# Determination of Transient Longitudinal Vibration Response of Intermediate Supports of a Structure Simulating a Short Length Bridge Subjected to Vehicular Traffic 

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## ARTICLE INFO

## Article history:

Received: 18 April 2014;
Received in revised form:
25 May 2014;
Accepted: 4 June 2014;

## Keywords

Bridge,
Columns,
Matrix Method,
Transient Vibrations,
Vehicle Bridge interaction.


#### Abstract

The objective of the investigation is to mathematically simulate dynamics and vibrations of a small structure. This structure is simulating a short length bridge with 3 spans, two end fixed supports and 2 intermediate supports. The top bridge portion of the structure is subjected to a concentrated moving load simulating a vehicle moving on the bridge. Structural Analysis as regards determination of support moments and reactions is done by Matrix Method of Structural Analysis. All supports are subjected to time varying reactions. Considering intermediate supports as a single DOF system, the transient vibration response is ascertained.


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## Introduction

Upon performing fairly detailed literature search, it is concluded that though considerable research has taken place towards Vehicle-bridge interaction from the point of view of dynamics[1]• of entire structure of the bridge, as far as deciding Vibration Response and some few investigations throw light on upper beam portion of the bridge [2]. Surprisingly not much information is available regarding estimation of vibration response of the intermediate columns of the bridge.

In view of this status the authors of this paper began making some head way. The first paper is contributed [3] towards deciding influence line for vertical members of a simple portal frame when on upper beam portion a concentrated load is changing its position at a constant assumed speed. This is as if a short single span bridge is simulated by a portal frame that paper [3] the structural analysis is done based on likely shape of deformation of a portal frame and making some other over simplifying assumptions. Hence, the deduced shape of the Influence line came out to be a straight line. In order to overcome these limitations of first investigations [3], the analysis again of the same structure with same loading condition is done adopting matrix method of structural analysis [4]. The results precipitated non-linear variation of INFLUENCE LINE [5]. This obviously appeared to be more realistic. In the subsequent paper [6] the authors attempted investigation of vibration response of vertical members of the portal frame subjected to time wise non-linear axial loading [5] and considered complete vertical members represented by a Single Mass - Single Spring - Single damper system (i.e. SDOF system) adopting the approach of LAPLASE TRANSORM FOR SOLUTION OF ordinary differential equations with the constant co-efficient [7].

Continuing on the same lines, in this paper, the authors have analyzed a small structure as depicted schematically in Figure 1 and detailed in next section of the paper, wherein the structure in the real is simulating a short bridge but have two end supports as fixed-cum simply supported whereas there are two intermediate supports at B,C which are simple supports. The Vibration Response of these intermediate supports B and C is estimated, after estimating time varying reactions due to various location of concentrated load (simulating travel of a vehicle) by Matrix Method Of Structural Analysis [8] and adopting the approach of Laplace Transformation of time domain ordinary differential equations with constant co-efficient into S or complex domain.


Figure 1: Schematics of a Structure Simulating a Short Length Bridge under Consideration

## Specific scope of the present investigations

A vehicle with a weight $\mathrm{W}=1000 \mathrm{KN}$ is traversing a bridge 3 m long. The vehicle is moving with a velocity of $30 \mathrm{Km} / \mathrm{hr}$. The two ends of the bridge namely $\mathrm{A} \& \mathrm{D}$ are fixed whereas there are two intermediate supports at B and C as shown in

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Figure 1. The cross section of upper beam portion of the bridge is as shown in the end elevation of Figure 1. The length of intermediate supports at $\mathrm{B} \& \mathrm{C}$ is 2 m and cross section is round with diameter 0.01 m (i.e. 10 cm outside dia and 1 cm radial thickness). The material of the bridge is assumed to be mild steel. The value of E is $2.0 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}\left(2.0 \times 10^{6} \mathrm{Kgf} / \mathrm{m}^{2}\right)$.

The analysis of the bridge is done adopting a Matrix Method of Structural Analysis[8]. This analysis has revealed the magnitudes of support moments as detailed in Table 1. Based on the magnitudes of these support moments the free body
diagrams of individual spans $\mathrm{AB}, \mathrm{BC}, \mathrm{CD}$, leads to the deduction of support reactions at supports A, B, C, D applying the basic conditions of equilibrium. Obviously, as the vehicle is moving on the bridge at a velocity of $30 \mathrm{Km} / \mathrm{hr}$, the reactions at supports A, B, C, D are changing with time. This variation of reactions with time is presented in Table 2. In Table 2, RA, RB, $R C, R D$ stand for the time varying reactions at supports $A, B, C$, D. Support at two ends A \& B are fixed cum simply supported whereas the intermediate supports B and C are simple support. In this paper it is therefore decided to show how the longitudinal vibration response of $B$ and $C$ can be decided treating off-course the supports B and C as if are represented by SDOF (i.e. Single Degree of Freedom System comprising of Single Mass, Single Spring Single Damper System).Table 1 Estimation of joint moments as per Matrix Method of Structural Analysis. The Member End Moments and Reactions are determined and lastly span CD is considered. The same procedure is repeated when vehicle is at a distance of 3 m from ' C ' and at a distance of 6 m from support ' C '. The Results are tabulated in Table 1.
Table 1 : Results of Matrix Method to Decide Part Moments.

|  | Sr . No. | Location | Memb Mome In KN | End |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Span } \\ & \text { AB } \end{aligned}$ | 01 | Load at a dis. Of 0.3 m from left support A | MAB | 163.8 |
|  |  |  | MBA | -29.6 |
|  |  |  | MBC | 29.4 |
|  |  |  | MCB | 8.4 |
|  |  |  | MCD | -8.4 |
|  |  |  | MDC | -4.2 |
|  | 02 | Load at a distance of 0.6 m from | MAB | 134.4 |
|  |  | left support A | MBA | -67.2 |
|  |  |  | MBC | 67.2 |
|  |  |  | MCB | 19.2 |
|  |  |  | MCD | -19.2 |
|  |  |  | MDC | -9.6 |
|  | 03 | Load at a distance of 0.3 m from | MAB | -43.4 |
|  |  | support B | MBA | -86.8 |
|  |  |  | MBC | 86.8 |
|  |  |  | MCB | -53.2 |
|  |  |  | MCD | 53.2 |
| Span |  |  | MDC | 26.6 |
| BC | 04 | Load at a distance of 0.6 m from | MAB | -35.2 |
|  |  | support B | MBA | -70.4 |
|  |  |  | MBC | 70.4 |
|  |  |  | MCB | -89.6 |
|  |  |  | MCD | 89.6 |
|  |  |  | MDC | 44.8 |
|  | 05 | Load at a distance of 0.3 m from | MAB | -9.8 |
|  |  | support C | MBA | -19.6 |
|  |  |  | MBC | 19.6 |
|  |  |  | MCB | 68.6 |
|  |  |  | MCD | 225.4 |
|  |  |  | MDC | -23.8 |
| CD | 06 | Load at a distance of 0.6m from | MAB | -6.4 |
|  |  | support C | MBA | -12.8 |
|  |  |  | MBC | 12.8 |
|  |  |  | MCB | 44.8 |
|  |  |  | MCD | 147.2 |
|  |  |  | MDC | -118.4 |

## Approach to decide the vibration response:

The procedure involved in deciding this vibration response is stated step wise as under :
(1) Estimate the mass of the column, M
(2) Determine it's longitudinal stiffness, K
(3) Determine it's damping co-efficient C , assuming damping ratio $\xi=0.01$ because the type of damping is simple hysteresis damping [8]
(4) Determine variation of opposite of reactions B \& C. This is the external force is a function of time. These are the external excitation for B and C .

In view of specifications of the supports at $B$ and $C$ given earlier these $\mathrm{M}, \mathrm{K}, \mathrm{C}$ came out to be as under :
$M=\frac{45.216}{981.0}=0.046 ; K=28.26 \times 10^{4} \mathrm{Kgf} / \mathrm{cm} ;$
$\mathrm{W}=$ Px Area x Length $=8 \times(0.785)(100-64) \times 200=45.216$ Kgf
$K=\frac{E}{L} \times 0.785(100-64)=\frac{2.0 \times 10^{6}}{2.0 \times 10^{2}} \times(0.785)(100-64)$

$$
=28.26 \times 10^{4} \mathrm{Kgf} / \mathrm{cm}
$$

$A s \xi=\frac{C}{c_{c}}=0.01 ; C=0.01 c_{c}=0.01\{2 \sqrt{K M}\}$
$=0.02 \sqrt{28.26 \times 10^{4} \times 0.046}$

$$
=0.02 \sqrt{1.3 \times 10^{4}}
$$

$$
\begin{aligned}
& =0.02 \sqrt{1.3 \times 10^{4}} \\
& \therefore C=2.28 \mathrm{Kgf} / \mathrm{cm} / \mathrm{sec}
\end{aligned}
$$

Table 2 : Estimation of Support Reactions in view of obtained results of joint moments and reactions. as per Matrix Method of Structure Analysis reported in Table 1.

| Time ind <br> milliseconds | $\mathbf{R A}$ in <br> $\mathbf{K N}$ | $\mathbf{R B}$ in <br> $\mathbf{K N}$ | $\mathbf{R C}$ in <br> $\mathbf{K N}$ | $\mathbf{R D} \quad \mathbf{K N}$ <br> $\mathbf{K N}$ <br> in <br> meters |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 36 | 834.4 | 203.4 | -50.4 | 12.6 | 0.3 |
| 72 | 467.2 | 619.2 | -115.2 | 28.8 | 0.6 |
| 156 | -130.2 | 863.8 | -186.6 | -79.8 | 1.3 |
| 192 | -105.6 | 486.4 | +753.6 | -134.4 | 1.6 |
| 276 | -29.4 | 117.6 | +813.4 | +323.8 | 2.3 |
| 317 | -19.2 | 76.8 | +371.2 | +571.2 | 2.6 |



Figure 2: variation of RB with respect to time
The variation of RB with respect to time as shown graphically through Figure 2 can be obtained in polynomial form as stated in Eq.(1).
$\mathrm{RB}=608.368+27.546 \mathrm{t}-0.156 \mathrm{t}^{2}+2.17 \times 10^{-4} \mathrm{t}^{3}+7.23 \mathrm{x}$
$18^{8} \mathrm{t}^{4} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$
Similarly, Variation of reaction of support at $\mathrm{C}, \mathrm{RC}$ in a graphical form be presented as shown in Figure 3. For time instants 36,72 and 156 ms the reaction is negative, i.e. it's inducing tension in the support C .


Figure 3: Variation of RC with respect to time
In order not to complicate the mathematical process of deducing the equation for RC from the graphic plot, it is desirable to shift the origin, to $-200,0$ point as shown in Figure 3. This makes addition of +200 KN at all time instants and hence all values of RC become positive. The mathematical form of variation of RC with respect to time $t$ in ms with shifted origin is a shown in Eq. (2).
$R C=431.958-14.755 \mathrm{t}+0.063 \mathrm{t}^{2}+2.77 \times 10^{-4} \mathrm{t}^{3}-1.05 \times 10^{-}$ ${ }^{6} \mathrm{t}^{4}$.
Equation (2) changes to the form of Equation (3) with original or real origin as under.
$\mathrm{RC}=\mathrm{A} 0+\mathrm{A} 1 \mathrm{t}+\mathrm{A} 2 \mathrm{t}^{2}+\mathrm{A} 3 \mathrm{t}^{3}+\mathrm{A} 4 \mathrm{t}^{4}+\mathrm{A} 5 \mathrm{t}^{5}=1050-61 \mathrm{t}+45 \mathrm{t}^{2}$
$-32 t^{3}+27 t^{4}-12 t-20 t^{5}$

## Determination of Vibration Response

## Vibration Response of Column OB

Treating the column OB as a SDOF system, the governing equation for the vibration response of the column $O B$ can be written down as
$M \ddot{x}+c \dot{x}+K x=R_{B}$
Substituting for $\mathrm{R}_{\mathrm{B}}$ in detailed form as given in Equation (1) one gets

$$
\begin{align*}
& M \ddot{x}+c \dot{x}+K x=-608.368+27.546 t-0.156 t^{2}+ \\
& 2.17 x 10^{4} t^{3}+7.23 x 10^{8} t^{4} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{3}
\end{align*}
$$

Equation (3) is an ordinary differential equation with constant co-efficient. Substituting for earlier decided values of M, K, C in Equation (3), Equation (3) is changed as
$0.046 \ddot{x}+2.28 \dot{x}+28.26 x 10^{4} x=608.368+27.546 t+$ $0.156 t^{2}+2.17 x 10^{-4} t^{3}+$ 7. $23 \times 10^{8} t^{4}$.

There are five terms on R.H.S. of Eq. (4). As the time to traverse the complete length of 3 m of top horizontal portion of the structure by the concentrated load of 1 KN at the speed of 30 $\mathrm{km} / \mathrm{hr}$ is only 317 ms the higher powers of t on RHS of Eq. (4) contribute very little to the reaction RB. In view of this reasoning Eq. (4) is approximated as
$0.046 \ddot{x}-2.28 \dot{x}-28.26 x 10^{4} x=-608.368-$
$27.546 t$.
(5)

Treating Eq. (5) as an ordinary differential equation with constant co-efficients and adopting the approach of LAPLASE TRANSFORMATION of solution of differential equation [7] the solution of Eq. (5) is obtained off-course with initial conditions as at $\mathrm{t}=0 ; \mathrm{x}=0$ and $\dot{\boldsymbol{x}}=0$. The mathematical detailed calculation of solution of differential equation (5) is based on Laplace Transformation and partial fraction expansion technique. The solution of Equation (5), the vibration response of support OB then comes out to be as under
$x(t)=K_{1} e^{-0(t)}+K_{2} e^{-\alpha_{1}(t)}+K_{3} e^{-\alpha_{2}(t)}+K_{1}^{\prime} e^{-0(t)}+K_{2} e^{-\alpha_{1}(t)}+$ $K_{3}{ }^{\prime} e^{-\alpha_{2}(t)}$

In Eq. (6) the values of $K_{1}, K_{2}, K_{3}, K^{\prime}{ }_{1}, K^{\prime}{ }_{2}, K^{\prime}{ }_{3} \quad \alpha_{1} \& \alpha_{2}$ come out to be as under
$K_{1}=9.902 \times 10^{-5} ; \quad K_{2}=-608.368(e)^{\left(122842.88 i+1.22 \times 10^{7}\right)}$;
$K_{3}=608.368 x e^{\left[-1.22 \times 10^{5} i-1.22 \times 10^{7}\right]}$;
$K_{1}^{\prime}=-4.483 \times 10^{-6} ; K_{2}^{\prime}$

$$
=\left(-4.487 \times 10^{6}\right.
$$

$$
\left.+i 12.367 x 10^{-17}\right) e^{-(-24.782+i(2478.47) t}
$$

$K_{3}^{\prime}=\left(-0.180 \times 10^{-10}+i 9.044 \times 10^{-10}\right) e^{[24.782+i(2478.47)} ;$
$\alpha_{1}=-24.782+i(2478.47)$,

$$
\alpha_{2}=-24.782-i(2478.47)
$$

Substituting these values of $\mathrm{K}_{1}, \mathrm{~K}_{2}, \mathrm{~K}_{3}, \mathrm{~K}_{1}, \mathrm{~K}^{\prime}, \mathrm{K}^{\prime}{ }_{3} \alpha_{1} \& \alpha_{2}$ in equation 6 , the final form for transient vibration response of quolumn OB would come out to be as under.

$$
\begin{align*}
& x(t)=9.902 \times 10^{-5}-608.368 e^{\left[1.22 \times 10^{7}+i 122842.88\right] x t} \\
& \quad 6.08 .368(e)^{\left[1.22 \times 10^{7}-1.22 \times 10^{5} i\right]} \\
&+4.483 \times 10^{-6 t}+\left(-4.487 \times 10^{6}\right. \\
&\left.+i 12.367 \times 10^{-12}\right) x e^{-(-24.782-i 2478.47) t} \\
&+\left(-0.180 \times 10^{-10}+\right.
\end{align*}
$$

## Vibration Response of Column OC

The way in which the vibration response of column $O B$ is decided as detailed in Art. 4.1 on the same lines it is decided for column OC. The salient features of this estimation are given below.
The governing equation of vibratory motion of OC is
$M \ddot{x}+c \dot{x}+K x=431.958-14.577 t+0.063 t^{2}+$ $2.77 x 1^{-4} t^{3}-1.05 x 1^{6} t^{4}$.
Substituting for numerical values of M, K, C in Eq. (8), it gets changed to
$0.046 \ddot{x}+2.28 \dot{x}+28.26 x 10^{4} x=431.958-14.755 t+$ $0.063 t^{2}+2.77 x 10^{-4} t^{3}-$
$1.05 \times 10^{6} t^{4}$.
The reasons for which Eq. (4) for column OB was approximated, the same reasoning holds true in this case also. Hence, the Eq. (9) is approximated as under:
$0.046 \ddot{x}+2.28 \dot{x}+28.26 x 10^{4} x=431.958-$
14.755t ..(10)

The solution of Eq. (10) is obtained again using LAPLACE TRANSFORMATION technique with the initial conditions as at $\mathrm{t}=0 ; \mathrm{x}=\dot{\boldsymbol{x}}=0$ \& the technique of partial fraction expansion.
The vibration response of column OC is then,
$x(t)=K_{1} e^{-0(t)}+K_{2} e^{-\alpha_{1}(t)}+K_{3} e^{-\alpha_{2}(t)}+K_{1}^{\prime} e^{-0(t)}+$
$K_{2}{ }^{\prime} e^{-\alpha_{1}(t)}+K_{3}{ }^{\prime} e^{-\alpha_{2}(t)}$
In Eq. (11) the values of $\mathrm{K}_{1}, \mathrm{~K}_{2}, \mathrm{~K}_{3}, \mathrm{~K}^{\prime}{ }_{1}, \mathrm{~K}^{\prime}{ }_{2}, \mathrm{~K}^{\prime}{ }_{3}, \alpha_{1} \& \alpha_{2}$ are as under.
$K_{1}=7.031 \times 10^{-5} ; \quad K_{2}=-35.172 \times 10^{-6}+$
$i\left(0.349 \times 10^{-6}\right)$;

$$
\begin{aligned}
& K_{3}=\frac{431.958}{\left(-12.3 \times 10^{6}+i 12.25 \times 10^{4}\right)} \\
& K_{1}^{\prime}=-2.401 \times 10^{-6} ; K_{2}^{\prime} \\
& =\left[-0.000484 \times 10^{-6} i+0.600 \times 10^{-6} ;\right. \\
& K_{3}^{\prime}=-2.422 \times 10^{-8} i+0.048 \times 10^{-8} ; \\
& \alpha_{1}=-24.782+i(2478.47), \\
& \alpha_{2}=-24.782-i(2478.47)
\end{aligned}
$$

## Effect of vibration response of intermediate supports

Articles 4.1 and 4.2 detailed estimation of vibration response of columns OB and OC. The Equation (6) and (11) describes variation of elongation of columns OB and OC respectively. In both these equations $2^{\text {nd }}, 3^{\text {rd }}, 5^{\text {th }}$ and $6^{\text {th }}$ terms give highly complex transient damped vibration response corresponding to highly under damping system since $C$ is very
small and that of K is very high. Hence, in this paper it is decided to evaluate influence of change in elongation of columns OB and OC only due to $1^{\text {st }} \& 4^{\text {th }}$ terms of Equations (6) \& (11) for columns OB and OC respectively. As shown in details of calculations towards influence of vibrations of OB and OC given in next two subsections 5.1 and 5.2 with this over simplifying assumptions as the additional stress under vibrations is evaluated.

## Determination of stress under Vibrations of Column OB.

The column would be subjected to maximum compressive load of 1 KN . Considering this as $\mathrm{W}=1 \mathrm{KN}$ the static maximum compression of column OB is estimated as detailed below.
Area of cross section of column $=\mathrm{A}=0.785\left(10^{2}-8^{2}\right)=30.6$ sq.cm. with value of Modules of Elasticity E $=2.0 \times 10^{6}$
$\mathrm{Kgf} / \mathrm{cm}^{2}$. The static maximum compression is estimated as detailed below.
Induced stresses $S=100.0 / 30.6=3.26 \mathrm{Kgf} / \mathrm{cm}^{2}$
$\therefore$ Induced Strain $=3.26 / 2.0 \times 10^{6}=1.63 \times 10^{-6}$
$\therefore$ Maximum static compression $=$ Original Length x strain $=2.0$ $\times 10^{2} \times 1.633 \times 10^{-6}=32.6 \times 10^{-5} \mathrm{~cm}$ compressive.
Approximate elongation considering vibrations of OB i.e. approximating Eq. (6) as under
$\mathrm{x}(\mathrm{t})=9.902 \times 10^{-5}+14.483 \times 10^{-6} \times 0.317=24.102 \times 10^{-5} \mathrm{~cm}$. $=9.902 \times 10^{-5}+14.483 \times 10^{-6} \times 0.317=24.102 \times 10^{-5} \mathrm{~cm}$.
In other words due to vibrations of OB, additional stress but tensial gets superimposed on OB which is detailed as below.
Additional stress under vibration $=$
$\left[\frac{(32.6-24.102) \times 10^{-5}}{\left(32.6 \times 10^{-5}\right)}\right] x \mathbf{1 0 0}=26.6 \%$

## Determination of Stress under Vibration of Column OC

As decided above in Article 5.1, the elongation/compression under maximum static dead load of $1 \mathrm{KN}=100.0 \mathrm{Kgf}$ would come out to be again same as that for column OB i.e. $32.6 \times 10^{-5}$ cm compressive. Now consider the effect of vibrations of column OC, then
$\mathrm{x}(\mathrm{t})=7.031 \times 10^{-5}-(-0.02401) \times 10^{-6}$
$=7.007 \times 10^{-5}$
$7.031 \times 10^{-5}-0.761 \times 10^{-6}=6.954 \times 10^{-5}$
Thus original compression is reduced by $32.6 \times 10^{-5}-6.954 \mathrm{x}$ $10^{-5}=25.646 \times 10^{-5}$. In other words the effect of vibration of column OC is to induce $25.646 / 32.6 \times 100=78.67 \%$ of additional tensile stress.

## Conclusion

The paper details the effect of vibrations of intermediate supports of a structure simulating short length three span bridges subjected to a moving concentrated load simulating a moving vehicle on the bridge. The equations for transient vibration response of intermediate columns are established, of course treating a column as SDOF system subjected to time varying reactions. Making some over simplifying assumptions transient vibrations response of intermediate supports is established. Based on this vibration response additional stress under vibrations is also estimated. The findings shows for the first support the additional stress is $25 \%$ whereas for the other it is 78.67\%

## Acknowledgment:

This Research is executed at Manoharbhai Patel Institute of Engineering and Technology, Gondia (MS) for partial fulfillment of the degree of Doctor of Philosophy $(\mathrm{PhD})$ in Civil Engineering of RTM Nagpur University of first author under the supervision of $2^{\text {nd }}$ and $3^{\text {rd }}$ author. The facilities for the research
are provided by Dept. of Civil Engineering, Manoharbhai Patel Institute of Engineering and Technology, Gondia (MS)

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## Annexure - I \& II

## Determination of vibration response

## Vibration Response of Column OB

Treating the column OB as a SDOF system, the governing equation for the vibration response of the co.umn OB can be written down as

$$
M \ddot{x}+c \dot{x}+K x=R_{B}
$$

Substituting for $\mathrm{R}_{B}$ in detailed form as given in Equation (1) one gets

$$
M \ddot{x}+c \dot{x}+K x=-608.368+27.546 t-0.156 t^{2}+
$$

$$
\begin{equation*}
2.17 x 10^{-4} t^{3}+7.23 x 10^{8} t^{4} \tag{4}
\end{equation*}
$$

Equation (4) is an ordinary differential equation with constant co-efficient. Substituting for earlier decided values of M, K, C values in Equation (4), and regarding the equation after transforming it to ' S ' domain from time domain, one would get with initial condition at $\mathrm{t}=0 ; \mathrm{x}=0$ and $\dot{\mathrm{x}}=0$.

$$
\begin{aligned}
& 0.046 \mathrm{~S}^{2} \mathrm{x}(\mathrm{~S})+2.28 \mathrm{sx}(\mathrm{~s})+28.26 \times 10^{4} \mathrm{x}(\mathrm{~s}) \\
&=-\frac{608.368}{\mathrm{~S}}+\frac{27.546}{\mathrm{~S}^{2}}-\frac{0.156(2)}{\mathrm{S}^{3}}+\frac{2.17 \times 10^{-4}(3)}{\mathrm{S}^{4}} \\
&+\frac{7.23 \times 10^{8}(4)}{\mathrm{S}^{5}} \ldots \ldots \ldots(5)
\end{aligned}
$$

Now, roots of $\left[0.0465 \mathrm{~S}^{2}+2.28 \mathrm{~S}+28.26 \times 10^{4}\right]=0$ are $\alpha_{1}$ \& $\alpha_{2}$ such that

$$
\left[0.0465 S^{2}+2.28 S+28.26 \times 10^{4}\right]=0=\left(S+\alpha_{1}\right)\left(S+\alpha_{2}\right)
$$

Solving the above quadratic Equation we get values of $\alpha_{1} \&$ $\alpha_{2}$ as,

$$
\begin{aligned}
& \alpha_{1}=565,150.4348 \\
& \alpha_{2}=565,200
\end{aligned}
$$

Now considering only first term on R.H.S. of Equations,

$$
\therefore X(S)=\frac{-608.368}{(S)\left(S+\alpha_{1}\right)\left(S+\alpha_{2}\right)}=\frac{K_{1}}{S}+\frac{K_{2}}{\left(S+\alpha_{1}\right)}+\frac{K_{3}}{\left(S+\alpha_{2}\right)}
$$

The above is adopting the approach of partial fraction expansion performing necessary calculation.

To get $K_{1}$, Multiply both sides by ' S ', we get

$$
\begin{gathered}
X(S)=\frac{-608.368(s)}{(S)\left(S+\alpha_{1}\right)\left(S+\alpha_{2}\right)}=\frac{K_{1}(s)}{S}+\frac{K_{2}(s)}{\left(S+\alpha_{1}\right)}+\frac{K_{3}(s)}{\left(S+\alpha_{2}\right)} \\
\therefore X(S)=\frac{-608.368(s)}{\left(S+\alpha_{1}\right)\left(S+\alpha_{2}\right)}=K_{1}+\frac{K_{2}(s)}{\left(S+\alpha_{1}\right)}+\frac{K_{3}(s)}{\left(S+\alpha_{2}\right)}
\end{gathered}
$$

$\therefore$ Put $\mathrm{S}=0$; we get

$$
\begin{aligned}
& X(S)= \frac{-608.368}{\left(\alpha_{1}\right)\left(\alpha_{2}\right)}=K_{1}+\frac{K_{2}(0)}{\alpha_{1}}+\frac{K_{3}(0)}{\alpha_{2}} \\
& \therefore X(S)= \frac{-608.368}{(565,150.4348)(-565,200)}=K_{1} \\
& \therefore K_{1}=1.9045 \times 10^{-9}
\end{aligned}
$$

By similar calculations, we obtained the values of $\mathrm{K}_{2}$ and $\mathrm{K}_{3}$ as follows

$$
K_{2}=0.9523 \times 10^{-9} \text { and } K_{3}=0.9522 \times 10^{-9}
$$

Now consider 2nd term of Equation (5)

$$
\therefore X(S)=\frac{27.546}{(S)^{2}\left(S+\alpha_{1}\right)\left(S+\alpha_{2}\right)}=\frac{\boldsymbol{K}_{1}^{\prime}}{S^{2}}+\frac{K_{2}^{\prime}}{\left(S+\alpha_{1}\right)}+\frac{\left.K_{3}^{\prime}\right)}{\left(S+\alpha_{2}\right)}
$$

By doing the calculations similar to the previous case for its form, values of $K^{\prime} 1, K^{\prime} 2, K^{\prime} 3$ are found as under.

The values are

$$
\begin{gathered}
K_{\mathbf{1}}^{\prime}=-0.862 \times 10^{-10} \\
K_{2}^{\prime}=7.629 \times 10^{-17} \\
K_{3}^{\prime}=7.628 \times 10^{-17}
\end{gathered}
$$

In Equation (5), on RHS the influence of $3^{\text {rd }}, 4^{\text {th }}$ and $5^{\text {th }}$ terms is very negligible. Hence they are dropped in further analysis.

