Available online at www.elixirpublishers.com (Elixir International Journal)

Discrete Mathematics

Elixir Dis. Math. 71 (2014) 25158-25159

ISSN: 2229-712X

Fuzzy generalized super continuous mappings

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ARTICLE INFO

Article history:

Received: 9 April 2014;

Received in revised form:

12 June 2014;

Accepted: 26 June 2014;

Keywords

Fuzzy super closure ,

Fuzzy super interior,

Fuzzy super open set,

Fuzzy super closed set,

Fuzzy continuity,

Fuzzy super continuity.

ABSTRACT

In this paper we introduced the concepts of fuzzy generalized super continuous mappings and explore some of its basic properties in fuzzy topological spaces

Introduction

1. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta (y) = \beta$ for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$ .A $\leq$ B if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1 [5]:- Let (X,τ) fuzzy topological space and $A\subseteq X$ then

1. Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. Fuzzy Super interior $sint(A) =\{x\in X:cl(U)\leq A\neq\phi\}$

Definition 1.2[5]: -A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) Fuzzy super closed if $scl(A) \leq A$.

(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$, then $A \cup B$ is fuzzy super closed.

Remark 1.3[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy super closed.

Definition 1.3[3, 8,9,10, 11]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:

1. Fuzzy g- super closed if $cl(A) \leq G$ whenever $A \leq G$ and G is super open.
2. Fuzzy g- super open if its complement 1-A is fuzzy g- super closed.

Definition1.4. [3,8,9,10,11]:- A fuzzy set A of (X, τ) is called:

(1) Fuzzy generalized super closed (briefly, Fg-super closed) if $cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy super open set in X;

(2) Generalized fuzzy super open (briefly, gFs- super open) if its compliment is fuzzy super closed.

Definition1.5. [3,8,9,10, 11]:- A fuzzy point $x_p\in A$ is said to be quasi-coincident with the fuzzy set A denoted by x_pqA iff $p + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by A_qB iff there exists $x \in X$ such that $A(x) + B(x) > 1$. If A and B are not quasi-coincident then we write A_qB. Note that $A \leq B$, $Aq(1-B)$.

2. Fuzzy g-Super Continuous Mappings

Definition 2.1: A mapping f: $(X,\tau) \to(Y,\sigma)$ is said to be fuzzy g-super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy g- super closed in X.

Theorem 2.1:A mapping f: $(X,\tau) \to(Y, \sigma)$ is fuzzy g- super continuous if and only if the inverse image of every fuzzy super open set of Y is fuzzy g- super open in .

Proof: It is obvious because $f^{-1}(1 - U) =1 - f^{-1}(U)$ for every fuzzy set U of Y.

Remark 2.1: Every fuzzy super continuous mapping is fuzzy g- super continuous, but the converse may not be true. For,

Example 2.1: Let X = {a,b}, Y={x,y} and the fuzzy sets $U \subseteq X$, $V \subseteq Y$ defined as follows

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U(a) = 0.5 U(b) = 0.7, V(x) = 0.3 V(y) = 0.2,

Let $\tau = \{0,U,1\}$ and $\sigma = \{0,V,1\}$ be topologies on X and Y respectively. Then the mapping f: $(X,\tau) \to (Y, \sigma)$ defined by f(a) = x and f(b) =y is fuzzy. g- Super continuous but not fuzzy super continuous.

Theorem 2.2: If f: $(X,\tau) \to (Y, \sigma)$ is fuzzy g- super continuous then for each fuzzy point x_β of X and each fuzzy super open set $f(x_\beta) \in V$ there exists a fuzzy g- super open set U such that $x_\beta \in U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V be a fuzzy super open set such that $f(x_\beta) \in V$ put $U = f^{-1}(V)$ then by hypothesis U is a fuzzy g- super open set of X such that $x_\beta \in U$ and $f(U) = (f^{-1}(V)) \leq V$.

Theorem 2.3: If f: $(X,\tau) \to (Y,\sigma)$ is fuzzy g- super continuous, then for each fuzzy point $x_\beta \in X$ and each fuzzy super open set V of Y such that $f(x_\beta)q$ V, there exists a fuzzy g- super open set U of X such that $x_\beta qU$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V be a fuzzy super open set such that $f(x_\beta)qV$. Put $U = f^{-1}(V)$.Then by hypothesis U is a fuzzy g- super open set of X such that x_β qU and $f(U) = f(f^{-1}(V)) \leq V$.

Definition 2.2: Let (X,τ) be a fuzzy topological space. The generalized super closure of a fuzzy set A of X denoted by gscl(A) is defined as follows:

gscl(A) = inf {B: B $\geq$ A, B is fuzzy g- super closed set of (X,τ))}

Remark 2.1: It is clear that, A $\leq$ gscl(A) $\leq$sclA) for any fuzzy set A of X.

Theorem 2.4: If f: $(X,\tau) \to (Y,\sigma)$ is fuzzy g- super continuous, then f(gscl(A)) $\leq$scl(f(A)) for every fuzzy set A of X.

Proof: Let A be a fuzzy set of X. Then scl(f(A)) is a fuzzy super closed set of Y. Since f is fuzzy g- super continuous f^{-1}(sclf(A)) is fuzzy g- super closed in X. Clearly A $\leq f^{-1}$(scl(f(A))). Therefore gscl(A) $\leq$ gscl(f^{-1} (scl(f(A)))) = f^{-1} (scl(f(A))). Hence f(gscl(A))$\leq$ scl(f(A)).

Remark 2.2: The converse of theorem 2.4 may not be true. For

Example 2.1: Let X={a,b,c}, Y={x,y,z} and the fuzzy set U and V are defined as U(a) = 1, U(b) = 0, U(c) = 0 ,V(x) = 1, V(y) = 0, V(z) = 1 ,Let $\tau = \{0,U,1\}$ and $\sigma = \{0,V,I\}$ be fuzzy topologies on X and Y respectively and f: $(X,\tau) \to (Y,\sigma)$ be a mapping defined by f(a)= y, f(b) = x, f(c) = z. Then f(gscl(A)) $\leq$ scl(f(A)) holds for every fuzzy set A of X, but I is not fuzzy g- super continuous.

Definition 2.2: A fuzzy topological space (X,τ) is said to be fuzzy $T_{1/2}$ if every fuzzy g- super closed set in X is fuzzy super closed in X.

Theorem 2.5: A mapping f from a fuzzy $T_{1/2}$ space (X,τ) to a fuzzy topological space (Y, σ) is fuzzy super continuous if and only if it is fuzzy g- super continuous.

Proof: Obvious.

Remark 2.4: The composition of two fuzzy g- super continuous mappings may not be fuzzy g- super continuous. For,

Example 2.3 : Let X = {a, b}, Y = {x, y} and Z = {p, q} and the fuzzy sets $U \subseteq X, V \subseteq Y$ and $W \subseteq Z$ are defined as follows U(a) = 0.5,. U(b) = 0.7,V V(x) = 0.3, V(y) = 0.2,,W(p) = 0.6, W(q) = 0.4,Let $\tau = \{0,U,l\}$, $\sigma = \{0,V,I\}$ and $\eta = \{0,W,l\}$ be fuzzy topologies on X, Y and Z respectively. Let the mapping f: $(X,\tau) \to (Y,\sigma)$ be defined by f(a)=x, f(b)=y and the mapping g : $(Y,\sigma) \to (Z,\eta)$ be defined by g(x)=p and g(y)=q. Then f and g are fuzzy g- super continuous but gof is not fuzzy g- super continuous. However,

Theorem 2.6: If f: $(X,\tau) \to (Y,\sigma)$is fuzzy g- super continuous and g:$(Y,\sigma) \to (Z,\eta)$ is fuzzy super continuous. Then gof: $(X,\tau) \to (Z,\eta)$ is fuzzy g- super continuous.

Proof: If A is fuzzy closed in Z, then $g^{-1}(A)$ is fuzzy super closed in Y because g is fuzzy super continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy g- super closed in X. Hence gof is fuzzy g- super continuous.

Theorem 2.7: If f: $(X,\tau) \to (Y,\sigma)$ and g : (Y,σ) — (Z,η) are two fuzzy g- super continuous mappings and (Y,σ) is fuzzy $T_{1/2}$ then gof: $(X,\tau) \to (Z,\eta)$ is fuzzy g- super continuous.

Proof: Obvious.

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