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Modeling of Zero Injection Busses in PMU Placement for Power System Observability

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ABSTRACT

In this paper, an improved model for complete observability of electrical power networks in optimal PMU placement is presented. Here a goal function is introduced based on the Integer Linear Programming (ILP) to find the optimal number and position for PMUs. Hitherto, modeling of zero injection busses constraints had been a challenge due to the intrinsic nonlinearity associated with it. We show that zero injection constraints can also be modeled as linear constraints. Also in this paper, a novel and topology based method is suggested when zero injection busses are considered in power grid. The ILP approach is implemented to solve the models. The models is tested in IEEE 14, 30, 57 and 118 busses systems. The simulation results are confirmed with the schemes concluded from authoritative publications.

Introduction

Phasor Measurement Unit (PMU) based on GPS technique is widely used to monitor the state of a power system. The PMU measurement is received by time sampling based on the same time reference synchronized by the GPS, so it could provide power engineers with immediate and precise measurements. By applying the PMU measurements in different areas in power systems such as state estimation, protection, load shedding, voltage collapses etc., the reliability and stability in power system are expected to be improved [1]. However, due to a high cost of PMUs or nonexistence of communication facilities in certain busses, it is impossible to place a PMU on every bus in the network, either as a stand-alone unit or relay-based function. So an optimal placement of PMUs is required for better power quality.

Before the discussion of PMU placement problem, the basic PMU placement rules should be mentioned. A PMU installed on a certain bus is able to measure the voltage magnitude and phase angle of the local bus and the branch current phasor of all branches emerging from this bus. The voltage magnitude and phase angle of the neighboring bus can be computed using voltage drop equations. Thus the busses monitored by a PMU are directly observable, the neighboring busses connected to the PMU busses are indirectly observable and the other busses which are not associated with the PMU busses are unobservable. PMUs are capable of providing power engineers with a snapshot of the power system, which reflects the real states in a power system. Through the phasor measurements taken at different locations at the same time, the operators could obtain a more accurate value of angle difference between different places which is useful for voltage stability analysis in a power system. In short, by utilizing PMUs, the reliability and stability in a power system are expected to be improved [2]. In this paper the conditions to find the number and position of PMU is considered. Up to now, various plans have been suggested to find the optimal placement of PMU. In references [3, 4] the authors applied electroplating to find the optimal position of PMUs.

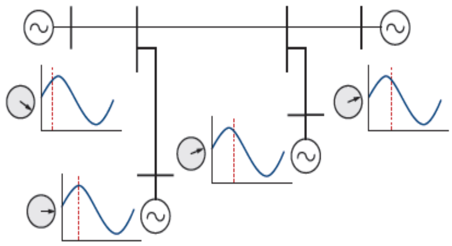

Figure 1. Snapshot of the Power System

This method requires an initial accidental guess. The calculation is a heavy work and does not necessarily result in optimal values. In reference [5] GA (genetic algorithm) is used to find the optimal position of PMU. The advantage of GA is that it gives a number of solutions that gives the companies the chance of choosing one. The shortcoming of this method is that it cannot give the minimum number of PMU to make the system displayable and the optimal number of PMUs should be given first. PSO (Particle Swarm Optimization) and BPSO (Binary Particle Swarm Optimization) are among the other methods used by the researchers [6].

Zero injections busses, which are analogous to transshipment nodes, have the potential to reduce the number of PMUs required for complete system observability. Ref. [7] considers modeling of zero injection constraints in an otherwise ILP framework. In the resulting formulation, observability constraints arising out of zero injection busses turn out to be non-linear. This increases the complexity of the discrete optimization problem.

We show that modeling of zero injection busses in the optimal PMU placement problem can be achieved by using linear constraints and a topology-based method.

In this paper, placement the PMU with the minimum number of them and making them displayable and considering the zero injections busses is done with Integer Linear Programming (ILP) .In this paper, branch and bound (B&B) method is used to

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optimal PMU placement. This method is simple, understandable and applicable with high calculation speed. ILP gives the best total answers for all the systems. This method is simulated on standard IEEE with MATLAB and it is compared with other placement methods for PMU. These comparisons show that this method is a sufficient and acceptable one.

PMU Placement Problem Formulation

A PMU is able to measure the voltage phasor of the installed bus and the current phasors of all the lines connecting to this bus. That is to say, a PMU can make the installed bus and its neighboring busses observable. The objective of placing PMUs in power systems is to determine a minimal set of PMUs such that the whole system is observable.

Now the optimal placement of PMUs can be formulated as a problem of Integer Linear Programming (ILP) [7]:

$$Min \sum_{i=1}^{n} w_i . x_i \qquad (1)$$

$$s.t \;\; y = A_{n\times n} X_{n\times 1} \geq b_{n\times 1}$$

Where n is total number of busses in the network and w is the cost function for the installed PMUs or the weight matrix for the busses that can vary based on the importance of every bus. w is normally equal to unit matrix. In this equation, x, A and b are defined as bellow:

$$A_{n\times n}(i,j) = \begin{cases} 1 & i=j \\ 1 & \text{if buses } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{n\times 1}(i) = \begin{cases} 1 & \text{if PMU installed in bus } i \\ 0 & \text{otherwise} \end{cases}$$

$$b_{n\times 1} = [1 \;\; 1 \;\; 1 \;\; \ldots \;\; 1 \;\; 1]^T$$

The inequality in function (1) is used for complete monitoring the system. The ith row in Ax matrix is the number of times that the ith bus is monitored which should be at least one.

Consider the IEEE-14 bus system shown in Figure 2. Let x_i be a binary decision variable associated with the bus i. Variable x_i is set to one if a PMU is installed at bus i, else it is set to zero.

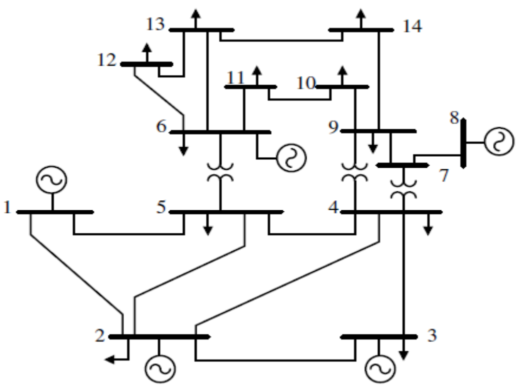

Figure 2. IEEE 14 Bus Test System

(seventh bus is a zero injection bus) Now if we arrange the equations (1) for IEEE-14 bus system in matrix form the results can be formulated as function (2).

The A matrix can be directly obtained by making the admittances matrix a binary one.

Solving the function (2) in ILP solver leads to an optimal number of 4 PMUs for making the system observable, with location of PMUs being on busses 2, 6, 7 and 9.

$$\min \sum_{i=1}^{14} w_i . x_i \qquad (2)$$

$$\underbrace{\begin{bmatrix} 1&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&1&0&1&0&1&0&0&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0&1&1&1&0 \\ 0&0&0&1&0&0&1&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&1&0&1&1&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \end{bmatrix}}_{x} \geq \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{b}$$

Modeling of Zero Injection Busses

Zero injection bus is a bus such that no current or power is injected into the system through this bus, which means no active or reactive load is associated with this bus. Figure 3 shows a zero injection bus in a network [2].

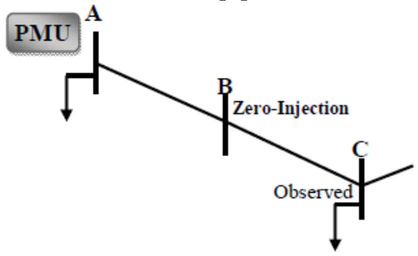

Figure 3. Zero Injection Case

In the network above, bus A is a PMU bus, bus B is a zero-injection bus and bus C is a PQ bus in power system. Bus A is directly measured by the PMU installed at bus A, so bus A is directly observable; bus B which is connected to the PMU bus (bus A) is as well an observable bus by computing the voltage information with the voltage drop equations. Because bus B is a zero-injection bus, the current flowing through line A-B equals the current flowing through line B-C. Knowing the voltage information on bus B and the current phasor on line B-C, the voltage data on bus C can be calculated using Ohm's law. So in conclusion, bus A, B and C are all observable when bus B happens to be a zero injection bus. In contrast, if bus B is not a zero-injection bus, the assumption that the current phasor on line A-B equals that on line B-C will be invalid and thus the voltage information on bus C cannot be calculated without the current information on line B-C.

In this case, only bus A and B are observable when bus B is not a zero injection bus. In short, considering the influence of zero injection busses in a power system, the number of observed busses is expected to be increased and the optimal number of PMUs required will be further minimized.

To understand this subject, 4 busses network that is shown in figure 4 is investigated [8].

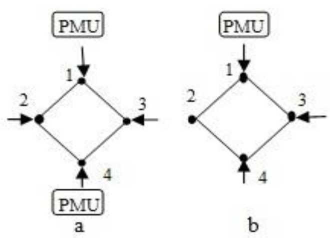

Figure 4. PMU placement for a 4 busses system

Figure 4(a) depicts the system with injections in all the busses. Figure 4(b) shows a similar system with zero injection in bus 2 and injections in bus 1, 3 and 4. For system in figure 4(a) it can be easily known that at least 2 PMUs are required for full system monitoring. These two PMUs can be installed on any of these four busses. But in figure 4(b), if bus number 2 is considered as the zero injection bus, the current in branch 4_2 is equal to the current in branch 2_1 ($I_{24}=I_{12}$). Accordingly, knowing the line parameters, voltage can be calculated in bus 4 ($V_4=V_2-I_{12}Z_{24}$). Finally there is no necessity to install a separate PMU in bus number 4. Accordingly, investigating the zero injections busses helps in reduction of the number of required PMUs to monitor the system.

Modeling of Zero Injection Busses in the ILP Framework

For modeling of zero injection busses in the ILP framework a new variable that is called U_i is defined to confirm the monitoring ability for bus *i*. if $U_i=1$, it means that the $_i$ bus can be monitored and $U_i=0$ means that *I* bus cannot be monitored. The set of busses that are connected to zero injection bus are called A_i and the A_i with zero injection bus are called B_i ($B_i = A_i \cup \{i\}$). Any zero injections cause a new constraint that this condition is formulated in ILP as function (3) [8].

$$\begin{aligned}
&Objective:\ \min \sum_{i=1}^{n} w_i . x_i \\
&Subject\ to: \\
&\qquad Ax \geq u \\
&and \qquad\qquad\qquad\qquad\qquad\qquad (3)\\
&\qquad u_j = 1 \ \ \forall j \notin B_1 \cup B_2 ... \cup B_z \\
&and \\
&\qquad \sum_{k \in B_i} u_k \geq |A_i| \ \ \forall i \in Z \\
&or \\
&\qquad a_i u \geq |A_i|
\end{aligned}$$

Here, $B_i = A_i \cup \{i\}$ and $|A_i|$ is the size of A_i set.

Here we consider IEEE 14-bus system in figure 2, This system has bus 7 as zero injection bus. Thus, Z={7}, A_7={4,8,9} and B_7={4,7,8,9}. Thus, Additional constraints on zero injection busses modeling in the ILP will be as follows:

$$u_4 + u_7 + u_8 + u_9 \geq 3$$

Solving the above formulation in ILP solver leads to an optimal number of 3 PMUs for making the system observable, with location of PMUs being on busses 2, 6 and 9.

Modeling of Zero Injection Busses by a Topology-Based Method

In a power system network, if bus *i* is a zero injection bus and k busses are connected to bus *i*, then there are k+1 busses including the zero injection bus and all it's neighboring busses. According to Kirchhoff Current Law (KCL), if any k among the k+1 busses are observable, the last one bus is automatically observable or we can call it automatically satisfied bus. Thus, the optimal number of PMUs can be reduced if considering the effect of zero injection busses in a power grid [9].

In the following figure, a zero injection bus *i* is incident to bus 1, 2, 3 and 4.

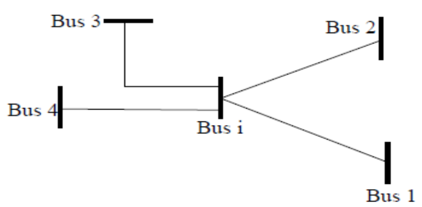

Figure 5. Sub-network Including a Zero Injection Bus

In this paper, auxiliary branches are added into the network in order to reflect the effect of zero injection bus. The auxiliary branches are only effective in solving PMU placement problem and have no partical significance in power flow. Two scenarios are discussed as follows.

Scenario I

Considering this scenario that busses {2, 3, 4, i} are observable and bus 1 is an automatically satisfied bus, auxiliary branches are added between nodes {1, 2}, {1, 3} and {1, 4}. If no PMU is located at zero injection bus I, one of the busses {2, 3, 4} should have a PMU in order to make bus I observable. Due to the auxiliary branches 1-2, 1-3 and 1-4, bus 1 could be reached by a PMU located at bus 2, 3, or 4. If a PMU exists at the zero injection bus i, busses {1, 2, 3, 4} will be reached simultaneosly. In addition, the auxiliary branches will not influence other busses in the external network. The similary process is applicable to the scenario that bus 2, 3 or 4 is an automatically satisfied bus.

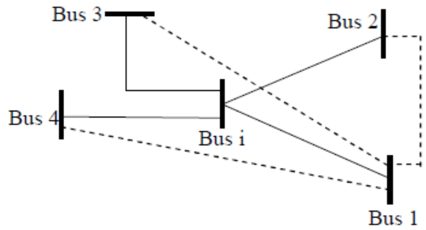

Figure 6. Scenario I*Scenario II*

Considering the scenario that busses {1, 2, 3, 4} are observable and bus I is the automatically satisfied bus, the zero injection bus I is suggested to be removed from the network since it has no correlation with other busses except busses {1, 2, 3, 4}. Removing the zero injection bus is equivalent to adding auxiliary negative branches between busses {i, 1}, {i, 2}, {i, 3} and {i, 4}.

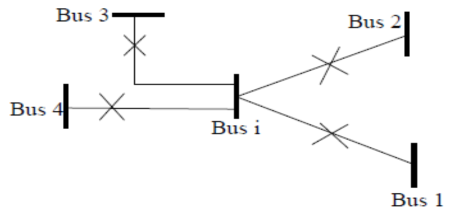

Figure 7. Scenario II

It is concluded that the effect of zero injection bus could be reflected by modifying the network topology and the original problem is transformed to a new PMU placement problem without zero injection busses based on the modified network topology. If the automatically satisfied bus is a neighboring bus of the zero injection bus, add auxiliary positive branches between this bus and any other neighboring busses; if the automatically satisfied bus is the zero injection bus, add auxiliary negative branches between the zero injection bus and it's neighboring busses.

Assuming that in a power grid r busses are zero injection busses and each zero injection bus has n_i (1≤ i ≤r) neighboring busses, there are n_i+1scenarios designating different automatically satisfied bus for zero injection bus i. consequently the total number of scenarios for the entire network is $\prod_{i=1}^{r} n_i$.

The global optimal solution should be the relatively optimal one among the locally optimal solutions in all scenarios. Hence, this proposed model is complete and accurate. The formulations for this approach are shown as function (4).

$$\min \sum_{i=1}^{n} w_i . x_i$$

$$Subject\ to: (A_{n\times n} + C_{n\times n})X_{n\times 1} \geq b_{n\times 1} \quad (4)$$

$$X_{n\times 1} = [x_1 \quad x_2 \quad ... \quad x_n]^T$$

$$x_i \in \{0,1\}$$

The definition of matrix $A_{n\times n}$, vector $b_{n\times 1}$ and vector $X_{n\times 1}$ are the same as the basic model shown in equations (1).

The definition of the auxiliary matrix $C_{n\times n}$ is: if bus *i* is an automatically satisfied bus and bus *i* and bus *j* are incident to the same zero injection bus, $C_{ij}=C_{ji}=1$; if bus *i* is an automatically satisfied bus as well as a zero injection bus, bus *j* is incident to bus *i*, $C_{ij}=C_{ji}=-1$; otherwise, $C_{ij}=C_{ji}=0$.

Case Studies

The integer programming models in this study are solved using the function *bintprog* in MATLAB software package. The proposed models are tested in IEEE 14, 30, 57 and 118 busses systems. Table 2 has the properties of the zero injection busses that are studied. The information related to these busses is obtained from reference [12].

Conclusions

In this paper, the optimal placement of phasor measuring units (PMUs) for the condition with minimum application for monitoring of the complete network was studied. In addition a novel method in PMU placement considering zero injection busses is proposed. If zero injection busses are modeled in the placement problem, the total number of PMUs is reduced for monitoring the power system. In the presented method here the inequalities related to the monitoring constraints was presented fully linear.

The PMU placement results using the integer linear programming (ILP) on the sample IEEE and a comparison with other methods was presented. It was shown that this method gives very accurate results for different networks and also it is a simple method. Simulation results show that the proposed algorithm is computational efficiency and can be used in different networks.

Table 1: Optimal Number and Locations of PMUs in Test Systems

Test system	No. of PMUs	Locations
14-bus IEEE	4	2,6,7,9
30-bus IEEE	10	1,7,9,10,12,18,24,25, 27,28
57-bus IEEE	17	1,4,9,15,20,24,27,29,31,32,36,38,39,41,47,51,54
118-bus IEEE	32	3,7,9,11,12,17,21,25,28,34,37,41,45,49,53,62,63,68,70,71,76,79,85,86,89,92,96,100,105,110,114

Table 2: Zero Injection Busses for the Test Systems

Test system	No. of zero injection busses	Zero Injection Busses Locations
14-bus IEEE	1	7
30-bus IEEE	5	6,9,11,25,29
57-bus IEEE	15	4,7,11,21,22,24,26,34,36,37,39,40,45,46,48
118-bus IEEE	10	5,9,30,37,38,63,64,67,71,81

Table 3: Optimal Number and Locations of PMUs Including Zero Injection Busses in Test Systems

Test system	No. of PMUs	Locations
14-bus IEEE	3	2,6,9
30-bus IEEE	7	1,5,10,12,18,23,27
57-bus IEEE	14	1,4,9,20,25,27,28,32,36,38,47,50,53,56
118-bus IEEE	29	2,5,10,12,15,17,20,23,28,34,40,45,49,52,56,62,65,71,75,77,80,85, 86,90,94,101,105,110, 114

Table 4: Comparison of the Number of PMUs With and Without Modeling Zero Injection Busses in Test Systems

Test system	No. of PMUs Without Modeling Zero Injection Busses	No. of PMUs With Modeling Zero Injection Busses
14-bus IEEE	4	3
30-bus IEEE	10	7
57-bus IEEE	17	14
118-bus IEEE	32	29

Table 5: Comparison of the Results the Proposed Models with Different Methods

Method	ILP (B&B)	Genetic algorithm[5]	Graph search[3]	Tabu search[10]	BPSO[11]
14-bus IEEE	3	3	5	3	3
30-bus IEEE	7	7	11	-	7
57-bus IEEE	14	12	19	12	13
118-bus IEEE	29	29	38	-	29

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