Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

ISSN: 2229-712X

Elixir Appl. Math. 71 (2014) 25116-25124

Cycle related mean graphs

B. Gayathri[1,*] and R. Gopi[2]

[1]Department of Mathematics, Periyar E.V.R. College, Tiruchirappalli – 620 023, India.

[2]Department of Mathematics, M.A.R. College of Engineering and Technology, Viralimalai – 621 316, India.

ARTICLE INFO

Article history:

Received: 2 April 2014;

Received in revised form:

12 June 2014;

Accepted: 21 June 2014;

Keywords

Mean labeling,

Mean graph.

ABSTRACT

Mean labeling of graphs was discussed in [24-28].Different kinds of mean labeling were discussed in [17]. In this paper, we have proved the Cycle related mean graphs.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [20]. The symbols *V* (*G*) and *E* (*G*) will denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications [2-4].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [23].

Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [5].

Mean labeling of graphs was discussed in [24-28].

Vaidya [32-35] and et al. have investigated several new families of mean graphs. Nagarajan [31] and et al. have found some new results on mean graphs.

Ponraj, Jayanthi and Ramya extended the notion of mean labeling to super mean labeling in [21].

Gayathri and Tamilselvi [18-19, 30] extended super mean labeling to *k*-super mean, (*k*, *d)*-super mean, *k*-super edge mean and (*k*, *d)*-super edge mean labeling. Manickam and Marudai [22] introduced the concept of odd mean graph.

Gayathri and Amuthavalli [1, 6-8] extended this concept to *k*-odd mean and (*k*, *d*)-odd mean graphs. Gayathri and Gopi[9-17] extended this concept to *k*-even mean and (*k*, *d*)-Even mean graphs.

Sundaram and Ponraj [29] introduced the mean number of a graph and obtained several results.

In this paper, we have proved the Necessary Condition for mean labeling.

2. Main Results

Definition 2.2.1

A **double triangular snake** is obtained from a path $v_1, v_2, ..., v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, ..., n - 1$ and to a new vertex u_i for $i = 1, 2, ..., n - 1$.

Theorem 2.2.2

The double triangular snake is a mean graph.

Tele:

E-mail addresses: maduraigayathri@gmail.com

Proof

Let $\{v_i, 1 \le i \le n, u_i, 1 \le i \le n-1$ and $w_i, 1 \le i \le n-1\}$ be the vertices and $\{e_i, 1 \le i \le n-1, a_i, 1 \le i \le 2n-2$ and $b_i, 1 \le i \le 2n-2\}$ be the edges of double triangular snake which are denoted as in Figure 2.1.

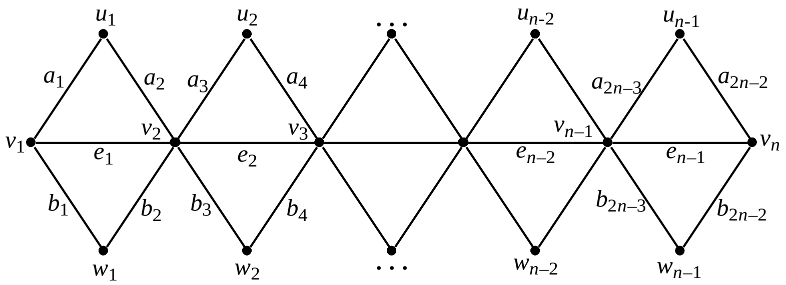

Figure 2.1: Ordinary labeling of double triangular snake

First we label the vertices as follows:

Define $f : V \to \{0, 1, 2, ..., q\}$ by

For $1 \le i \le n$; $f(v_i) = 5(i-1)$

For $1 \le i \le n-1$; $f(u_i) = 5i-3$; $f(w_i) = 5i-1$

Then the induced edge labels are:

For $1 \le i \le n-1$; $f^*(e_i) = 5i-2$

For $1 \le i \le 2n-2$,

$$f^*(a_i) = \begin{cases} \frac{5i-3}{2} & i \text{ is odd} \\ \frac{5i-2}{2} & i \text{ is even} \end{cases} \quad ; \quad f^*(b_i) = \begin{cases} \frac{5i-1}{2} & i \text{ is odd} \\ \frac{5i}{2} & i \text{ is even} \end{cases}$$

The above defined function f provides mean labeling of the double triangular snake.

Mean labeling of double triangular snake is given in Figure 2.2.

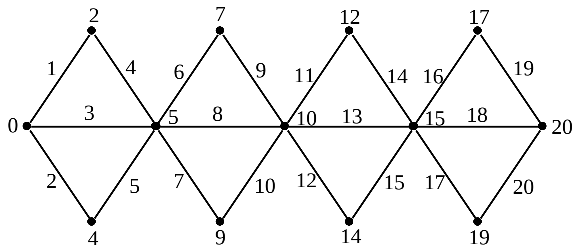

Figure 2.2: Mean labeling of double triangular snake

Definition 2.2.3

A **double quadrilateral snake** is obtained from a path $v_1, v_2, ..., v_n$ by joining each of the vertices v_i and v_{i+1} ($i = 1, 2, ..., n-1$) to new vertices u_i and u_i' and to the new vertices w_i and w_i' respectively and adding an edge between each pair of vertices (u_i, w_i) and (u_i', w_i').

Theorem 2.2.4

The double quadrilateral snake is a mean graph.

Proof

Let $\{v_i, 1 \le i \le n, u_i, 1 \le i \le n-1, w_i, 1 \le i \le n-1, u_i^{'}, 1 \le i \le n-1$ and $w_i^{'}, 1 \le i \le n-1\}$ be the vertices and $\{e_i, 1 \le i \le n-1, a_i, 1 \le i \le 2n-2, b_i, 1 \le i \le 2n-2, c_i, 1 \le i \le n-1$ and $d_i, 1 \le i \le n-1\}$ be the edges of double quadrilateral snake which are denoted as in Figure 2. 3.

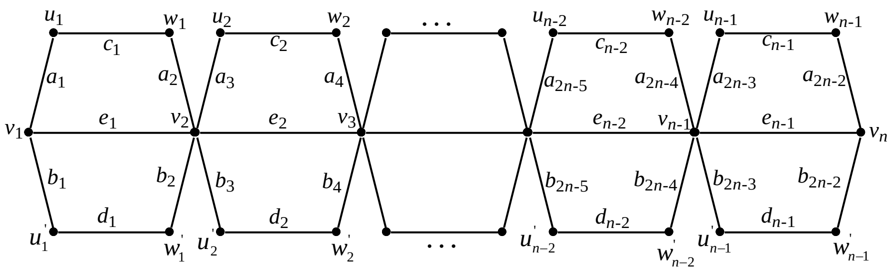

Figure 2. 3: Ordinary labeling of double quadrilateral snake

First we label the vertices as follows:

Define $f : V \rightarrow \{0, 1, 2, ..., q\}$ by

$f(v_1) = 1$; For $2 \le i \le n$, $f(v_i) = 7i - 8$,

$f(u_1) = 2$; For $2 \le i \le n-1$, $f(u_i) = 7i - 4$,

$f(w_1) = 7$; For $2 \le i \le n-1$, $f(w_i) = 7i - 3$,

$f(u_1^{'}) = 0$; For $2 \le i \le n-1$, $f(u_i^{'}) = 7i - 2$

$f(w_1^{'}) = 5$; For $2 \le i \le n-1$, $f(w_i^{'}) = 7i$.

Then the induced edge labels are:

$f^*(e_1) = 4$; For $2 \le i \le n-1$, $f^*(e_i) = 7i - 4$

$f^*(a_1) = 2$; $f^*(a_2) = 7$

For $3 \le i \le 2n-2$,

$$f^*(a_i) = \begin{cases} \dfrac{7i-5}{2} & i \text{ is odd} \\ \dfrac{7i-4}{2} & i \text{ is even} \end{cases}$$

$f^*(b_1) = 1$; $f^*(b_2) = 6$

For $3 \le i \le 2n-2$,

$$f^*(b_i) = \begin{cases} \dfrac{7i-3}{2} & i \text{ is odd} \\ \dfrac{7i}{2} & i \text{ is even} \end{cases}$$

$f^*(c_1) = 5$; For $2 \le i \le n-1$, $f^*(c_i) = 7i - 3$

$f^*(d_1) = 3$; For $2 \le i \le n-1$, $f^*(d_i) = 7i - 1$

The above defined function f provides mean labeling of the double quadrilateral snake.

Mean labeling of double quadrilateral snake is given in Figure 2. 4.

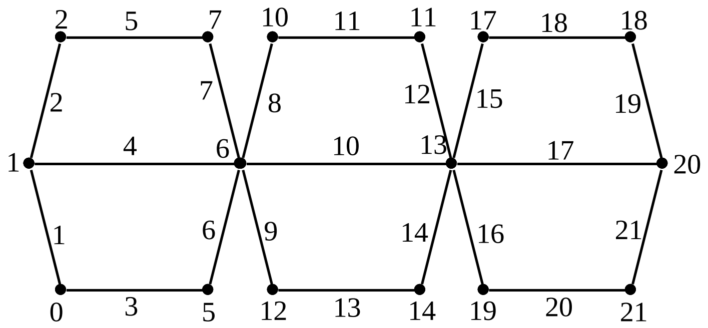

Figure 2.4: Mean labeling of double quadrilateral snake

Theorem 2.2.5

The graph $K_{2,n} \cup \{e\}$ is a mean graph.

Proof

Let $\{u, v, u_i, 1 \le i \le n\}$ be the vertices and $\{e, e_i, 1 \le i \le 2n\}$ be the edges which are denoted as in Figure 2.5.

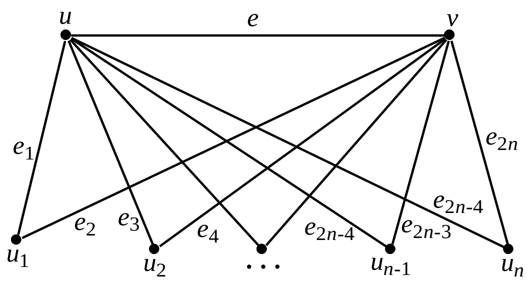

Figure 2.5: Ordinary labeling of $K_{2,n} \cup \{e\}$

First we label the vertices as follows:

Define $f : V \to \{0, 1, 2, ..., q\}$ by

$$f(u) = 0\ ;\ f(v) = 2n + 1$$

For $1 \le i \le n$, $f(u_i) = 2i$

Then the induced edge labels are:

$$f^*(e) = n + 1$$

For $1 \le i \le 2n$,

$$f^*(e_i) = \begin{cases} \dfrac{i+1}{2} & i \text{ is odd} \\ \dfrac{i+2n+2}{2} & i \text{ is even} \end{cases}$$

The above defined function f provides mean labeling of the graph $K_{2,n} \cup \{e\}$.

Mean labeling of the graph $K_{2,5} \cup \{e\}$ is given in Figure 2.6.

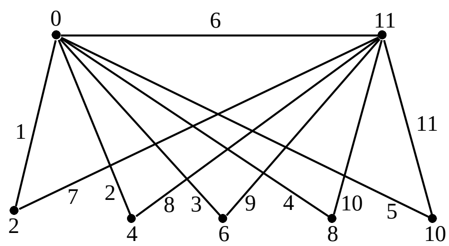

Figure 2.6: Mean labeling of $K_{2,5} \cup \{e\}$

Definition 2.2.6

The **generalized antiprism** A_m^n is obtained from $C_m \times P_n$ by inserting the edges $\{v_{i,j+1}, v_{i+1,j}\}$ for $1 \le i \le m$ and $1 \le j \le n-1$ where the subscripts are taken modulo m.

Theorem 2.2.7

The generalized antiprism A_3^n $(n \ge 3)$ is a mean graph.

Proof

Let $\{u_{ij}, 1 \le i \le 3, 1 \le j \le n\}$ be the vertices and $\left\{a_{ij} : 1 \le i \le 2, 1 \le j \le n,\ b_{ij}, 1 \le i \le 2, 1 \le j \le n-1,\ c_{ij}, 1 \le i \le 3, 1 \le j \le n-1,\ d_j, 1 \le j \le n\right\}$ be the edges which are denoted as in Figure 2.7.

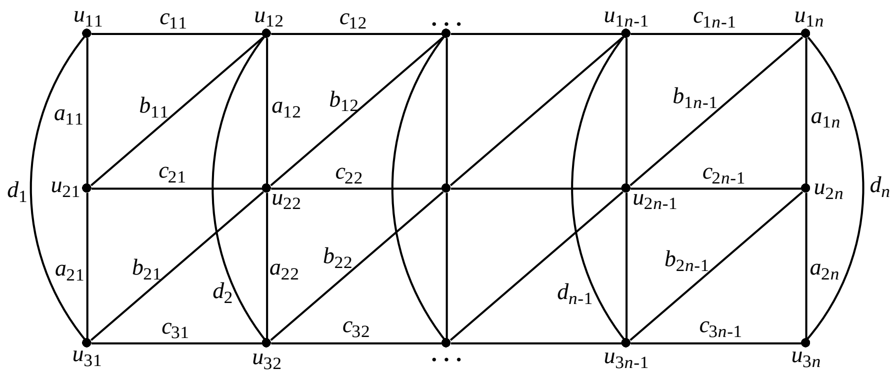

Figure 2.7: Ordinary labeling of A_3^n

First we label the vertices as follows:

Define $f : V \to \{0, 1, 2, ..., q\}$ by

For $1 \le i \le n-1$, $f(u_{1j}) = 8j - 4$; $f(u_{1n}) = 8n - 5$

For $2 \le i \le 3$, $1 \le j \le n$, $f(u_{ij}) = 8j - 2i - 2$

Then the induced edge labels are:

For $1 \le i \le 2$, $1 \le j \le n$, $f^*(a_{ij}) = 8j - 2i - 3$

For $1 \le i \le 2$, $1 \le j \le n-1$, $f^*(b_{ij}) = 8j - 2i + 1$

For $1 \le i \le 3$, $1 \le j \le n-1$, $f^*(c_{ij}) = 8j - 2i + 2$

For $1 \le j \le n$, $f^*(d_j) = 8j - 6$

The above defined function f provides mean labeling of the graph A_3^n.

Mean labeling of the graph A_3^5 is given in Figure 2.8.

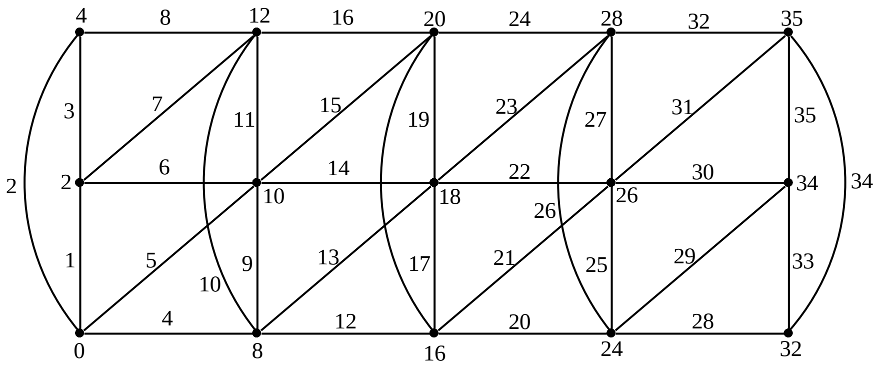

Figure 2.8: Mean labeling of A_3^5

Definition 2.2.8

The **graph PC_n** ($n \geq 5$) is obtained from $C_n = v_1v_2, ..., v_nv_1$ by adding the chords v_i and v_{n-i+2} for $2 \leq i \leq l$ where $l = \frac{n}{2}$ or $\frac{n-1}{2}$ when n is even or odd.

Theorem 2.2.9

The graph PC_n ($n \geq 5$) is a mean graph.

Proof

Case (i): *n* is even

Let $\{v_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n, e_i^{'}, 1 \leq i \leq \frac{n-2}{2}\}$ be the edges which are denoted as in Figure 2.9.

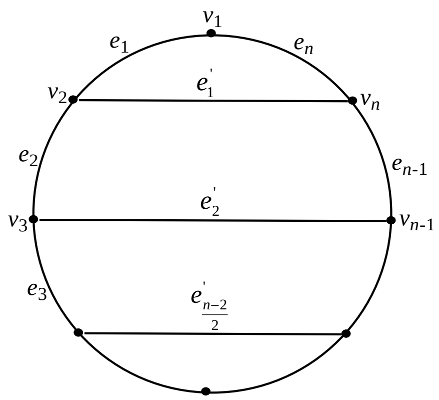

Figure 2.9: Ordinary labeling of PC_n

First we label the vertices as follows:

Define $f : V \rightarrow \{0, 1, 2, ..., q\}$ by

$f(v_1) = 0$

For $2 \leq i \leq \frac{n+2}{2}$, $f(v_i) = 3i - 4$; $f\left(v_{\frac{n+4}{2}}\right) = \frac{3n-4}{2}$

For $\frac{n+6}{2} \leq i \leq n$, $f(v_i) = 3(n - i + 1)$

Then the induced edge labels are:

For $1 \leq i \leq \frac{n}{2}$, $f^*(e_i) = 3i - 2$

For $\frac{n+2}{2} \le i \le n$, $f^*(e_i) = 3(n-i)+2$; For $1 \le i \le \frac{n-2}{2}$, $f^*(e_i^{'}) = 3i$

Case (ii): *n* is odd

Let $\{v_i, 1 \le i \le n\}$ be the vertices and $\{e_i, 1 \le i \le n, e_i^{'}, 1 \le i \le \frac{n-3}{2}\}$ be the edges which are denoted as in Figure 2.10.

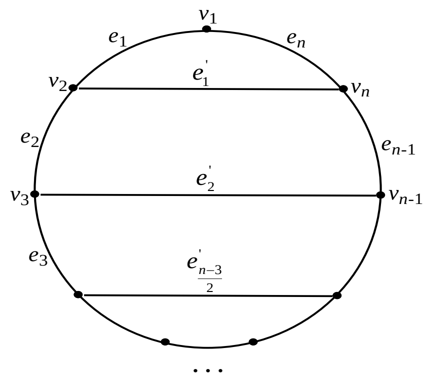

Figure 2.10: Ordinary labeling of PC_n

First we label the vertices as follows:

Define $f : V \to \{0, 1, 2, ..., q\}$ by

$f(v_1) = 0$

For $2 \le i \le \frac{n+1}{2}$, $f(v_i) = 3i-4$

For $\frac{n+3}{2} \le i \le n$, $f(v_i) = 3(n-i+1)$

Then the induced edge labels are:

For $1 \le i \le \frac{n-1}{2}$, $f^*(e_i) = 3i-2$; $f^*\left(e_{\frac{n+1}{2}}\right) = \frac{3(n-1)}{2}$

For $\frac{n+3}{2} \le i \le n$, $f^*(e_i) = 3(n-i)+2$

For $1 \le i \le \frac{n-3}{2}$, $f^*(e_i^{'}) = 3i$

The above defined function *f* provides mean labeling of the graph PC_n.

Mean labeling of the graph PC_7 and PC_{10} are given in Figure 2.11 and Figure 2.12 respectively.

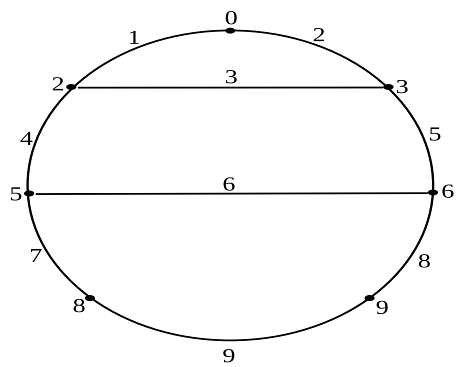

Figure 2.11: Mean labeling of PC_7

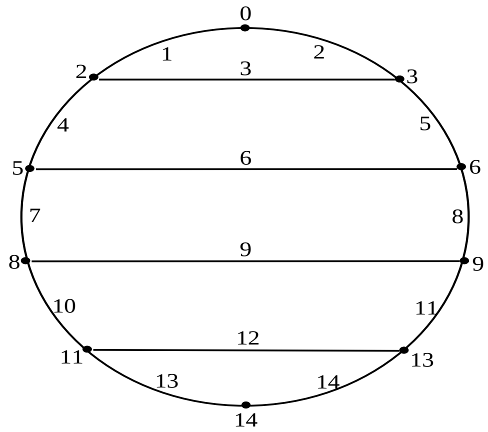

Figure 2.12: Mean labeling of PC_{10}

References

1. K. Amuthavalli, Graph labeling and it’s Application’s some generalizations of odd mean labeling, Ph.D. Thesis, Mother Theresa Women’s University, Kodaikanal, July (2010).
2. G.S. Bloom, S.W. Golomb, Applications of numbered undirected graphs, Proc. IEEE, 65 (1977), 562 – 570.
3. G.S. Bloom, S.W. Golomb, Numbered complete graphs unusual rulers and assorted applications, Theory and Applications of Graphs – Lecture notes in Math., Springer – Verlag, New York, 642 (1978), 53- 65.
4. G.S. Bloom, D.F. Hsu, On graceful digraphs and a problem in network addressing, Congressus Numerantium, 35 (1982)91 – 103.
5. J.A. Gallian, A dynamic survey of graph labeling, Electronic Journal of Combinatorics, 18 (2012) # DS6.
6. B. Gayathri and K. Amuthavalli, *k*-odd mean labeling of crown graphs, *International Journal of Mathematics and Computer Science*, **2(3)** (2007) 253-259.
7. B. Gayathri and K. Amuthavalli, (*k, d)*-odd mean labeling of some graphs, *Bulletin of Pure and Applied Sciences*, **26E(2)** (2007) 263-267.
8. B. Gayathri and K. Amuthavalli, *k*-odd mean labeling of $\langle K_{1,n}, K_{1,m} \rangle$, *Acta Ciencia Indica*, **34(2)** (2008) 827-834.
9. B. Gayathri, R. Gopi, *k*-even mean labeling of $D_{m,n}$ @ C_n, International Journal of Engineering Science, Advanced Computing and Bio-Technology, Vol. 1, No. 3, July – September 2010, pp.137 – 145.
10. B. Gayathri, R. Gopi, *k*-even mean labeling of $D_{m,n}$, Acta Ciencia Indica, Vol.XXXVII, No. 2, 291 – 300, 2011.
11. B. Gayathri and R. Gopi, *k*-even mean labeling of $C_n \cup P_m$, Elixir International Journal of Applied Sciences, No. 36, 2011, P. No. 3102-3105.
12. B. Gayathri and R. Gopi, *k*-even mean labeling of $T_{n,m,t}$, International Journal of Engineering Science, Advanced Computing and Bio-Technology, Vol. 2, No. 2, April-June 2011, P. No. 77-87.
13. B. Gayathri, R. Gopi, International Conference on Mathematics and Computer Science, *k*-even mean labeling of $C_m \odot \overline{K_n}$, Loyola College, Chennai, January 7-8, 2011, Proc. Page No:519-524.
14. B. Gayathri, R. Gopi, (*k*, *d*)-even mean labeling of $P_m \odot nK_1$, International Journal of Mathematics and soft computing, Vol. 1, No. 1, 17 – 23, August 2011.
15. B. Gayathri, R. Gopi, *k*-even mean labeling of some graphs, Heber International Conference on Applications of Mathematics and Statistics, Bishop Heber College (Autonomous), January 5-7, 2012 Proc. Page No:420-425.
16. B. Gayathri, R. Gopi, International Conference on Mathematics in Engineering and Business Management, *k*-even mean labeling of some trees, Stella Maris College (Autonomous), Chennai, March 9-10, 2012, Proc. Page No:99-102.
17. R. Gopi, A Study on Different kinds of Mean Labeling, Ph.D. Thesis, Bharathidasan University, Trichy, February (2013).

18. B. Gayathri, M. Tamilselvi and M. Duraisamy, (*k*, *d*)-super mean labeling of some graphs, *International Journal of Mathematics and Computer Science*, **2(3)** (2007) 245-252.

19. B. Gayathri, M. Tamilselvi, *k*-Super mean labeling of some trees and cycle related graphs, *Bulletin of Pure and Applied Sciences*, Volume **26E(2)** (2007) 303-311.

20. F. Harary Graph Theory, Addision – Wesley, Reading Massachusetts, 1972.

21. R. Ponraj, Jayanthi and D. Ramya, On super mean graphs of order ≤ 5, *Bulletin of Pure and Applied Sciences*, **25E** (2006) 143-148.

22. K. Manickam and M. Marudai, Odd mean labeling of graphs, *Bulletin of Pure and Applied Sciences*, **25E(1)** (2006) 149-153.

23. Rosa, On certain valuations of the vertices of a graph Theory of Graphs (Internet Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Duhod, Paris (1967) 349 – 355.

24. S. Somasundaram, R. Ponraj, Mean labeling of graphs, National Academy Science Letter, 26 (7 – 8) (2003), 10 – 13.

25. S. Somasundaram and R. Ponraj, Mean labeling of graphs, *Natl. Acad. Sci. Let.*, **26** (2003), 10-13.

26. S. Somasundaram and R. Ponraj, Non-existence of mean labeling for a wheel, *Bull. Pure and Appl. Sciences (Mathematics & Statistics)*, **22E** (2003) 103-111.

27. S. Somasundaram and R. Ponraj, Some results on mean graphs, *Pure and Applied Mathematical Sciences*, **58** (2003) 29-35.

28. S. Somasundaram and R. Ponraj, On mean graphs of order ≤ 5, *J. Decision and Mathematical Sciences*, **9** (2004) 47-58.

29. M. Sundaram and R. Ponraj, Mean number of a graph, *Pure and Applied Mathematica Sciences*, **LXV(1-2)** (2007), 93-102.

30. M. Tamilselvi, A study in Graph Theory-Generalization of super mean labeling, Ph.D. Thesis, Vinayaka Mission University, Salem, August (2011).

31. R. Vasuki and A. Nagarajan, Further results on mean graphs, *Scientia Magna*, **6(2)** (2010) 26-39.

32. S.K. Vaidya and Lekha Bijukumar, Mean labeling for some new families of graphs, *Journal of Pure and Applied Sciences*, **1(18)** (2010) 115-116.

33. S.K. Vaidya and Lekha Bijukumar, Some new families of mean graphs, *Journal of Mathematics Research*, **2(3)** (2010) 169-176.

34. S.K. Vaidya and Lekha Bijukumar, New mean graphs, *International J. Math. Combin.*, **3** (2011) 107-113.

35. S.K. Vaidya and Kanani, Some new mean graphs, *International Journal of Information Science and Computer Mathematics*, **1(1)** (2010) 73-80.