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Fineutro sets and fineutro topological spaces

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ABSTRACT

This paper introduces the new form of set namely Fineutro set and lays a foundation to discuss its behavior. Also we have defined Fineutro topological spaces.

1. Introduction

In 1965 , Zadeh [12] initiated the concept of fuzzy sets and Chang[4] introduced fuzzy topology in 1968 .After which there have been a number of generalizations on this fundamental concept .The notion of intuitionistic fuzzy sets introduced by Atanassov[2] is one among them . Using the notion of intuitionistic fuzzy sets Coker[5] defined intuitionistic fuzzy topological space.The theory of neutrosophic set, which is the generalization of the classical sets ,conditional fuzzy set and interval valued fuzzy set was introduced by Smarandache [10].This concept has been applied in many fields such as databases, Medical diagnosis problem, Decision making problem, Topology and control theory and so on. The concept of Neutrosophic set handle indeterminate data whereas fuzzy set theory and intuitionistic fuzzy set theory failed when the relations are indeterminate.

From philosophical point of view , the neutrosophic set takes the value from real standard or non-standard subset of $]^-0, 1^+[$ .But in real life application in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

In 2009 Manoranjan Bhowmik[6] introduced Intuitionistic Neutrosophic set. We now see the birth of a new form of set namely Fineutro set which elucidates the property that sum of the membership,indeterminacy membership and non membership functional values lie between 0 and 2. Furthur we discuss their characterizations.

2.Preliminaries

Definition 2.1:[7]

A Neutrosophic set A on the universe of discourse X is defined as

A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition2.2:[6]

A truth vaue based Neutrosophic set A on the universe of discourse X is defined as

A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

Definition2.3:[6]

An intuitionistic neutrosophic set is defined by $A^* = \left\langle x, T_{A^*}(x), I_{A^*}(x), F_{A^*}(x)\right\rangle$ where

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$\min\{T_{A^*}(x), F_{A^*}(x)\} \le 0.5$, $\min\{T_{A^*}(x), I_{A^*}(x)\} \le 0.5$ *and* $\min\{F_{A^*}(x), I_{A^*}(x)\} \le 0.5$ *forall* $x \in X$

With the condition $0 \le T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \le 2$

3. Fineutro Sets

Definition 3.1:

A Fineutro set A on the universe of discourse X is defined as A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle$ for all x∈X where $T_A, I_A, F_A : X \to [0,1]$ and it must satisfy the following conditions

(i) $I_A(x) \le 1 - \min(T_A(x), F_A(x))$

(ii) $0 \le T_A(x) + I_A(x) + F_A(x) \le 2$

Definition 3.2:

A Fineutro set A is a subset of a Fineutro set B (i.e.,) A ⊆ B for all x if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$

Definition 3.3 :

Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fineutro sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

Note:

1.Arbitrary union of a Fineutro sets is a Fineutro set

2. Arbitrary intersection of a Fineutro sets is a Fineutro set

Definition 3.4:

The difference between two fineutro sets A and B is defined as

A\B (x)= $\langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 3.5:

A Fineutro set A over the universe X is said to be null or empty Fineutro set if $T_A(x) = 0$

$I_A(x) = 0$, $F_A(x) = 1$ for all x ∈ X. It is denoted by 0_N

Definition 3.6:

A Fineutro set A over the universe X is said to be absolute (universe) Fineutro set if $T_A(x) = 1$,$I_A(x) = 1$, $F_A(x) = 0$ for all x ∈ X. It is denoted by 1_N

Definition 3.7:

The complement of a Fineutro set A is denoted by A^c and is defined as

A^c = $\langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x)\rangle$ where $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x)$, $F_{A^c}(x) = T_A(x)$

The complement of a Fineutro set A can also be defined as A^c = $1_N - A$.

Proposition 3.8:

Let A_i 's and B be Fineutro sets in X (i ∈ J) then $A_i \subseteq B$ for each $i \in J \Rightarrow \bigcup A_i \subseteq B$

Proof:

Let $A_i \subseteq B$ (i.e.,) $A_1 \subseteq B, A_2 \subseteq B, \ldots\ldots\ldots, A_n \subseteq B$

$$\Rightarrow T_{A_1}(x) \le T_B(x), I_{A_2}(x) \le I_B(x), F_{A_1}(x) \ge F_B(x)$$

$$T_{A_2}(x) \le T_B(x), I_{A_2}(x) \le I_B(x), F_{A_2}(x) \ge F_B(x) \qquad \dots\dots\dots\dots(1)$$

....

$$T_{A_n}(x) \le T_B(x), I_{A_n}(x) \le I_B(x), F_{A_n}(x) \ge F_B(x)$$

$$\bigcup A_i = \left\langle x, \max(T_{A_1}, T_{A_2}, \dots T_{A_n}), \max(I_{A_1}, I_{A_2}, \dots I_{A_n}), \min(F_{A_1}, F_{A_2}, \dots F_{A_n}) \right\rangle$$

$$\max(T_{A_1}(x), T_{A_2}(x), \dots T_{A_n}(x)) \le T_B(x)$$

$$\max(I_{A_1}(x), I_{A_2}(x), \dots I_{A_n}(x)) \le I_B(x)$$

$$\min(F_{A_1}(x), F_{A_2}(x), \dots F_{A_n}(x)) \ge F_B(x)$$

$$\Rightarrow \bigcup A_i \subseteq B$$

by (1)

Proposition 3.9:

Let A_i 's and B be Fineutro sets in X ($i \in J$) then $B \subseteq A_i$ for each $i \in J \Rightarrow B \subseteq \bigcap A_i$

Proof:

Let $B \subseteq A_i$ (i.e.,) $B \subseteq A_1, B \subseteq A_2, \dots B \subseteq A_n$

$$\Rightarrow T_B(x) \le T_{A_1}(x),\ I_B(x) \le I_{A_1}(x), F_B(x) \ge F_{A_1}(x)$$

$$T_B(x) \le T_{A_2}(x),\ I_B(x) \le I_{A_2}(x), F_B(x) \ge F_{A_2}(x)$$

....

$$T_B(x) \le T_{A_n}(x),\ I_B(x) \le I_{A_{n1}}(x), F_B(x) \ge F_{A_n}(x)$$

$$\Rightarrow T_B(x) \le \min(T_{A_1}(x), T_{A_2}(x), \dots T_{A_n}(x))$$

$$I_B(x) \le \min(I_{A_1}(x), I_{A_2}(x), \dots I_{A_n}(x))$$

$$F_B(x) \le \max(F_{A_1}(x), F_{A_2}(x) \dots F_{A_n}(x))$$

$\Rightarrow B \subseteq \bigcap A_i \quad \forall i \in J$ where $\bigcap A_i = \left\langle x, \min(T_{A_1}, T_{A_2}, \dots T_{A_n}), \min(I_{A_1}, I_{A_2}, \dots I_{A_n}), \max(F_{A_1}, F_{A_2}, \dots F_{A_n}) \right\rangle$

Proposition 3.10 :

Let A_i's be Fineutro sets in X , $i \in J$ then $(i)\ \left(\bigcup A_i\right)^c = \bigcap A_i^c \quad (ii) \left(\bigcap A_i\right)^c = \bigcup A_i^c$

Proof:

$$\bigcup A_i = \left\langle x, \max(T_{A_1}, T_{A_2}, \dots T_{A_n}), \max(I_{A_1}, I_{A_2}, \dots I_{A_n}), \min(F_{A_1}, F_{A_2}, \dots F_{A_n}) \right\rangle$$

$$\left(\bigcup A_i\right)^c = \left\langle x, \min(F_{A_1}, F_{A_2}, \dots F_{A_n}), 1 - \max(I_{A_1}, I_{A_2}, \dots I_{A_n}), \max(T_{A_1}, T_{A_2}, \dots T_{A_n}) \right\rangle$$

$$= \left\langle x, \min(F_{A_1}, F_{A_2}, \dots F_{A_n}), \min(1 - I_{A_1}, 1 - I_{A_2}, \dots 1 - I_{A_n}), \max(T_{A_1}, T_{A_2}, \dots T_{A_n}) \right\rangle \quad \dots\dots\dots(1)$$

$$A_i^c = \left\langle x, (F_{A_1}, F_{A_2}, \dots F_{A_n}), (1 - I_{A_1}, 1 - I_{A_2}, \dots 1 - I_{A_n}), (T_{A_1}, T_{A_2}, \dots T_{A_n}) \right\rangle$$

$$\bigcap A_i^c = \left\langle x, \min(F_{A_1}, F_{A_2}, \dots F_{A_n}), \min(1 - I_{A_1}, 1 - I_{A_2}, \dots 1 - I_{A_n}), \max(T_{A_1}, T_{A_2}, \dots T_{A_n}) \right\rangle \quad \dots\dots\dots(2)$$

From (1) and (2) $\left(\bigcup A_i\right)^c = \bigcap A_i^c$

Proof of (ii) is similar.

Proposition 3.11 :

Let A and B be Fineutro sets then $A \subseteq B \Leftrightarrow B^c \subseteq A^c$

Proof:

Let A and B be Fineutro sets then

$$A \subseteq B \Leftrightarrow T_A(x) \le T_B(x), I_A(x) \le I_B(x), F_A(x) \ge F_B(x)$$
$$\Leftrightarrow F_B(x) \le F_A(x), 1 - I_B(x) \le 1 - I_A(x), T_B(x) \ge T_A(x)$$
$$\Leftrightarrow B^c \subseteq A^c$$

Proposition 3.12 :

Let A be a Fineutro set in X then $(A^c)^c = A$

Proof:

Let A= $\langle x, T_A(x), I_A(x), F_A(x) \rangle$ be a Fineutro set in X then $A^c = \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle$

Hence $(A^c)^c = \langle x, T_A(x), I_A(x), F_A(x) \rangle = A$

Note:

$(0_N)^c = 1_N$, $(1_N)^c = 0_N$

Proposition 3.13 :

Let A be a Fineutro set in X then the following properties hold:

$(i)\ A \cup 0_N = A\ \ (ii)\ A \cup 1_N = 1_N\ \ (iii)\ A \cap 0_N = 0_N\ \ (iv)\ A \cap 1_N = A$

Proof:

Let A= $\langle x, T_A(x), I_A(x), F_A(x) \rangle$, $0_N = \langle x, 0, 0, 1 \rangle, 1_N = \langle x, 1, 1, 0 \rangle$

$$(i)\quad A \cup 0_N = \langle x, \max(T_A(x), 0), \max(I_A(x), 0), \min(F_A(x), 1) \rangle$$
$$= \langle x, T_A(x), I_A(x), F_A(x) \rangle$$
$$= A$$

$$(ii)\quad A \cup 1_N = \langle x, \max(T_A(x), 1), \max(I_A(x), 1), \min(F_A(x), 0) \rangle$$
$$= \langle x, 1, 1, 0 \rangle = 1_N$$

$$(iii)\quad A \cap 0_N = \langle x, \min(T_A(x), 0), \min(I_A(x), 0), \max(F_A(x), 1) \rangle$$
$$= \langle x, 0, 0, 1 \rangle = 0_N$$

$$(iv)\quad A \cap 1_N = \langle x, \min(T_A(x), 1), \min(I_A(x), 1), \max(F_A(x), 0) \rangle$$
$$= \langle x, T_A(x), I_A(x), F_A(x) \rangle = A$$

Proposition 3.14 :

Let A and B be two Fineutro sets in X then $A \cup B = A$ if and only if $B \subseteq A$

Proof:

Let A and B be two Fineutro sets in X such that $A \cup B = A$. (i.e.,)

$$\langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle = \langle x, T_A(x), I_A(x), F_A(x) \rangle$$

$\Leftrightarrow \max(T_A(x), T_B(x)) = T_A(x)$, $\max(I_A(x), I_B(x)) = I_A(x)$, $\min(F_A(x), F_B(x)) = F_A(x)$

$\Leftrightarrow T_B(x) \le T_A(x)$, $I_B(x) \le I_A(x)$, $F_B(x) \ge F_A(x)$

$\Leftrightarrow B \subseteq A$

Proposition 3.15:

Let A and B be two Fineutro sets in X then A \ B = $B^c \setminus A^c$

Proof:

Let A = $\langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$

$$(A \setminus B)(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$$

$$A^c = \langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \text{ , } B^c = \langle x, F_B(x), 1 - I_B(x), T_B(x)\rangle$$

$$(B^c \setminus A^c)(x) = \langle x, \min(F_B(x), T_A(x)), \min(1 - I_B(x), I_A(x)), \max(T_B(x), F_A(x))\rangle$$

$$(A \setminus B)(x) = (B^c \setminus A^c)(x)$$

Hence

Proposition 3.16:

Let A , B and C be Fineutro sets in X then

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

(iii) $A \cup (B \cup C) = (A \cup B) \cup C$

(iv) $A \cap (B \cap C) = (A \cap B) \cap C$

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(vi) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vii) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$(viii)$ $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

4. Fineutro Topological Spaces

Definition 4.1:

A Fineutro topology on a nonempty set X is a τ of Fineutro sets in X satisfying the following axioms

(i) $0_N, 1_N \in \tau$

(ii) $A_1 \cap A_2 \in \tau$ *for any* $A_1, A_2 \in \tau$

(iii) $\cup A_i \in \tau$ *for any arbitrary family* $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ)is called Fineutro topological space and any Fineutro set in τ is known as Fineutro open set in X .

Example 4.2 :

Let X= {a,b,c}and consider the family τ ={0_N , 1_N,A_1,A_2,A_3,A_4} where

$A_1 = \{\langle a,0.6,0.5,0.3\rangle, \langle b,0.8,0.6,0.4\rangle, \langle c,0.4,0.3,0.5\rangle\}$

$A_2 = \{\langle a,0.7,0.6,0.2\rangle, \langle b,0.6,0.2,0.5\rangle, \langle c,0.9,0.7,0.2\rangle\}$

$A_3 = \{\langle a,0.7,0.6,0.2\rangle, \langle b,0.8,0.6,0.4\rangle, \langle c,0.9,0.7,0.2\rangle\}$

$A_4 = \{\langle a,0.6,0.5,0.3\rangle, \langle b,0.6,0.2,0.5\rangle, \langle c,0.4,0.3,0.5\rangle\}$

Then (X, τ) is called Fineutro topological space on X

Definition 4.3:

The complement A^c of a Fineutro set A in a Fineutro topological space (X, τ) is called a Fineutro closed set in X .

Now we define closure and interior operations in Fineutro topological spaces

Definition 4.4:

Let (X, τ) be a Fineutro topological space and A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a Fineutro set in X. Then the closure and interior of A are defined by

$$Fine\ \text{int}(A) = \bigcup \{G : G\ is\ a\ Fineutro\ open\ set\ in\ X\ and\ G \subseteq A\}$$
$$Fine\ cl(A) = \bigcap \{G : G\ is\ a\ Fineutro\ closed\ set\ in\ X\ and\ A \subseteq G\}$$

Proposition 4.5:

Let (X,τ) be a Fineutro topological space over X .Then the following properties hold.

$$(i)\ Fine\ cl\ (A^c) = (Fine\ \text{int}\ A)^c \quad (ii)\ Fine\ \text{int}\ A^c = (Fine\ \ cl\ A)^c$$

Proof:

Let A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle$

Suppose that the family of Fineutro open sets G_i contained in A are indexed by the family $\{\langle x, T_{G_i}(x), I_{G_i}(x), F_{G_i}(x)\rangle : i \in J\}$

$$Fine\ \ \text{int}\ A = \langle x, \max(T_{G_i}(x)), \max(I_{G_i}(x)), \min(F_{G_i}(x))\rangle$$

Then we see that $\Rightarrow (Fine\ \ \text{int}\ A)^c = \langle x, \min(F_{G_i}(x)), 1-\max(I_{G_i}(x)), \max(T_{G_i}(x))\rangle$ ....(1)

$$A^c = \langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle$$

$$\because G_i \subseteq A \quad \forall i \in J$$
$$T_{G_i}(x) \le T_A(x), I_{G_i}(x) \le I_A(x), F_{G_i}(x) \ge F_A(x) \qquad \forall i \in J$$

We obtain that $\{\langle x, F_{G_i}(x), 1 - I_{G_i}(x), T_{G_i}(x)\rangle : i \in J\}$ is the family of Fineutro closed sets containing A^c .(i.e.,)

$$Fine\ \ cl\ (A^c) = \{ < x, \min F_{G_i}(x), \min(1 - I_{G_i}(x)), \max(T_{G_i}(x)) >: i \in J\}$$
$$= \{ < x, \min F_{G_i}(x), 1 - \max I_{G_i}(x), \max(T_{G_i}(x)) >: i \in J\} \qquad ....(2)$$
$$Hence\ \ Fine\ \ cl\ (A^c) = (Fine\ \ \text{int}\ A)^c$$

Similarly we can prove (ii).

Proposition 4.6:

Let $(X, \tau_1)\ and\ (X, \tau_2)$ be two Fineutro topological spaces. Denote $\tau_1 \cap \tau_2 = \{A : A \in \tau_1\ and\ A \in \tau_2\}$ then $\tau_1 \cap \tau_2$ is a Fineutro topological space.

Proof:

Obviously $0_N, 1_N \in \tau_1 \cap \tau_2$

Let $A_1, A_2 \in \tau_1 \cap \tau_2 \Rightarrow A_1, A_2 \in \tau_1\ ,\ A_1, A_2 \in \tau_2$

$\tau_1\ and\ \tau_2$ are Fineutro topological spaces on X .Then

$$A_1 \cap A_2 \in \tau_1\ \ and\ \ A_1 \cap A_2 \in \tau_2 \Rightarrow A_1 \cap A_2 \in \tau_1 \cap \tau_2$$

Let $\{A_i : i \in J\} \subseteq \tau_1 \cap \tau_2 \ \Rightarrow\ A_i \in \tau_1$ and $A_i \in \tau_2 \quad \forall i \in J$

Since $\tau_1\ and\ \tau_2$ are Fineutro topological spaces on X

$$\bigcup\{A_i : i \in J\} \in \tau_1\ \ and\ \ \bigcup\{A_i : i \in J\} \in \tau_2 \Rightarrow \bigcup\{A_i : i \in J\} \in \tau_1 \cap \tau_2$$

Therefore $\tau_1 \cap \tau_2$ is a Fineutro topological space.

Remark:

$\tau_1 \cup \tau_2$ is not a Fineutro topological space can be seen by the following example.

Example 4.7:

Let X={a,b} , $\tau_1 = \{0_N, 1_N, A\}$ and $\tau_1 = \{0_N, 1_N, B\}$ where

$A = \{\langle a,0.6,0.7,0.2\rangle, \langle b,0.9,0.7,0.2\rangle\}$

$B = \{\langle a,0.7,0.6,0.3\rangle, \langle b,0.8,0.5,0.4\rangle\}$

Here $\tau_1 \cup \tau_2 = \{0_N, 1_N, A, B\}$, Since $A \cup B, A \cap B \notin \tau_1 \cup \tau_2$, $\tau_1 \cup \tau_2$ is not a Fineutro topological space

Definition 4.8:

Let (X, τ) be a Fineutro topological space on X .

(i) A family $\beta \subseteq \tau$ is called a base for (X, τ) if and only if each member of τ can be written as the elements of β.

(ii) A family $\gamma \subseteq \tau$ is called a sub base for (X, τ) if and only if the family of finite intersections of elements in γ forms a base for (X, τ).In this case the Finite topology τ is said to be generated by γ.

Proposition 4.9:

Let (X,τ) be a Fineutro topological space over X .Then the following properties hold.

(i) 0_N, 1_N are Fineutro closed sets over X

(ii) The intersection of any number of Fineutro closed sets is a Fineutro closed set over X

(iii) The union of any two Fineutro closed sets is a Fineutro closed set over X

Proof: It is obvious.

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