{X}, \mathrm{M}, \mathrm{N},{ }^{*}, \diamond\right)$ be the complete intuitionistic fuzzy metric spaces and let A, B, S, T be self mappings of X . Let the pairs $(\mathrm{A}, \mathrm{S})$ and $(\mathrm{B}, \mathrm{T})$ be OWC and $\mathrm{k} \in(0,1)$ then
$M(A x, B y, k t) \geq \min \{M(S x, T y, t), M(S x, A x, t), M(T y, B y, t), M(S x, B y, t), M(T y, A x, t)$,

$$
\left.\frac{\mathrm{aM}(\mathrm{Ax}, \mathrm{Ty}, \mathrm{t})+\mathrm{bM}(\mathrm{By}, \mathrm{Sx}, \mathrm{t})+\mathrm{cM}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\}
$$

$\mathrm{N}(A x, B y, k t) \leq \max \{\mathrm{N}(S x, T y, t), N(S x, A x, t), N(T y, B y, t), N(S x, B y, t), N(T y, A x, t)$,

$$
\begin{equation*}
\left.\frac{\mathrm{aN}(\mathrm{Ax}, \mathrm{Ty}, \mathrm{t})+\mathrm{bN}(\mathrm{By}, \mathrm{Sx}, \mathrm{t})+\mathrm{cN}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\} \tag{1}
\end{equation*}
$$

for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{t}>0$ such that $\mathrm{Aw}=\mathrm{Sw}=\mathrm{w}$ and a unique point $\mathrm{z} \in \mathrm{X}$ such that $\mathrm{Bz}=\mathrm{Tz}=\mathrm{z}$ moreover $\mathrm{z}=\mathrm{w}$, so that there is a unique common fixed point of $A, B, S$, and $T$.
Proof: Let the pairs (A, S ) and (B, T) are OWC so there are points $x, y \in X$ such that $A x=S x$ and $B y=T y$ we claim that $A x=B y$. If not then by inequality (1)
$M(A x, B y, k t) \geq \min \{M(S x, T y, t), M(S x, A x, t), M(T y, B y, t), M(S x, B y, t), M(T y, A x, t)$,

$$
\left.\frac{\mathrm{aM}(\mathrm{Ax}, \mathrm{Ty}, \mathrm{t})+\mathrm{bM}(\mathrm{By}, \mathrm{Sx}, \mathrm{t})+\mathrm{cM}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\}
$$

$\mathrm{N}(A x, B y, k t) \leq \max \{N(S x, T y, t), N(S x, A x, t), N(T y, B y, t), N(S x, B y, t), N(T y, A x, t)$,

$$
\left.\frac{\mathrm{aN}(\mathrm{Ax}, \mathrm{Ty}, \mathrm{t})+\mathrm{bN}(\mathrm{By}, \mathrm{Sx}, \mathrm{t})+\mathrm{cN}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\}
$$

$\mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{kt}) \geq \min \{\mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{t}), 1,1, \mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{t}), \mathrm{M}(\mathrm{By}, \mathrm{Ax}, \mathrm{t})\}$
$\mathrm{N}(A x, B y, k t) \leq \max \{\mathrm{N}(\mathrm{Ax}, \mathrm{By}, \mathrm{t}), 0,0, \mathrm{~N}(\mathrm{Ax}, \mathrm{By}, \mathrm{t}), \mathrm{N}(\mathrm{By}, A x, \mathrm{t})\}$
Then by lemma (2.1) $\quad \mathrm{Ax}=\mathrm{By}$
Suppose that there is another point z such that $\mathrm{Az}=\mathrm{Sz}$. Then by inequality (1) we have
$A z=S z=B y=T y$, so $A x=A z$ and $w=A x=S x$ is the unique point of coincidence of $A$ and $S$. By Lemma (2.2) $w$ is the only common point of $A$ and $S$. Similarly there is a unique point $z \in X$ such that $z=B z=T z$. Assume that $w \neq z$ then by (1)
$\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{kt})=\mathrm{M}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt})$ and $\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{kt})=\mathrm{N}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt})$
$\mathrm{M}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt}) \geq \min \{\mathrm{M}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t}), \mathrm{M}(\mathrm{Sw}, \mathrm{Aw}, \mathrm{t}), \mathrm{M}(\mathrm{Tz}, \mathrm{Bz}, \mathrm{t}), \mathrm{M}(\mathrm{Sw}, \mathrm{Bz}, \mathrm{t}), \mathrm{M}(\mathrm{Tz}, \mathrm{Aw}, \mathrm{t})$,
$\left.\frac{\mathrm{aM}(\mathrm{Aw}, \mathrm{Tz}, \mathrm{t})+\mathrm{bM}(\mathrm{Bz}, \mathrm{Sw}, \mathrm{t})+\mathrm{cm}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\}$
$\mathrm{N}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t}), \mathrm{N}(\mathrm{Sw}, \mathrm{Aw}, \mathrm{t}), \mathrm{N}(\mathrm{Tz}, \mathrm{Bz}, \mathrm{t}), \mathrm{N}(\mathrm{Sw}, \mathrm{Bz}, \mathrm{t}), \mathrm{N}(\mathrm{Tz}, \mathrm{Aw}, \mathrm{t})$,

$$
\left.\frac{\mathrm{aN}(\mathrm{Aw}, \mathrm{Tz}, \mathrm{t})+\mathrm{bN}(\mathrm{Bz}, \mathrm{Sw}, \mathrm{t})+\mathrm{cN}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\}
$$

$\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \geq \min \{\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t}), 1,1, \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \mathrm{M}(\mathrm{z}, \mathrm{w}, \mathrm{t})\}$
$\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), 0,0, \mathrm{~N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{z}, \mathrm{w}, \mathrm{t})\}$. Then by lemma (2.1)
Therefore $\mathrm{w}=\mathrm{z} . \mathrm{z}$ is a common fixed point of $\mathrm{A}, \mathrm{B}, \mathrm{S}$ and T .
Uniqueness: Let $u$ be another common fixed point of $A, B, S$ and $T$. Then put $x=z$ and $y=u$ in (1) $M(A z, B u, k t) \geq \min \{M(S z, T u$, t), M(Sz, Az, t), M(Tu, Bu, t), M(Sz, Bu, t), M(Tu, Az, t),

$$
\begin{gathered}
\left.\frac{\mathrm{aM}(\mathrm{Az}, \mathrm{Tu}, \mathrm{t})+\mathrm{bM}(\mathrm{Bu}, \mathrm{~S} z, \mathrm{t})+\mathrm{cM}(\mathrm{Sz}, \mathrm{Tu}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\} \\
\mathrm{N}(\mathrm{Az}, \mathrm{Bu}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{Sz}, \mathrm{Tu}, \mathrm{t}), \mathrm{N}(\mathrm{Sz}, \mathrm{Az}, \mathrm{t}), \mathrm{N}(\mathrm{Tu}, \mathrm{Bu}, \mathrm{t}), \mathrm{N}(\mathrm{Sz}, \mathrm{Bu}, \mathrm{t}), \mathrm{N}(\mathrm{Tu}, \mathrm{Az}, \mathrm{t}), \\
\left.\frac{\mathrm{aN}(\mathrm{Az}, \mathrm{Tu}, \mathrm{t})+\mathrm{bN}(\mathrm{Bu}, \mathrm{Sz}, \mathrm{t})+\mathrm{cN}(\mathrm{Sz}, \mathrm{Tu}, \mathrm{t})}{\mathrm{a}+\mathrm{b}+\mathrm{c}}\right\}
\end{gathered}
$$

$\mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{kt}) \geq \min \{\mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t}), 1,1, \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t}), \mathrm{M}(\mathrm{u}, \mathrm{z}, \mathrm{t})\}$
$\mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{t}), 0,0, \mathrm{~N}(\mathrm{z}, \mathrm{u}, \mathrm{t}), \mathrm{N}(\mathrm{u}, \mathrm{z}, \mathrm{t})\}$
$M(z, u, k t) \geq M(z, u, t)$ and $N(z, u, k t) \leq N(z, u, t)$ Then by lemma (2.1) $z=u$. Hence the proof.

Remark 3.2 : Theorem (3.1) reduces to three maps $A, B$ and $S$ and the pairs $(A, S)$ and $(B, S)$ are $O W C$ and take $A x=S x$ and $B y=$ Sy. This proof is same as the theorem (3.1) which gives the following corollary (3.3)

Corollary 3.3 : Let A, B and $S$ be three self-maps on an Intuitionistic fuzzy metric spaces
(X, M, N, *, $\rangle$ ) satisfying: the pairs (A, S) and (B, S) occasionally weakly compatible
$M(A x, B y, k t) \geq \min \{M(S x, S y, t), M(A x, S x, t), M(B y, S y, t), M(A x, S y, t)$,

$$
\left.\mathrm{M}(\mathrm{By}, \mathrm{Sx}, \mathrm{t}), \frac{\mathrm{am}(\mathrm{Ax}, \mathrm{Sx}, \mathrm{t})+\mathrm{b} \mathrm{M}(\mathrm{By}, \mathrm{Sy}, \mathrm{t})}{\mathrm{a}+\mathrm{b}}\right\}
$$

$\mathrm{N}(A x, B y, k t) \leq \operatorname{ma}\{N(S x, S y, t), N(A x, S x, t), M(B y, S y, t), N(A x, S y, t)$,

$$
\left.N(B y, S x, t), \frac{a N(A x, S x, t)+b N(B y, S y, t)}{a+b}\right\}
$$

for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{t}>0$ such that $\mathrm{Aw}=\mathrm{Sw}=\mathrm{w}$ and a unique point $\mathrm{z} \in \mathrm{X}$ such that $\mathrm{Bz}=\mathrm{Sz}=\mathrm{z}$ moreover $\mathrm{z}=\mathrm{w}$, so that there is a unique common fixed point of $A, B$ and $S$.

Theorem 3.4: Let $\left(X, M, N,{ }^{*}, \diamond\right)$ be the complete intuitionistic fuzzy metric spaces and let $A, B, S, T$ be self mappings of $X$. Let the pairs ( $\mathrm{A}, \mathrm{S}$ ) and ( $\mathrm{B}, \mathrm{T}$ ) be OWC and $\mathrm{k} \in(0,1)$ and $\alpha+\beta=1$, then
$M(A x, B y, k t) \geq \min \{M(S x, T y, t), M(S x, A x, t), M(T y, B y, t), M(S x, B y, t), M(T y, A x, t)$,
$\{\alpha \mathrm{M}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{By}, \mathrm{Ty}, \mathrm{t}), \mathrm{M}(\mathrm{Sx}, \mathrm{Ax}, \mathrm{t}), \mathrm{M}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})\}\}\}$
$\mathrm{N}(A x, B y, k t) \leq \max \{N(S x, T y, t), N(S x, A x, t), N(T y, B y, t), N(S x, B y, t), N(T y, A x, t)$,

$$
\begin{equation*}
\{\alpha \mathrm{N}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{By}, \mathrm{Ty}, \mathrm{t}), \mathrm{N}(\mathrm{Sx}, \mathrm{Ax}, \mathrm{t}), \mathrm{N}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})\}\}\} \ldots( \tag{2}
\end{equation*}
$$

for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{t}>0$ such that $\mathrm{Aw}=\mathrm{Sw}=\mathrm{w}$ and a unique point $\mathrm{z} \in \mathrm{X}$ such that $\mathrm{Bz}=\mathrm{Tz}=\mathrm{z}$ moreover $\mathrm{z}=\mathrm{w}$, so that there is a unique common fixed point of $A, B, S$, and $T$.
Proof: Let the pairs $(A, S)$ and $(B, T)$ are $O W C$ so there are points $x, y \in X$ such that $A x=S x$ and $B y=T y$, we claim that $A x=$ By . If not then by inequality (2)
$M(A x, B y, k t) \geq \min \{M(A x, B y, t), M(A x, A x, t), M(B y, B y, t), M(A x, B y, t), M(B y, A x, t)$, $\{\alpha \mathrm{M}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{By}, \mathrm{By}, \mathrm{t}), \mathrm{M}(\mathrm{Ax}, \mathrm{Ax}, \mathrm{t}), \mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})\}\}\}$
$\mathrm{N}(A x, B y, k t) \leq \max \{N(A x, B y, t), N(A x, A x, t), N(B y, B y, t), N(A x, B y, t), N(B y, A x, t)$, $\{\alpha \mathrm{N}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{By}, \mathrm{By}, \mathrm{t}), \mathrm{N}(\mathrm{Ax}, \mathrm{Ax}, \mathrm{t}), \mathrm{N}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})\}\}\}$
$M(A x, B y, k t) \geq \min \{M(A x, B y, t), 1,1, M(A x, B y, t), M(B y, A x, t)$,
$\{\alpha \mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})+\beta \min \{1,1, \mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})\}\}\}$
$\mathrm{N}(A x, B y, k t) \leq \max \{N(A x, B y, t), 0,0, N(A x, B y, t), N(B y, A x, t)$,

$$
\{\alpha \mathrm{N}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})+\beta \max \{0,0, \mathrm{~N}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})\}\}\}
$$

$\mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{kt}) \geq \mathrm{M}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})$ and $\mathrm{N}(\mathrm{Ax}, \mathrm{By}, \mathrm{kt}) \leq \mathrm{N}(\mathrm{Ax}, \mathrm{By}, \mathrm{t})$. Then by lemma (2.1) $\mathrm{Ax}=\mathrm{By}$
Suppose that there is another point z such that $\mathrm{Az}=\mathrm{Sz}$.
Then by inequality (2) we have $\mathrm{Az}=\mathrm{Sz}=\mathrm{By}=\mathrm{Ty}$ so $\mathrm{Ax}=\mathrm{Az}$ and $\mathrm{w}=\mathrm{Ax}=\mathrm{Sx}$ is the unique point of coincidence of A and S . By Lemma (2.2) w is the only common point of A and S. Similarly there is a unique point $z \in X$ such that $z=B z=T z$
Assume that $\mathrm{w} \neq \mathrm{z}$ then by (2) $\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{kt})=\mathrm{M}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt})$ and $\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{kt})=\mathrm{N}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt})$
$M(A x, B y, k t) \geq \min \{M(S x, T y, t), M(S x, A x, t), M(T y, B y, t), M(S x, B y, t), M(T y, A x, t)$,
$\{\alpha \mathrm{M}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{By}, \mathrm{Ty}, \mathrm{t}), \mathrm{M}(\mathrm{Sx}, \mathrm{Ax}, \mathrm{t}), \mathrm{M}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})\}\}\}$
$\mathrm{N}(A x, B y, k t) \leq \max \{N(S x, T y, t), N(S x, A x, t), N(T y, B y, t), N(S x, B y, t), N(T y, A x, t)$,

$$
\{\alpha \mathrm{N}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{By}, \mathrm{Ty}, \mathrm{t}), \mathrm{N}(\mathrm{Sx}, \mathrm{Ax}, \mathrm{t}), \mathrm{N}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})\}\}\}
$$

Put $\mathrm{x}=\mathrm{w}$ and $\mathrm{y}=\mathrm{z}$ inequality (2)
$M(A w, B z, k t) \geq \min \{M(S w, T z, t), M(S w, A w, t), M(T z, B z, t), M(S w, B z, t), M(T z, A w, t)$,
$\{\alpha \mathrm{M}(\mathrm{Sw}, \mathrm{Bz}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{Bz}, \mathrm{Tz}, \mathrm{t}), \mathrm{M}(\mathrm{Sw}, \mathrm{Aw}, \mathrm{t}), \mathrm{M}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t})\}\}\}$
$\mathrm{N}(\mathrm{Aw}, \mathrm{Bz}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t}), \mathrm{N}(\mathrm{Sw}, \mathrm{Aw}, \mathrm{t}), \mathrm{N}(\mathrm{Tz}, \mathrm{Bz}, \mathrm{t}), \mathrm{N}(\mathrm{Sw}, \mathrm{Bz}, \mathrm{t}), \mathrm{N}(\mathrm{Tz}, \mathrm{Aw}, \mathrm{t})$,
$\{\alpha \mathrm{N}(\mathrm{Sw}, \mathrm{Bz}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{Bz}, \mathrm{Tz}, \mathrm{t}), \mathrm{N}(\mathrm{Sw}, \mathrm{Aw}, \mathrm{t}), \mathrm{N}(\mathrm{Sw}, \mathrm{Tz}, \mathrm{t})\}\}\}$
$M(w, z, k t) \geq \min \{M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, t), M(z, w, t)$,

$$
\{\alpha \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}(\mathrm{w}, \mathrm{w}, \mathrm{t}), \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})\}\}\}
$$

$\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{w}, \mathrm{w}, \mathrm{t}), \mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{z}, \mathrm{w}, \mathrm{t})$,

$$
\{\alpha \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{w}, \mathrm{w}, \mathrm{t}), \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t})\}\}\}
$$

$\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \geq \min \{\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t}), 1,1, \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \mathrm{M}(\mathrm{z}, \mathrm{w}, \mathrm{t}),\{\alpha \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})+\beta \min \{1,1, \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})\}\}\}$
$\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), 0,0, \mathrm{~N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{z}, \mathrm{w}, \mathrm{t}),\{\alpha \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t})+\beta \max \{0,0, \mathrm{~N}(\mathrm{w}, \mathrm{z}, \mathrm{t})\}\}\}$
$\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \geq \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})$ and $\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{kt}) \leq \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t})$ Then by lemma (2.1) $\mathrm{w}=\mathrm{z}$
Uniqueness: Let $u$ be another common fixed point of $A, B, S$ and T.Then put $x=z \& y=u$ in (2)
$M(A z, B u, k t) \geq \min \{M(S z, T u, t), M(S z, A z, t), M(T u, B u, t), M(S z, B u, t), M(T u, A z, t)$,
$\{\alpha \mathrm{M}(\mathrm{Sz}, \mathrm{Bu}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{Bu}, \mathrm{Tu}, \mathrm{t}), \mathrm{M}(\mathrm{Sz}, \mathrm{Az}, \mathrm{t}), \mathrm{M}(\mathrm{Sz}, \mathrm{Tu} \mathrm{t})\}\}\}$
$\mathrm{N}(\mathrm{Az}, \mathrm{Bu}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{Sz}, \mathrm{Tu}, \mathrm{t}), \mathrm{N}(\mathrm{Sz}, \mathrm{Az}, \mathrm{t}), \mathrm{N}(\mathrm{Tu}, \mathrm{Bu}, \mathrm{t}), \mathrm{N}(\mathrm{Sz}, \mathrm{Bu}, \mathrm{t}), \mathrm{N}(\mathrm{Tu}, \mathrm{Az}, \mathrm{t})$, $\{\alpha \mathrm{N}(\mathrm{Sz}, \mathrm{Bu}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{Bu}, \mathrm{Tu}, \mathrm{t}), \mathrm{N}(\mathrm{Sz}, \mathrm{Az}, \mathrm{t}), \mathrm{N}(\mathrm{Sz}, \mathrm{Tu}, \mathrm{t})\}\}\}$
$M(z, u, k t) \geq \min \{M(z, u, t), M(z, z, t), M(u, u, t), M(z, u, t), M(u, z, t)$,
$\{\alpha \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t})+\beta \min \{\mathrm{M}(\mathrm{u}, \mathrm{u}, \mathrm{t}), \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t})\}\}\}$
$N(z, u, k t) \leq \max \{N(z, u, t), N(z, z, t), N(u, u, t), N(z, u, t), N(u, z, t)$,
$\{\alpha \mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{t})+\beta \max \{\mathrm{N}(\mathrm{u}, \mathrm{u}, \mathrm{t}), \mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t}), \mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{t})\}\}\}$
$\mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{kt}) \geq \min \{\mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t}), 1,1, \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t}), \mathrm{M}(\mathrm{u}, \mathrm{z}, \mathrm{t}),\{\alpha \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t})+\beta \min \{1,1, \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t})\}\}\}$
$\mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{kt}) \leq \max \{\mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{t}), 0,0, \mathrm{~N}(\mathrm{z}, \mathrm{u}, \mathrm{t}), \mathrm{N}(\mathrm{u}, \mathrm{z}, \mathrm{t}),\{\alpha \mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{t})+\beta \max \{0,0, \mathrm{~N}(\mathrm{z}, \mathrm{u}, \mathrm{t})\}\}\}$
$\mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{kt}) \geq \mathrm{M}(\mathrm{z}, \mathrm{u}, \mathrm{t})$ and $\mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{kt}) \leq \mathrm{N}(\mathrm{z}, \mathrm{u}, \mathrm{t})$. Then by lemma (2.1) $\mathrm{z}=\mathrm{u}$. Hence the proof
Remark 3.5: By taking $\min \{a, b\}=a \star b$ and $\max \{a, b\}=a \diamond b$ in the above theorem (3.4), we get the following corollary.
Corollary 3.6: Let $\left(X, M, N,{ }^{*}, \diamond\right)$ be the complete intuitionistic fuzzy metric space and let A, B, S, T be self mappings of X. Let the pairs ( $\mathrm{A}, \mathrm{S}$ ) and ( $\mathrm{B}, \mathrm{T})$ be OWC and $\mathrm{k} \in(0,1)$ and $\alpha+\beta=1$, then
$M(A x, B y, k t) \geq\{M(S x, T y, t) \star M(S x, A x, t) \star M(T y, B y, t) \star M(S x, B y, t) \star M(T y, A x, t)$
$\star\{\alpha M \quad(S x, B y, t)+\beta\{M(B y, T y, t) \star M(S x, A x, t) \star M(S x, T y, t)\}\}\}$
$\mathrm{N}(\mathrm{Ax}, \mathrm{By}, \mathrm{kt}) \leq\{\mathrm{N}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Sx}, \mathrm{Ax}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Ty}, \mathrm{By}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Sx}, \mathrm{By}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Ty}, \mathrm{Ax}, \mathrm{t})$

$$
\diamond\{\alpha \mathrm{N}(\mathrm{Sx}, \mathrm{By}, \mathrm{t})+\beta\{\mathrm{N}(\mathrm{By}, \mathrm{Ty}, \mathrm{t} \diamond \mathrm{~N}(\mathrm{Sx}, \mathrm{Ax}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Sx}, \mathrm{Ty}, \mathrm{t})\}\}\} \ldots .(3)
$$

for all $x, y \in X$ and $t>0$ such that $A w=S w=w$ and a unique point $z \in X$ such that $B y=T z=z$ moreover $z=w$, so that there is a unique common fixed point of $A, B, S$, and $T$.

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