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Frequency of Failures of a System and Confidence-interval

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ABSTRACT

This paper presents Maximum likelihood (M L) estimation approach to derive reliability measures of a two component identical system in the presence of Common Cause Shock failures (CCS) as well as human errors. The M L estimates of system reliability measures like frequency of failures were derived for both series and parallel systems. We also developed the confidence interval in the present study. The approach used is empirical one with Monte Carlo simulation.

Introduction

Progress in science and technology has made engineering systems more powerful than ever. The intensity of sophistication in high-tech industrial producers emerged with reliability problems. Therefore the problem of reliability continue to exist and more likely to require complex solutions. Consequently, the field of reliability analysis and statistical probability modeling of the systems and equipments/components were growing. Ever since the theory of reliability was formally recognized statistical and modeling of the components/systems analysis was used to develop various reliability measures that are important to assess the system performance. In Reliability theory, basically underlying phenomena of interest is "life or time to failure of the components"/equipments/systems which is treated as random phenomenon and probabilistic prediction of various indices of reliability were tried to be attempted to asses the component/system performance.

Conventionally the reliability analysts and researchers assumed that the component in the system will fail individually by inherent in capability and randomly. This type of failure is known as "intrinsic failures" in the reliability literature. During 1980's reliability analysts and researchers were encountered with yet another type of failure known as common cause shock failure (CCS) / common mode failures. CCS as defined by IEEE ATM Sub committee is those significant, which affect multiple component failures. The event may be out side of the component. In addition, to this another important failure is human error and it is defined as a failure to perform a prescribed task which could result in damage of equipment and property. Some of the reasons for occurrence of human errors are poor equipment design, inadequate training or skill of the concerned man power, improper tools etc. Billinton and Allan [1] and Reddy [4] discussed the role of CCS failures. Dhillon [2, 3] studied the concept of CCS failures as well as human errors.

Therefore, in the present research work, an attempt is made to find an approach of estimation method, which could establish a formal estimation procedure to estimate the reliability measures like frequency of failures of a system. Thus in the present research work it is attempted to develop estimates of the system reliability indices such as Frequency of failures for series system ($F_{chs}(T)$) and Frequency of failures for parallel system ($F_{chp}(T)$) practically under the influence of common cause shock failures, human errors as well as intrinsic failures.

Assumptions

The system has two components, which are stochastically independent

1. The system is affected by individual, common cause failures as well as human errors
2. The components in the system will fail singly at the constant rate λ_i and failure probability is P_1
3. The components may fail due to common causes at the constant rate λ_c and with failure probability is P_2
4. The components may fail due to human errors at the constant rate λ_h and with failure probability is P_3 s.t $P_1 + P_2 + P_3 = 1$
5. Time occurrences of CCS failures, human errors and individual failures follow Exponential law
6. The failed components are repaired singly and repair time follows exponential distribution with rate of service μ

Notations

λ_i, λ_c & λ_h: the failure rates of individual, CCS failures and human errors respectively.

P_1, P_2 & P_3: the chance of individual, CCS failures and human errors respectively.

μ : repair rate.

$F_{chs}(T)$: Steady state Frequency of Failure function of series system.

$\hat{F}_{chs}(T)$: M L Estimate of steady state Frequency of Failure function of series system.

$F_{chp}(T)$: Steady state Frequency of Failure function for parallel system.

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$\hat{F}_{chp}(T)$: M L Estimate of steady state frequency of Failure function for parallel system .

$\bar{x}, \bar{y} \& \bar{w}$: Sample means of the occurrence of individual, CCS failures and human errors respectively

$\bar{z}$: Sample mean of repair time of the components

$\hat{\bar{x}}, \hat{\bar{y}} \& \hat{\bar{w}}$: Sample estimates of individual failure rate, CCS failure rate and human errors respectively

$\hat{\bar{z}}$: Sample estimate of repair time of the components

n : Sample size

N : Number of simulated samples

θ : $(\lambda_i, \lambda_c, \lambda_h, \mu)$

M S E : Mean square error

The Model

Under the stated assumptions Markovian model can be formulated to derive the Frequency of failure function F(t) under the influence of individual, CCS as well as human errors and the Markovian graph is given in fig.1. The quantities λ_0, λ_1, λ_2, λ_3, μ_1&μ_2 are as follows $\lambda_0 = 2\lambda_i P_1$, $\lambda_1 = \lambda_i P_1$, $\lambda_2 = \lambda_c P_2$, $\lambda_3 = \lambda_h P_3$, $\mu_1 = \mu$ & $\mu_2 = 2\mu$

From the Markov graph the equations were formed and the probabilities of the various states of the systems i.e. P0(t), P1(t), P2(t) are derived by Sagar G. Y [5].

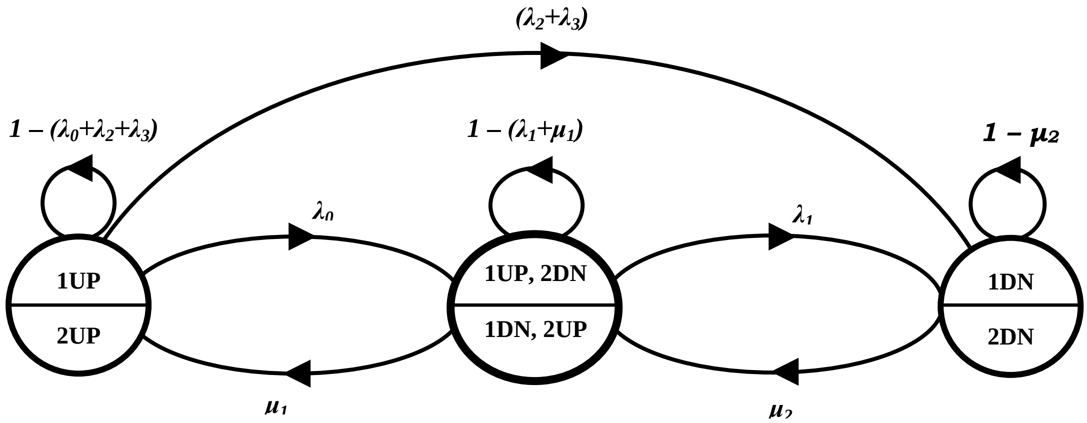

FIGURE 1. TWO COMPONENT SYSTEM WITH CCS FAILURES AND HUMAN ERRORS

Frequency Of Failures – M L Estimation

Let $x_1, x_2, \ldots\ldots x_n$ be a sample of 'n' number of times between individual failures which will obey exponential law.

Let $y_1, y_2, \ldots\ldots y_n$ be a sample of 'n' number of times between CCS failures which follow exponential as well.

Let $w_1, w_2, \ldots\ldots w_n$ be a sample of 'n' number of times between human errors which follow exponential as well.

Let $z_1, z_2, \ldots\ldots z_n$ be a sample of 'n' number of times repair of the components with exponential population law.

$\hat{\bar{x}}, \hat{\bar{y}}, \hat{\bar{w}} \& \hat{\bar{z}}$ are the maximum likelihood estimates of individual failure rate *(λ_i)*, CCS failure rate *(λ_c)*, human errors rate *(λ_h)* and repair rate 'μ' of the system respectively.

Where,

$$\hat{\bar{x}} = \frac{1}{\bar{x}};\ \hat{\bar{y}} = \frac{1}{\bar{y}};\ \hat{\bar{w}} = \frac{1}{\bar{w}};\ \hat{\bar{z}} = \frac{1}{\bar{z}} \quad \text{and}$$

$$\bar{x} = \frac{\sum x_i}{n};\ \bar{y} = \frac{\sum y_i}{n};\ \bar{w} = \frac{\sum w_i}{n};\ \bar{z} = \frac{\sum z_i}{n}$$

are the sample estimates of the rate of individual failure times, rate of CCS failure times, rate of human error times and rate of repair times of the components respectively.

Estimation of Frequency of Failure Function

The maximum likelihood estimates of frequency of failure function for series and parallel systems are derived in this section.

Series System

Thus, the expression of frequency of failure function for series system is give by

$$F_{chs}(T) = 2\mu^2[2\lambda_i P_1 + \lambda_c P_2 + \lambda_h P_3]/[2\mu\ (\mu\ + 2\lambda_i P_1 + \lambda_c P_2 + \lambda_h P_3)\ +\ \mu\lambda_c P_2 + \lambda_h P_3 + \lambda_i P_1 (2\lambda_i P_1 + \lambda_c P_2 + \lambda_h P_3)\ ] \qquad (1)$$

Where λ_i, λ_c, λ_h, μ, P_1, P_2 & P_3 are individual failures rate, Common cause failure rate, human error rate, repair rate and probability of occurrence of individual, CCS failures as well as human errors.

Therefore, the expression of maximum likelihood estimate of frequency of failure function for series system is given by

$$\hat{F}_{chs}(T) = 2\,\hat{\bar{z}}\,.\,\hat{\bar{z}}\,[2\,\hat{\bar{x}}\,P_1 + \hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3]/[2\,\hat{\bar{z}}\,(\hat{\bar{z}} + 2\,\hat{\bar{x}}\,P_1 + \hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3) + \hat{\bar{z}}\,\hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3 + \hat{\bar{x}}\,P_1(2\,\hat{\bar{x}}\,P_1 + \hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3)] \qquad (2)$$

Where, $\hat{\bar{x}}$, $\hat{\bar{y}}$, $\hat{\bar{w}}$ & $\hat{\bar{z}}$ are the maximum likelihood estimates of individual failure rate (λ_i), Common cause failures rate (λ_c), human error (λ_h) and repair rate (μ) of system respectively.

Parallel System

The expression of Frequency of failure function of parallel system is

$$F_{chp}(T) = 2\mu[(\lambda_c P_2 + \lambda_h P_3)(\mu + \lambda_i P_1) + 2\lambda_i^2 P_1^2]/[2\mu(\mu + 2\lambda_i P_1 + \lambda_c P_2 + \lambda_h P_3) + \mu(\lambda_c P_2 + \lambda_h P_3) + \lambda_i P_1(2\lambda_i P_1 + \lambda_c P_2 + \lambda_h P_3)\ ] \qquad (3)$$

Where, λ_i, λ_c, λ_h, μ, P_1, P_2 & P_3 are defined in (1)

Therefore, the expression of maximum likelihood estimate of Frequency failure function for parallel system is

$$\hat{F}_{chp}(T) = 2\,\hat{\bar{z}}\,[(\hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3)(\hat{\bar{z}} + \hat{\bar{x}}\,P_1) + 2\,\hat{\bar{x}}\,.\,\hat{\bar{x}}\,P_1^2]/[2\,\hat{\bar{z}}\,(\hat{\bar{z}} + 2\,\hat{\bar{x}}\,P_1 + \hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3) + \hat{\bar{z}}\,(\hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3) + \hat{\bar{x}}\,P_1(2\,\hat{\bar{x}}\,P_1 + \hat{\bar{y}}\,P_2 + \hat{\bar{w}}\,P_3)] \qquad (4)$$

Where, $\hat{\bar{x}}$, $\hat{\bar{y}}$, $\hat{\bar{w}}$ & $\hat{\bar{z}}$ are the sample estimates given in (section 5).

Confidence Interval

Obviously, the above estimates are functions of $\bar{x}, \bar{y}, \bar{w} \& \bar{z}$ which are differentiable. Now from multivariate central limit theorem

$\sqrt{n}[(\bar{x}, \bar{y}, \bar{w} \,\&\, \bar{z}) - (\lambda_i, \lambda_c, \lambda_h, \mu)] \sim N_4(0, \Sigma)$ for $n \rightarrow \infty$

Where $\Sigma = (\sigma_{ij})_{3\times3}$ co-variance matrix $\Sigma = dig(\lambda_i^2, \lambda_c^2, \lambda_h^2 \mu^2)$ Also we have $\sqrt{n}[F(T) - \hat{F}(T)] \sim N(0, \sigma_\theta^2)$ as $n \rightarrow \infty$ and θ is the vector. By the properties of M L method of estimation $\hat{F}(T)$ is CAN estimate of F(T) respectively. Also $\sigma^2_{(\hat{\theta})}$ be the estimator of $\sigma^2_{(\theta)}$ Where $(\hat{\theta}) = (\hat{\bar{x}}, \hat{\bar{y}}, \hat{\bar{w}} \,\&\, \hat{\bar{z}})$ and Let us consider $\psi = \sqrt{n}[(\hat{F}(T) - F(T)] / \sigma^2_\theta \sim N(0,1)$ from Slutsky theorem, we have $P[-Z_{\alpha/2} \le \psi \le Z_{\alpha/2}] = 1-\alpha$

Where $Z_{\alpha/2}$ are the $\alpha/2$ percentiles points of normal distribution and are available from normal tables. Hence $(1-\alpha)\%$ confidence interval for $F_{chs}(t)$, $F_{chp}(t)$ are given by

$$F_{chs}(T) \pm Z_{\alpha/2}\ \sigma^2_{(Fchs\ (T))} / \sqrt{n}$$

$$F_{chp}(T) \pm Z_{\alpha/2}\ \sigma^2_{(Fchp(T))} / \sqrt{n}$$

Monte Carlo Simulations and Validity

The maximum likelihood estimates of steady state frequency of failure measures of two component identical system, in the sections 5.1.1 & 5.1.2 the exact probability density function of these estimates are not known and not much literature is seen in this direction. Hence in the present work an attempt is made to develop empirical evidence of M L Estimation approach by Monte Carlo simulation procedure for validity of results. For a range of specified values of the rates of individual (λ_i), common cause failures(λ_c), human errors (λ_h) and repair rates (μ) and for the samples of sizes n = 5 (5) 30 are using computer package developed in this research work and M L Estimates are computed for N = 10,000 (20,000) 90,000 and mean square error (MSE) and confidence interval of the estimates for $F_{chs}(T)$ & $F_{chp}(T)$ were obtained and given in numerical illustration. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interesting to note that for a sample size as low as five i.e (n=5) M L estimate is still seem to be reasonably good giving near accurate estimate in this case. This shows that M L approach and estimators are quite useful in estimating Reliability indices like Frequency of Failures in both series and parallel systems.

TABLE 6.1

Results of the simulations for steady state Frequency of Failure function for Series system with λ_i= 0.5; λ_c = 0.6; λ_h= 0.2 μ.= 0.4; p_1 = 0.5; P_2 = 0.25; P_3 = 0.25

Sample Size n = 5

N	F_{chs} (T)	$\hat{F}_{chs}$ (T)	M S E	Confidence-Intervals (95%)
10000	0.200000	0.240541	0.015003	(0.000000, 0.487379)
30000	0.200000	0.239759	0.014510	(0.000000, 0.487379)
50000	0.200000	0.239307	0.013653	(0.000000, 0.487379)
70000	0.200000	0.239462	0.013716	(0.000000, 0.487379)
90000	0.200000	0.238927	0.013371	(0.000000, 0.487379)

Sample Size n = 10

N	F_{chs} (T)	$\hat{F}_{chs}$ (T)	M S E	Confidence-Intervals (95%)
10000	0.200000	0.222082	0.005664	(0.000000, 0.373918)
30000	0.200000	0.221247	0.005471	(0.000000, 0.373918)
50000	0.200000	0.221986	0.005522	(0.000000, 0.373918)
70000	0.200000	0.221076	0.005428	(0.000000, 0.373918)
90000	0.200000	0.221452	0.005446	(0.000000, 0.373918)

Sample Size n = 15

N	F_{chs} (T)	$\hat{F}_{chs}$ (T)	M S E	Confidence-Intervals (95%)
10000	0.200000	0.215527	0.003056	(0.000000, 0.323653)
30000	0.200000	0.215692	0.003997	(0.000000, 0.323653)
50000	0.200000	0.216062	0.003025	(0.000000, 0.323653)
70000	0.200000	0.216242	0.003051	(0.000000, 0.323653)
90000	0.200000	0.215995	0.003044	(0.000000, 0.323653)

Sample Size n = 20

N	F_{chs}(T)	$\hat{F}_{chs}$ (T)	M S E	Confidence-Intervals (95%)
10000	0.200000	0.212965	0.001890	(0.006310, 0.293689)
30000	0.200000	0.213704	0.002006	(0.006310, 0.293689)
50000	0.200000	0.213199	0.001990	(0.006310, 0.293689)
70000	0.200000	0.213010	0.001940	(0.006310, 0.293689)
90000	0.200000	0.212992	0.001949	(0.006310, 0.293689)

Sample Size n = 25

N	F_{chs}(T)	$\hat{F}_{chs}$(T)	M S E	Confidence-Intervals (95%)
10000	0.200000	0.211467	0.001322	(0.026759, 0.273241)
30000	0.200000	0.211460	0.001338	(0.026759, 0.273241)
50000	0.200000	0.211601	0.001348	(0.026759, 0.273241)
70000	0.200000	0.211521	0.001366	(0.026759, 0.273241)
90000	0.200000	0.211640	0.001365	(0.026759, 0.273241)

Sample Size n = 30

N	F_{chs}(T)	$\hat{F}_{chs}$(T)	M S E	Confidence-Intervals (95%)
10000	0.200000	0.210845	0.000977	(0.041853, 0.258147)
30000	0.200000	0.210558	0.000983	(0.041853, 0.258147)
50000	0.200000	0.210587	0.000991	(0.041853, 0.258147)
70000	0.200000	0.210525	0.000961	(0.041853, 0.258147)
90000	0.200000	0.210340	0.000965	(0.041853, 0.258147)

TABLE 6.2

Results of the simulations for steady state Frequency of failure function for parallel system with λ_i= 0.5; λ_c = 0.6; λ_h = 0.2 μ.= 0.4, P_1 = 0.6; P_2 = 0.3; P_3 = 0.1

Sample Size n =5

N	F_{chp}(T)	$\hat{F}_{chp}$(T)	M S E	Confidence-Intervals (95%)
10000	0.234627	0.274289	0.010612	(0.070251, 0.299003)
30000	0.234627	0.274519	0.010576	(0.070251, 0.299003)
50000	0.234627	0.275828	0.010761	(0.070251, 0.299003)
70000	0.234627	0.275351	0.010749	(0.070251, 0.299003)
90000	0.234627	0.275436	0.010791	(0.070251, 0.299003)

Sample Size n =10				
N	$F_{chp}(T)$	$\hat{F}_{chp}(T)$	M S E	Confidence-Intervals (95%)
10000	0.234627	0.264290	0.004314	(0.118395, 0.250858)
30000	0.234627	0.263539	0.004228	(0.118395, 0.250858)
50000	0.234627	0.263605	0.004210	(0.118395, 0.250858)
70000	0.234627	0.263705	0.004226	(0.118395, 0.250858)
90000	0.234627	0.263732	0.004255	(0.118395, 0.250858)

Sample Size n =15				
N	$F_{chp}(T)$	$\hat{F}_{chp}(T)$	M S E	Confidence-Intervals (95%)
10000	0.234627	0.256331	0.003116	(0.139724, 0.229529)
30000	0.234627	0.256655	0.003180	(0.139724, 0.229529)
50000	0.234627	0.256815	0.003137	(0.139724, 0.229529)
70000	0.234627	0.256479	0.003134	(0.139724, 0.229529)
90000	0.234627	0.256637	0.003164	(0.139724, 0.229529)

Sample Size n =20				
N	$F_{chp}(T)$	$\hat{F}_{chp}(T)$	M S E	Confidence-Intervals (95%)
10000	0.234627	0.253252	0.002205	(0.152439, 0.216815)
30000	0.234627	0.252979	0.002249	(0.152439, 0.216815)
50000	0.234627	0.253103	0.002217	(0.152439, 0.216815)
70000	0.234627	0.253053	0.002256	(0.152439, 0.216815)
90000	0.234627	0.253206	0.002243	(0.152439, 0.216815)

Sample Size n =25				
N	$F_{chp}(T)$	$\hat{F}_{chp}(T)$	M S E	Confidence-Intervals (95%)
10000	0.234627	0.251336	0.001781	(0.161116, 0.208138)
30000	0.234627	0.251171	0.001724	(0.161116, 0.208138)
50000	0.234627	0.250906	0.001729	(0.161116, 0.208138)
70000	0.234627	0.251138	0.001758	(0.161116, 0.208138)
90000	0.234627	0.250838	0.001718	(0.161116, 0.208138)

Sample Size n =30				
N	$F_{chp}(T)$	$\hat{F}_{chp}(T)$	M S E	Confidence-Intervals (95%)
10000	0.234627	0.249207	0.001348	(0.167521, 0.201733)
30000	0.234627	0.249918	0.001432	(0.167521, 0.201733)
50000	0.234627	0.249556	0.001408	(0.167521, 0.201733)
70000	0.234627	0.249573	0.001413	(0.167521, 0.201733)
90000	0.234627	0.249798	0.001426	(0.167521, 0.201733)

Conclusions

In the present research work, it is tried to evaluate estimation approach which could give formal estimation procedure of the reliability measures with specific reference to intrinsic, CCS failures as well as human errors. Estimation of Reliability measures such as frequency of failures for the two identical component system in the presence of the intrinsic, CCS as well as human errors. The estimates are proposed for both series and parallel systems. Therefore the M L estimation approach for obtaining estimates of the reliability measures like frequency of failures for various configuration with samples of size as low as n = 5 is reasonably appropriate as the MSE is as low as 0.001. Thus the research investigation identifies that M L estimation approach is satisfactory as evidence by empirical means in the absence of analytical approach.

References

[1] Billinton, R & Allan, R. N. 1983. Reliability Evaluation of Engineering Systems; Concepts and Techniques, Plenum Press, New York.

[2] Dhillon, B. S. 1978. On common cause failures – Bibliography, Microelectronics and Reliability, Vol. 18, No. 6, pp. 533-534.

[3] Dhillon, B. S. 1989. Stochastic analysis of a parallel system with common cause failures and critical human errors, Microelectronics and Reliability, Vol. 29, No. 4, pp. 627-637.

[4] Reddy, Y. R. 2003. Reliability analysis for two unit non-identical system with CCS failures, Ph.D thesis, S.K.University, Anantapur.

[5] Sagar, G. Y. 2007. Markovian approach to system reliability measures with common cause shock failures and human errors, Ph. D theis, S. K. University, Anantapur.