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Reliability Analysis and Performance Evaluation of an Automobile Assembly Plant with Risk Factor

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ABSTRACT

The purpose of this article is to compute the reliability measures of an automobile assembly plant. An automobile assembly plant consists of five sub-systems working in series as: Vendor (supply steel metal and body parts), Weld shop, Paint shop, Assembly shop and Quality department. Weld shop and Assembly shop has the configuration 2-out-of -3: G and 1-out-of-3: F. Considered system can completely fail due to failure of any of the subsystems. It is also assumed that the system can fail due to machine failures. Machine failures can be major or minor. All failures follow exponential time distribution whereas all repairs follow general time distribution. Preventive maintenance policy has been applied to reduce the failure in the system. Risk analysis has been made which may occur due to the mistake of quality department in quality checks. Various reliability characteristics such as transition state probabilities, steady state behaviour, reliability, availability, M.T.T.F, sensitivity analysis and the cost analysis have been obtained using supplementary variable technique and Gumbel-Hougaard copula methodology. We also perform a parametric investigation which provides numerical results to show the effects of different system parameters to the reliability and MTTF which may be helpful to managerial staff of the industry in the decision making.

Introduction

The auto industry is often thought of as one of the most global of all industries. Its products have spread around the world, and it is dominated by a small number of companies with worldwide recognition. However, in certain respect the industry is more regional than global, in spite of the globalizing trends evident in the 1990s. The spread of vehicle production in developing countries increased markedly in the boom years of rapid expansion in the emerging markets in the 1990s. One feature of the auto industry in the last 25 years was the way in which leading vehicle manufacturers extended their operations in developing countries. For the global producers, rapidly growing markets in developing countries were meant to provide for spreading vehicle development costs; for establishing cheap production sites for the production of selected vehicles and components; and for access to new markets for higher-end vehicles, which would still be produced in the triad economies.

Due to high rivalry and competitiveness, the venture search becomes advanced and higher performances at lower cost, so the optimization of the operations of the industrial system and its support systems such as maintenance system, assembly and production system is necessary. In a highly competitive industry, production management has to continually focus on achieving increased product performance, quality and efficiency in order to maintain a fair share of the available market and improve its customer base. To improve business performances, maintenance is thus directly related to risk analysis and dependability which allow forecasting the gaps between nominal and non-nominal operations of the system (degradation, failure, etc). As the complexity and automation of equipments increased, it resulted in severe problems of maintenance and repair. This put forward the tasks of developing a systematic approach to the study of any phenomena and process that can lead to failure free operation or render service for a good or at least reasonable period of time. Bazovsky, Igor [1] and Barlow and Proschan [2] have described various reliability aspects and its principles in the modern day life.

Further, real-time and embedded systems are now a central part of our lives. Reliable functioning of these systems is of paramount concern to the millions of users that depend on these systems every day. Unfortunately most embedded systems still fall

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short of user's expectation of reliability. Basically a system is a combination of elements forming a planetary whole i.e. there is a functional relationship between its components. The properties and behaviour of each component ultimately affects the properties of the system. Any system has a hierarchy of components that pass through the different stages of operations which can be operational, failure, degraded or in repair. Failure doesn't mean that it will always be complete; it can be partial as well. But both these types affect the performance of system and hence the reliability. Majority of the systems in the industries are repairable. The performance of these systems can influence the quality of product, the cost of business, the service to the customers, and thereby the profit of enterprises directly. Modern repairable systems tend to be highly complex due to increase in convolution and automation of systems.

As far as the production-operations are concerned, not only reliability but also steady state availability analysis is essential, again on account of increased complexity and cost of present day equipment. Also the markets are getting globalized and more competitive. Penalties for delayed deliveries have been increased. Sometimes the orders are cancelled and defaulting plants are not favoured with orders. To overcome these types of problems, reliability and steady state availability analysis is necessary for performance studies in the area of discrete manufacturing systems. Many researcher [3, 4, 5 and 6] discussed reliability and steady state analysis of manufacturing plant by using different approaches like Probabilistic rational model technique, Matlab tool, matrix method etc.

This paper proposes a methodology to develop a decision-making aid tool whose objective is to assess the dependability and performances of an industrial system. In practical situation data collected or available for the complex repairable industrial systems are vague, ambiguous, qualitative and imprecise in nature due to various practical constraints. So it is not easy to calculate reliability indices of such systems up to a desired accuracy. If reliability indices of these systems have been calculated, then they have high range of uncertainty. The purpose of the present paper is to compute the reliability characteristics of an automobile assembly plant. An automobile assembly plant consists of five sub-units working in series as: Vendor (supply steel metal and body parts), Weld shop, Paint shop, Assembly shop and Quality department. In the Weld shop and Assembly shop three parallel machines are involved in doing the same job. These subsystems follows 2-out -of-3:G and 1-out-of-3:F configurations which specifies that if at least two machines of the Weld shop and Assembly shop are working then the system is in operating state and able to fulfil the required target and if two machines are failed and only one machine is working then the system fails [7]. Also system can fail due to machine failures. Machine failures can be major or minor or both. Here two different groups of repairmen are involved in repairing of the system. Since these groups have different skills and expertise hence when both of the groups are involved in repairing for different failures then joint distribution is obtained with the help of Gumble-Hougaard copula. After assembly of the product, quality checks are done by the quality department. Any ignorance or unawareness of the quality experts can lead to the system into the risk state, which may result to the system failure. Preventive maintenance is one of the important aspects of production companies; it is possible by providing rest to all the machines one by one for a particular period of time as per maintenance schedule. The above facts i.e. preventive maintenance and risk analysis have been taken into consideration in the present study. The reliability block diagram, transition state diagram and state specification of the considered system is shown in Figures-1, 2 and table-1 respectively.

Nomenclatures

Pr	Probability.
$P_0(t)$	Pr{ at time t the system is in state S_0 }
$P_i(k,t)$	Pr {the system is in failed state due to the failure of the i^{th} subsystem at time t and elapsed repair time lies between k and $k+\Delta$},where i=V, P, M, QR ,Q, A,W and k= x, y, r, q, h, g, u.
$V/W/P/A/Q$	Vendor/Weld shop/Paint shop/Assembly shop/Quality department.
K	Elapsed repair time, where k= x, y , r, q, h, g ,u.
ψ_{1W} / ψ_{1A}	Failure rate of the one machine of the Weld shop/Failure rate of the one machine of the assembly shop.
ψ_M	Failure rate due to machine failure

$P_{1W}(t)$	Pr {the system is in operating state when one machine of the weld shop is failed and remaining two are working}
$P_{1A}(t)$	Pr{the system is in operating state when one machine of the assembly shop is failed and remaining two are working }
ψ_i	General failure rate of i^{th} subsystem, where i= V, P, M, Q,A,W
$\phi_i(k)$	General repair rate of i^{th} system in the time interval (k, $k+\Delta$), where i= V, P, M,Q,QR,A,W and k= x, y, r, q, h, g, u.
γ_Q	Risk factor or risk rate (that lead the system into the risk state).
$P_{i,W}(k, t)$	Pr (at time t system is in failed state due to the failure of the i^{th} Subsystem (from the state S_2) while one machine of the weld shop is not working). Elapsed repair time for i^{th} subsystem lies between (k, $k+\Delta$), where i= V, P, A and k= x, y, g.
$P_{i,A}(k, t)$	Pr (at time t system is in failed state due to the failure of the i^{th} Subsystem (from the state S_4) while one machine of the assembly shop is not working). Elapsed repair time for i^{th} subsystem lies between (k, $k+\Delta$), where i= V, P,W and k= x, y, u.
$P_{QR}(q, t)$	Pr (at time t system is in risk state due to negligence of quality Department)
K_1, K_2	Revenue per unit time and service cost per unit time respectively.

Let $u_1 = e^r$ and $u_2 = \phi_M(r)$ then the expression for joint probability according to Gumbel-Hougaard family of copula is given as

$$\phi_M = \exp\left[r^\theta + \left[\log \phi_M(r)\right]^\theta\right]^{1/\theta}.$$

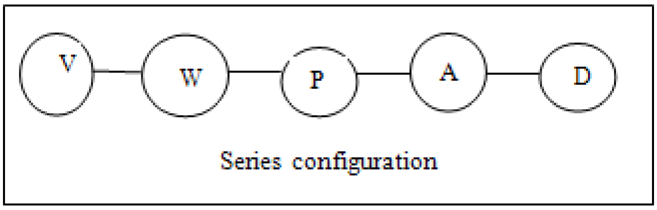


Figure. 1 Reliability block diagram

Methodology used

Cox [8] has done an analysis of Non-Markovian stochastic process by inclusion of supplementary variables. Supplementary variable technique is used to estimate the reliability measures of the considered industrial problem. Nelson and Ram, Singh [9, 10] incorporated Copula methodology to evaluate the joint probability distribution of repairs in case of machine failures. This method provides an easy way to estimate the variation in different system performance in terms of reliability with respect to time.

Approach

The mathematical model of an automobile assembly plant has been developed using Markov Process with the help of supplementary variable technique and copula methodology. The corresponding differential equations obtained are solved using Laplace Transforms. Maple -program have been developed to study the variations of reliability, MTTF, availability, sensitivity analysis and cost effectiveness of the system with respect to various parameters. Parametric investigations have also been performed with the help of numerical results to show the effects of various system parameters to the reliability and MTTF which may be helpful to managerial staff of the industry in the decision making.

Novelty

Industrial implications of the results have been discussed.

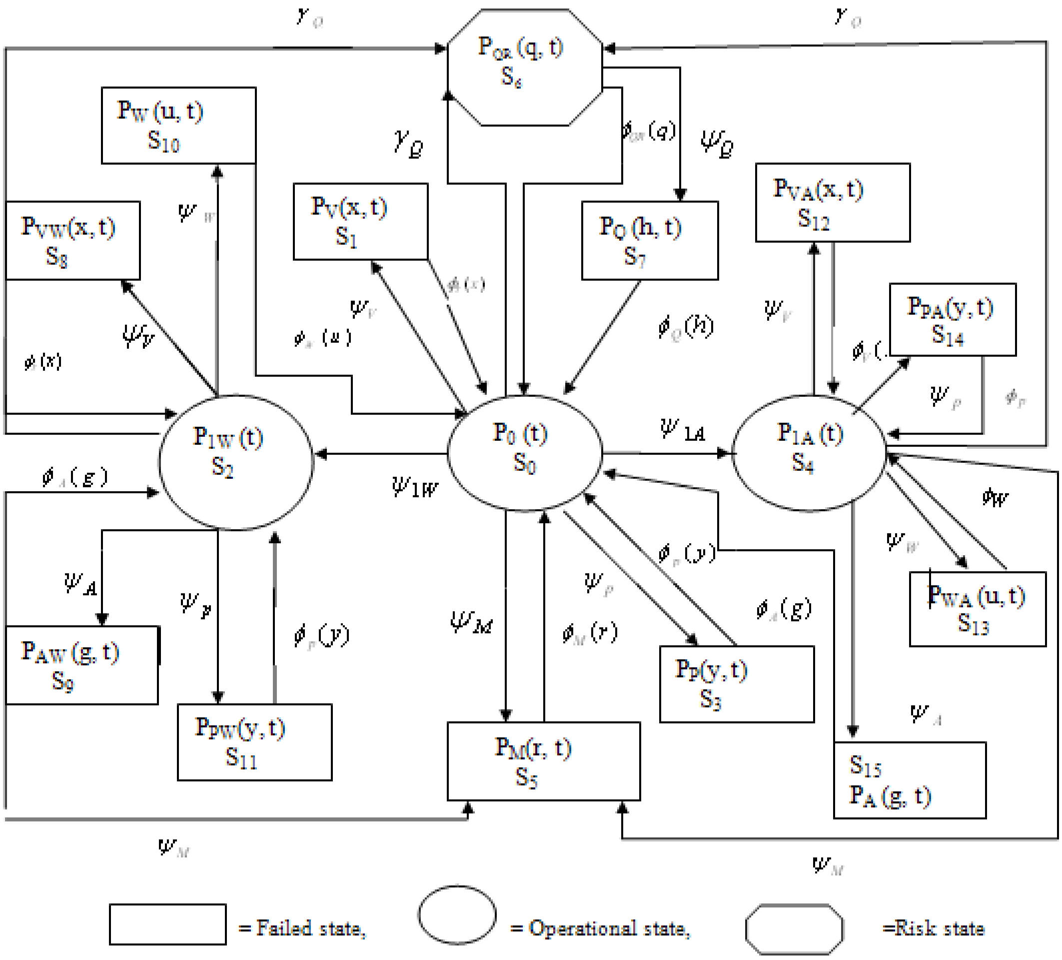


Figure 2. Transition State diagram

System description

An assembly line is a manufacturing process in which parts (usually interchangeable parts) are added to a product in a sequential manner using optimally planned logistics to create a finished product much faster than with handcrafting-type methods. Assembly lines are designed for a sequential organization of workers, tools or machines, and parts.

Assembling of a vehicle is very complex system, which includes various processes. Manufacturing process of vehicles mainly includes Vendor, Weld Shop, Paint Shop, Assembly Shop and Quality department.

Vendor The most important part of manufacturing is Vendor, since vehicle consist of thousands of small and big components and manufacturing of all parts are not viable for a single company, therefore one company assembles all the parts which is called OEM (Original Equipment Manufacturing) as like Tata Motors, Maruti, Fiat etc.

Weld Shop This is the first and basic process of vehicle manufacturing. In this shop the basic sheet metal material provided by vendors are converted to the usable form and different components of basic welded body, and then all the components welded to form vehicle body. Later it is sent to Paint Shop.

Paint Shop This is the second shop in the process of manufacturing. In this shop, body which received from Weld Shop is painted in different colours as per market requirement then it is sent to Assembly Shop.

Engine Shop This shop is indirectly connected to assembly shop and not taken into the consideration in the present system, where engine and gear box are assembled from the components provided by the vendors and then this engine along with gearbox supplied to assembly shop.

Assembly Shop This is one of the most important shops in vehicle manufacturing as the finished vehicle assembles from this shop only. In this shop painted body is equipped and assembled with different non identical components which are supplied by different vendors logically to form finished vehicle.

Quality Department Before sending finished vehicle to the dealer it goes through different quality checks, like it is tested on the test track by running it and PDI (Pre dispatch inspection) for any issues. All parts which are supplied by the vendors for vehicle manufacturing goes through different quality checks.

Assumptions

The following assumption has been taken into the considerations in this study.

- Initially at t=0, all subsystems are operating well.
- Failures are statistically independent.
- The repair time of the subsystems are assumed to be arbitrarily distributed.
- Repaired subsystem/ plant(s) works like new.
- All failures follow exponential time distribution.
- Weld shop and Assembly shop has 2-out-of-3: G and 1-out-of-3:F configuration.
- The whole system can also fail due to machine failures that may be either major or minor or both.
- Joint probability distribution has been obtained with the help of copula for repair when the system suffers from both major and minor machine failures [11, 12].

Formulation of mathematical model

Probabilistic considerations and limiting procedure yield the following integro-differential equations satisfying the model:

$$\left[\frac{d}{dt}+\psi_{1A}+\psi_{1W}+\psi_V+\psi_M+\psi_p+\gamma_Q\right]P_0(t)=\int_0^\infty\phi_V(x)P_V(x,t)dx+\int_0^\infty\phi_p(y)Pp(y,t)dy+\int_0^\infty\phi_M(r)P_M(r,t)dg+\int_0^\infty\phi_Q(q)P_Q(q,t)dq+\int_0^\infty\phi_A(g)P_A(g,t)dg+\int_0^\infty\phi_W(u)P_W(u,t)du+\int_0^\infty\phi_Q(h)P_Q(h,t)dh \quad \dots (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_V(x)\right]P_V(x,t)=0 \quad \dots (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_p(y)\right]P_p(y,t)=0 \quad \dots (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\exp\left[r^\theta+\{[\log\phi_M(r)]\}^\theta\right]^{1/\theta}\right]P_M(r,t)=0 \quad \dots (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial q}+\psi_Q+\phi_{QR}(q)\right]P_{QR}(q,t)=0 \quad \dots (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial h}+\phi_Q(h)\right]P_Q(h,t)=0 \quad \dots (6)$$

$$\left[\frac{d}{dt}+\psi_A+\psi_W+\psi_V+\psi_M+\psi_p+\gamma_Q\right]P_{1w}(t)=\int_0^\infty\phi_V(x)P_{VW}(x,t)dx+\int_0^\infty\phi_A(g)P_{AW}(g,t)dg+\int_0^\infty\phi_p(y)P_{P_W}(y,t)dy+\psi_{1W}P_0(t) \quad \dots (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_V(x)\right]P_{VW}(x,t)=0 \quad \ldots (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial g}+\phi_A(g)\right]P_{AW}(g,t)=0 \quad \ldots (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_p(y)\right]P_{PW}(y,t)=0 \quad \ldots (10)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+\phi_W(u)\right]P_W(u,t)=0 \quad \ldots (11)$$

$$\left[\frac{d}{dt}+\psi_A+\psi_W+\psi_V+\psi_M+\psi_p+\gamma_Q\right]P_{1A}(t)=\int_0^\infty \phi_V(x)P_{VA}(x,t)dx+\int_0^\infty \phi_p(y)P_{PA}(y,t)dy+\int_0^\infty \phi_W(u)P_{WA}(u,t)du+\psi_{1A}P_0(t) \quad \ldots (12)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_V(x)\right]P_{VA}(x,t)=0 \quad \ldots (13)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_p(y)\right]P_{PA}(y,t)=0 \quad \ldots (14)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+\phi_W(u)\right]P_{WA}(u,t)=0 \quad \ldots (15)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial g}+\phi_A(g)\right]P_A(g,t)=0 \quad \ldots (16)$$

Boundary conditions

$$P_V(0,t)=\psi_V P_0(t) \quad \ldots (17)$$

$$P_p(0,t)=\psi_p P_0(t) \quad \ldots (18)$$

$$P_M(0,t)=\psi_M P_0(t) \quad \ldots (19)$$

$$P_{QR}(0,t)=\gamma_Q\left[P_0(t)+P_A(t)+P_W(t)\right] \quad \ldots (20)$$

$$P_Q(0,t)=\psi_Q P_{QR}(t) \quad \ldots (21)$$

$$P_{VW}(0,t)=\psi_V P_W(t) \quad \ldots (22)$$

$$P_{AW}(0,t)=\psi_A P_W(t) \quad \ldots (23)$$

$$P_{PW}(0,t)=\psi_p P_W(t) \quad \ldots (24)$$

$$P_W(0,t)=\psi_W P_W(t) \quad \ldots (25)$$

$$P_{VA}(0,t)=\psi_V P_A(t) \quad \ldots (26)$$

$$P_{pA}(0,t)=\psi_p P_p(t) \quad \ldots (27)$$

$$P_{WA}(0,t) = \psi_W P_A(t) \quad \text{… (28)}$$

$$P_A(0,t) = \psi_A P_A(t) \quad \text{…(29)}$$

Initial Condition

$P_0(0) = 1$, otherwise zero. … (30)

Solution of the model

Solving equations (1) through (16) by taking Laplace transform and using initial and boundary conditions, one may obtain following transition state probabilities of the system.

$$\overline{Pup}(s) = \frac{1}{K(s)}\left[1 + B(s) + a(s)\right] \quad \text{... (31)}$$

$$\overline{Pdown}(s) = \overline{P_v}(s) + \overline{P_P}(s) + \overline{P_M}(s) + \overline{P_{QR}}(s) + \overline{P_Q}(s) + \overline{P_{VW}}(s) + \overline{P_{PW}}(s) + \overline{P_{AW}}(s) + \overline{P_W}(s) + \overline{P_{VA}}(s) + \overline{P_{PA}}(s) + \overline{P_{WA}}(s) + \overline{P_A}(s)$$

$$= \frac{1}{K(s)}[\psi_v J_v(s) + \psi_P J_P(s) + \psi_M J_M(s) + \gamma_Q[1 + B(s) + A(s) J_{QR}(s) + \gamma_Q \psi_Q[1 + B(s) + A(s) J_{QR}(s) J_Q(s)] + \psi_v J_v(s) B(s) + \psi_P J_P(s) B(s) + \psi_A J_A(s) B(s) + \psi_W J_W(s) B(s) + \psi_v J_v(s) A(s) + \psi_P J_P(s) A(s) + \psi_W J_W(s) A(s) + \psi_A J_A(s) B(s) \quad \text{... (32)}$$

where,

$$K(s) = s + \psi_V + \psi_M + \psi_P + \psi_{1A} + \psi_{1W} + \gamma_Q - \psi_V \overline{S}_V(s) - \psi_P \overline{S}_P(s) - \psi_M \overline{S}_M(s) - \gamma_Q \overline{S}_{QR}(s) - \left[\gamma_Q \overline{S}_{QR}(s) + \psi_A \overline{S}_A(s)\right]\frac{\psi_{1A}}{C_1} - \left[\gamma_Q \overline{S}_{QR}(s) + \psi_W \overline{S}_W(s)\right]\frac{\psi_{1w}}{c_2} - \psi_Q \gamma_Q \overline{S}_Q(s)\left[1 + \frac{\psi_{1W}}{C_1} + \frac{\psi_{1A}}{C_2}\right] J_{QR}(s) \quad \text{… (33)}$$

$$J_i(s) = \frac{1 - \overline{S_v(s)}}{s}, \text{for } i = V, P, M, QR, Q, A, W \quad \text{… (34)}$$

$$J_{QR}(s) = \frac{1 - \overline{S}_{QR}(s)}{s + \psi_Q} \quad \text{… (35)}$$

$$A(s) = \frac{\psi_{1A}}{C_1} \quad \text{… (36)}$$

$$B(s) = \frac{\psi_{1W}}{C_2} \quad \text{… (37)}$$

$$C_1 = s + \psi_V + \psi_M + \psi_P + \psi_A + \psi_W + \gamma_Q - \psi_V \overline{S}_V(s) - \psi_P \overline{S}_P(s) - \psi_W \overline{S}_W(s) \quad \text{… (38)}$$

$$C_2 = s + \psi_V + \psi_M + \psi_P + \psi_A + \psi_W + \gamma_Q - \psi_V \overline{S}_V(s) - \psi_P \overline{S}_P(s) - \psi_A \overline{S}_A(s) \quad \text{… (39)}$$

$$\overline{S}_i(j) = \int_0^\infty \phi_i(j) \exp[-s_j - \int_0^i \phi_i(j) dj] dj, \;\; for \;\; i = V, P, M, QR, Q, A, W \;\&\; j = x, y, r, q, h, g, u. \quad \ldots (40)$$

$$\phi_M = \exp\left[r^\theta + \left[\log \phi_M(r)\right]^\theta\right]^{1/\theta} \quad \ldots (41)$$

Verification

$$\overline{P}_{up}(s) + \overline{P}_{down}(s) = \frac{1}{s} \quad \ldots (42)$$

Steady state behaviour of the system

Using Abel's lemma in Laplace transforms, viz;

$$\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t) = f(say) \quad \ldots (43)$$

provided the limit on the right hand side exists, the time independent operational probabilities are obtained as follows.

$$P_0 = \frac{1}{K'(0)} \quad \ldots (44)$$

$$P_{1W} = \frac{B(0)}{K'(0)} \quad \ldots (45)$$

$$P_{1A} = \frac{A(0)}{K'(0)} \quad \ldots (46)$$

$$P_v = \frac{\psi_v}{K'(0)} T_v \quad \ldots (47)$$

$$P_M = \frac{\psi_M}{K'(0)} T_M \quad \ldots (48)$$

$$P_P = \frac{\psi_P}{K'(0)} T_P \quad \ldots (49)$$

$$P_{QR} = \frac{\gamma_Q}{K'(0)} \left[1 + B(0) + A(0)\right] \frac{T_{QR}}{\psi_Q} \quad \ldots (50)$$

$$P_Q = \frac{\gamma_Q \psi_Q}{K'(0)} \left[1 + B(0) + A(0)\right] \frac{T_{QR}}{\psi_Q} T_Q \quad \ldots (51)$$

$$P_{VW} = \frac{\psi_V B(0)}{K'(0)} T_V \quad \ldots (52)$$

$$P_{AW} = \frac{\psi_A B(0)}{K'(0)} T_A \quad \ldots (53)$$

$$P_{PW} = \frac{\psi_P B(0)}{K'(0)} T_P \quad \ldots (54)$$

$$P_W = \frac{\psi_W B(0)}{K'(0)} T_W \quad \ldots (55)$$

$$P_{VA} = \frac{\psi_V A(0)}{K^{'}(0)} T_V \quad \text{… (56)}$$

$$P_{PA} = \frac{\psi_P A(0)}{K^{'}(0)} T_P \quad \text{… (57)}$$

$$P_{WA} = \frac{\psi_W A(0)}{K^{'}(0)} T_W \quad \text{…(58)}$$

$$P_{A} = \frac{\psi_A A(0)}{K^{'}(0)} T_A \quad \text{…(59)}$$

where,

$$K^{'}(0) = \left[\frac{d}{ds} K(s)\right]_{s=0} \quad \text{… (60)}$$

$T_i = -\overline{S}_i(0)$ =Mean time to repair the i^{th} failure … (61)

$$A(0) = \frac{\psi_{1A}}{\psi_M + \psi_A + \gamma_Q} \quad \text{… (62)}$$

$$B(0) = \frac{\psi_{1W}}{\psi_M + \psi_A + \gamma_Q} \quad \text{… (63)}$$

$$\lim_{s \to 0} T_{QR}(s) = \frac{1}{\psi_Q} M_{QR}$$ (As s tends to 0) … (64)

$$S_{\phi_i}(j) = \frac{\phi_i}{j + \phi_i} \quad \text{… (65)}$$

Particular cases

When repairs follow exponential distribution then for all i and j in equations (31) and (32), one may obtain the following transition state probabilities.

$$\overline{P}_0(s) = \frac{1}{T(s)} \quad \text{… (66)}$$

$$\overline{P}_V(s) = \frac{\psi_V}{T(s)[s + \phi_V]} \quad \text{… (67)}$$

$$\overline{P}_P(s) = \frac{\psi_P}{T(s)[s + \phi_P]} \quad \text{… (68)}$$

$$\overline{P}_M(s) = \frac{\psi_M}{T(s)[s + \phi_M]} \quad \text{… (69)}$$

$$\overline{P}_{QR}(s) = \frac{\gamma_Q}{T(s)}[1 + D(s) + E(s)]\frac{1}{s + \phi_{QR}} \quad \text{… (70)}$$

$$\overline{P}_Q(s) = \frac{\gamma_Q \psi_Q}{T(s)}[1 + D(s) + E(s)]\left[\frac{1}{s + \phi_{QR}}\right]\left[\frac{1}{s + \phi_Q}\right] \quad \text{… (71)}$$

$$\overline{P}_{VW}(s)=\frac{\psi_V E(s)}{T(s)[s+\phi_V]} \qquad \ldots (72)$$

$$\overline{P}_{AW}(s)=\frac{\psi_A E(s)}{T(s)[s+\phi_A]} \qquad \ldots (73)$$

$$\overline{P}_{PW}(s)=\frac{\psi_P E(s)}{T(s)[s+\phi_P]} \qquad \ldots (74)$$

$$\overline{P}_{W}(s)=\frac{\psi_W E(s)}{T(s)[s+\phi_W]} \qquad \ldots (75)$$

$$\overline{P}_{VA}(s)=\frac{\psi_V D(s)}{T(s)[s+\phi_V]} \qquad \ldots (76)$$

$$\overline{P}_{PA}(s)=\frac{\psi_P D(s)}{T(s)[s+\phi_P]} \qquad \ldots (77)$$

$$\overline{P}_{WA}(s)=\frac{\psi_W D(s)}{T(s)[s+\phi_W]} \qquad \ldots (78)$$

$$\overline{P}_{A}(s)=\frac{\psi_A D(s)}{T(s)[s+\phi_A]} \qquad \ldots (79)$$

where,

$$T(s)=s+\psi_V+\psi_M+\psi_P+\psi_{1A}+\psi_{1W}+\gamma_Q-\frac{\psi_V\phi_V}{s+\phi_V}-\frac{\psi_P\phi_P}{s+\phi_P}-\frac{\psi_M\phi_M}{s+\phi_M}-\frac{\gamma_Q\phi_{QR}}{s+\phi_{QR}}$$
$$-\left[\frac{\gamma_Q\phi_{QR}}{s+\phi_{QR}}+\frac{\psi_A\phi_A}{s+\phi_A}\right]D(s)-\left[\frac{\gamma_Q\phi_{QR}}{s+\phi_{QR}}+\frac{\psi_W\phi_W}{s+\phi_W}\right]E(s)-$$
$$\psi_Q\gamma_Q\frac{\phi_Q}{s+\phi_Q}[1+D(s)+E(s)]\frac{1}{[s+\phi_{QR}]} \qquad \ldots (80)$$

$$D(s)=\frac{\psi_{1A}}{G-\left[\frac{\psi_V\phi_V}{s+\phi_V}+\frac{\psi_P\phi_P}{s+\phi_P}+\frac{\psi_W\phi_W}{s+\phi_W}\right]} \qquad \ldots (81)$$

$$E(s)=\frac{\psi_{1W}}{G-\left[\frac{\psi_V\phi_V}{s+\phi_V}+\frac{\psi_P\phi_P}{s+\phi_P}+\frac{\psi_A\phi_A}{s+\phi_A}\right]} \qquad \ldots (82)$$

$$G=s+\psi_V+\psi_M+\psi_P+\psi_A+\psi_W+\gamma_Q \qquad \ldots (83)$$

Non repairable system

If the system is non repairable then the repair rates will be zero and probabilities will be independent of *x*, then the reliability function is given by

$$\overline{\mathrm{R}}(s)=\frac{1}{s+\psi_v+\psi_P+\psi_{1A}+\psi_{1W}+\psi_M+\gamma_Q} \qquad \ldots (84)$$

where, R(s) is the Laplace transform of the reliability function.

By taking the inverse laplace transform of eq. (84), the reliability of the system in terms of time is obtained as:

$$R(t) = \exp[-(\psi_V + \psi_M + \psi_P + \psi_{1A} + \psi_{1W} + \gamma_Q)t] \quad \text{… (85)}$$

Consider $\psi_v = .005, \psi_P = .006, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02$ in equation (85) and varying values of t as 0, 1, 2, 3, 4…., one can obtain Figure-3 which shows how the reliability varies with respect to time.

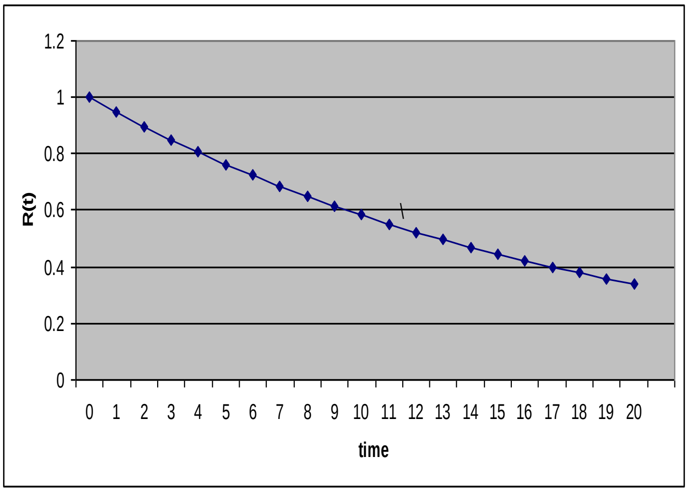


Figure 3. Reliability Vs Time

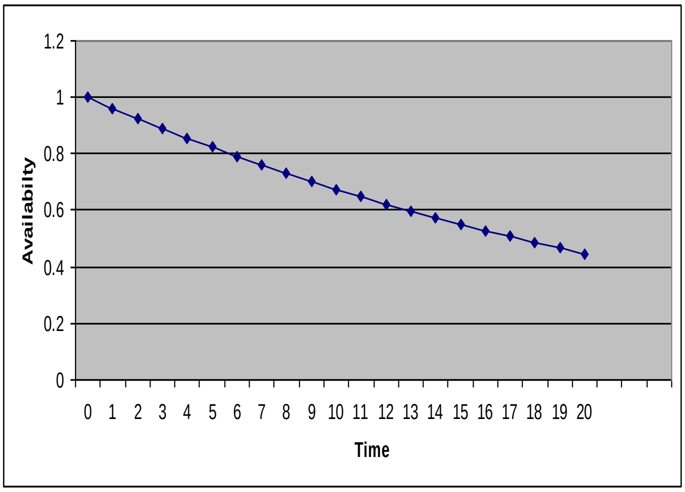


Figure 4. Availability Vs Time

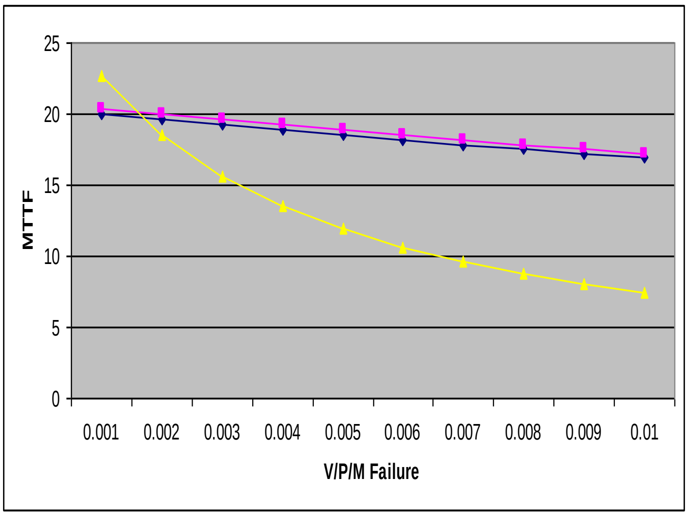


Figure 5. MTTF Vs Vendor, Paint shop and Machine failure

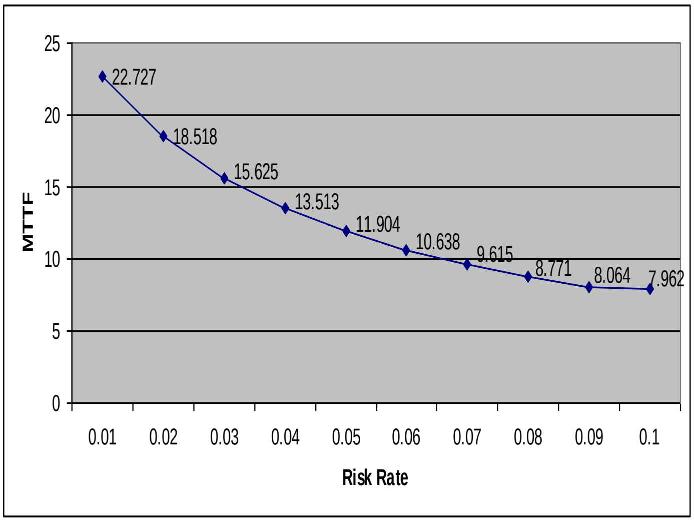
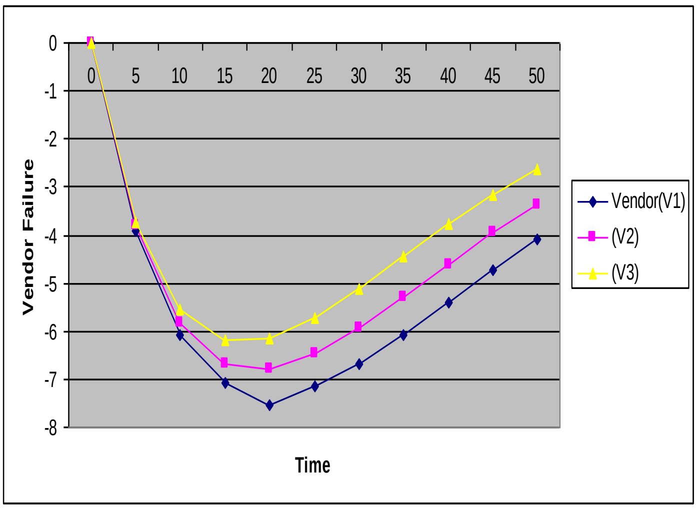


Figure 6. MTTF Vs Risk Rate



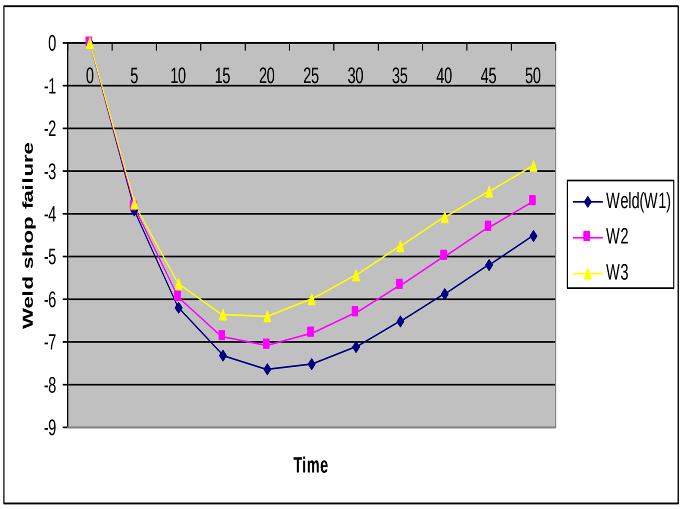
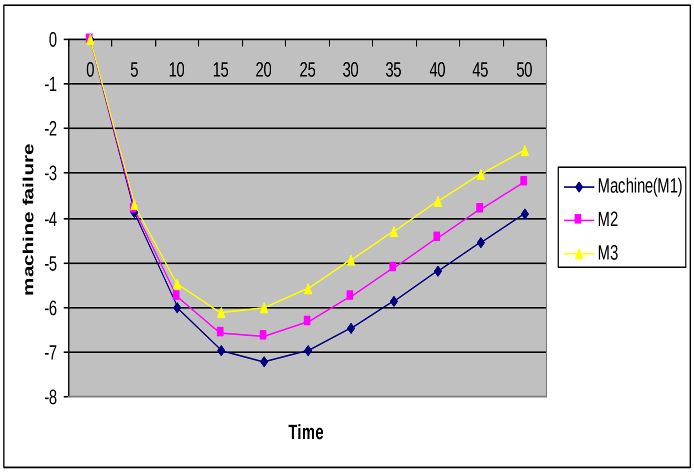


Figure 7. Sensitivity of system reliability w. r. t. Vendor failure **Figure 8: Sensitivity of system reliability w. r. t. Weld shop failure**



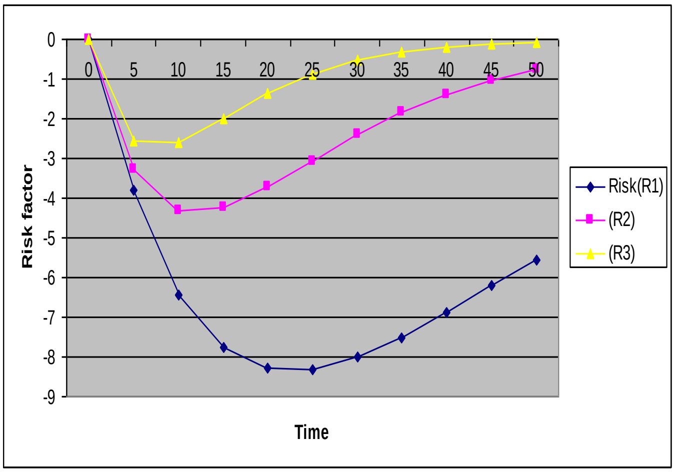
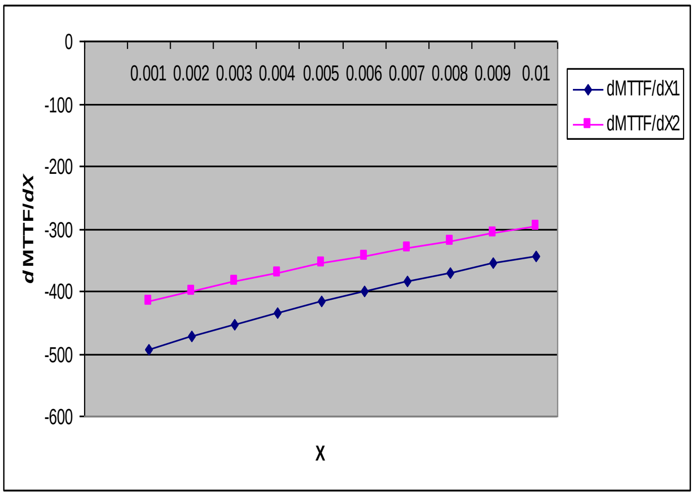


Figure9. Sensitivity of system reliability w. r. t. Machine failure **Figure 10. Sensitivity of system reliability w. r. t. Risk factor**



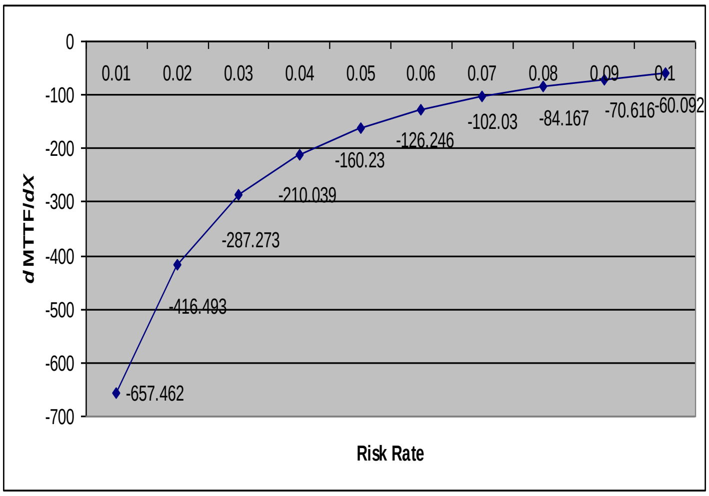


Figure 11 Sensitivity for MTTF with respect to X$_1$=ψ_V , X$_2$=ψ_P . (X=X$_1$=X$_2$) **Figure 12 Sensitivity for MTTF with respect to X= γ_Q (γ_Q =.01,.02,….,.1) ($\psi_V = \psi_P$ =.001, .002 ….01)**

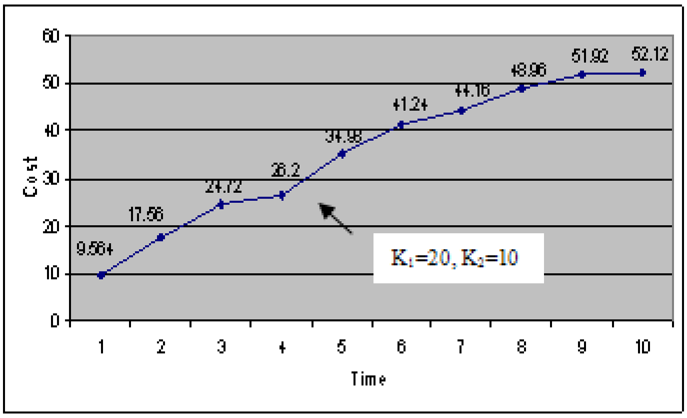


Figure 13. Cost Vs Time

Availability of the system

$$\overline{Pup}(s) = P_0(s) + P_{1W}(s) + P_{1A}(s)$$

$$= \frac{1}{K(s)}\left[1 + B(s) + A(s)\right] \quad \text{... (86)}$$

Setting, $\psi_v = .005, \psi_P = .006, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02,$ and repair rates ϕ_i =1, *i*= V, P, M, Q, QR, A, W and *x= y= r= q= h= g= u=1*. When repair rates follow exponential distribution, equation (86) becomes,

$$\overline{P}up(s) = \frac{1 + \frac{.016}{s + .055}}{s + .054}$$

Taking inverse Laplace transforms, we have

$$Pup(t) = -16.e^{(-.0550000000t)} + 17.e^{(-.0540000000t)} \quad \text{… (87)}$$

Now, varying, *t*=0, 1, 2, 3…….20, in equations (87), we get the change of availability of the system with respect to time which is given in Figure-4.

MTTF of the system

The mean time to failure (MTTF) is given as under:

$$\text{MTTF} = \lim_{s \to 0} \overline{P}\text{up}(s) = \int_0^\infty R(t)dt$$

$$= \frac{1}{\psi_V + \psi_M + \psi_P + \psi_{1A} + \psi_{1W} + \gamma_Q} \quad \text{… (88)}$$

a) Setting, $\psi_P = .006, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02$ and putting ψ_v =.001,.002,.003....in equation (88) one can get Figure-5 which exhibits the variation of MTTF for different values of Vendor failure.

b) Consider, $\psi_v = .005, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02$ and putting ψ_P =.001,.002,.003…in equation (88), one can obtain the MTTF with respect to different values of Paint shop failure as shown in Figure-5.

c) Let $\psi_v = .005, \psi_P = .006, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02$ and substituting $\psi_M = .001,.002,.003,.004,..,$ in equation (88), we get Figure-5 that gives us the changes of MTTF for various values of Machine failure.

d) Assuming, $\psi_v = .005, \psi_P = .006, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009$ and taking γ_Q =.01,.02, .03, .04 … in equation (88), we have Figure-6 which shows the variation of MTTF for a range of values of Risk failure.

Sensitivity analysis for system reliability and MTTF

First we perform sensitivity analysis for changes in the system reliability resulting from changes in system parameters ψ_V ,ψ_P and γ_Q [13]**.** Differentiating equation (85) with respect to ψ_V , we obtain

$$\frac{\partial R(t)}{\partial \psi_V} = -te^{(-(\psi_V + \psi_P + \psi_{1A} + \psi_{1W} + \psi_M + \gamma_Q)*t)} \quad \text{… (89)}$$

Using the same procedure we can get $\frac{\partial R(t)}{\partial \psi_W}$, $\frac{\partial R(t)}{\partial \psi_M}$ and $\frac{\partial R(t)}{\partial \gamma_Q}$.

Setting $\psi_P = .006, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02$ and putting ψ_v =.001, .005 and .01, in equation (89), we have Figure-7 which shows the sensitivity of system reliability with respect to Vendor.

Using the same procedure described above, we can get $\frac{\partial R(t)}{\partial \psi_W}$, $\frac{\partial R(t)}{\partial \psi_M}$ and $\frac{\partial R(t)}{\partial \gamma_Q}$. Numerical results of the sensitivity analysis for $\frac{\partial R(t)}{\partial \psi_W}$, $\frac{\partial R(t)}{\partial \psi_M}$ and $\frac{\partial R(t)}{\partial \gamma_Q}$ are presented in Figures-8, 9 and10 respectively.

Now we provide a sensitivity analysis of MTTF of the system with respect to the system parameters ψ_V, ψ_P and γ_Q. Differentiating equation (88) with respect to ψ_V, we obtain

$$\frac{\partial}{\partial \psi_V} MTTF = -\frac{1}{(\psi_v + \psi_P + \psi_{1A} + \psi_{1W} + \psi_M + \gamma_Q)^2} \quad \text{… (90)}$$

Setting $\psi_P = .006, \psi_M = .007, \psi_{1W} = .007, \psi_{1A} = .009, \gamma_Q = .02$ and putting ψ_v =.001, .005 and .01, in equation (90), we have Figure-11 which shows the sensitivity of MTTF with respect to Vendor. Similarly, by the same procedure we can obtain $\partial MTTF / \partial \psi_P$ and $\partial MTTF / \partial \gamma_Q$ as given in Figures-11 and 12 respectively.

Table 1 Shows the state specification of the transition diagram

States	Description	System State
S_0	The state when the system is in fully operational condition.	G
S_1	The state when the system is in failed state due to the failure of Vendor.	F_R
S_2	The state when the system is in operable state when only two machines (out of three) of the weld shop are working.	G
S_3	The state when the system is in failed state due to the failure of Paint shop.	F_R
S_4	The state when the system is in operable state when only two machines (out of three) of the assembly shop are working.	G
S_5	The state when the system is in failed state due to the machine failure (major and minor).	F_R
S_6	The state when the system is in risk state due the mistake of quality department.	RS
S_7	The state when the system is in failed state from the risk state due to the fault of quality department	F_R
S_8	The state when the system is in failed state from the state S_2 due to the failure of Vendor.	F_R
S_9	The state when the system is in failed state from the state S_2 due to the failure of assembly shop.	F_R
S_{10}	The state when the system is in failed state from the state S_2 due to the failure of two machines of the weld shop.	F_R
S_{11}	The state when the system is in failed state from the state S_2 due to the failure of Paint shop.	F_R
S_{12}	The state when the system is in failed state from the state S_4 due to the failure of Vendor.	F_R
S_{13}	The state when the system is in failed state from the state S_4 due to the failure of weld shop.	F_R
S_{14}	The state when the system is in failed state from the state S_4 due to the failure of Paint shop.	F_R
S_{15}	The state when the system is in failed state from the state S_4 due to the failure of two machines of the assembly shop.	F_R

Note: G= Good state; F_R= Failed state; RS = Risk state.

Cost effectiveness of the system

Cost function for considered system is given by

$$G(t) = K_1 \cdot \int_0^t P_{up}(t)dt - K_2 t$$

where, K_1 and K_2 are revenue and repair costs per unit time, respectively.

$$G(t) = K_1 \int_0^t e^{(-(\psi_v + \psi_P + \psi_{1A} + \psi_{1W} + \psi_M + \gamma_Q)t)} + \left[\frac{\psi_{1A} + \psi_{1W}}{\psi_{1A} + \psi_{1W} - \psi_W - \psi_A}\right] e^{(-(\psi_v + \psi_P + \psi_{1A} + \psi_{1W} + \psi_M + \gamma_Q)t)} -$$

$$\left[\frac{\psi_{1A}+\psi_{1W}}{\psi_{1A}+\psi_{1W}-\psi_{W}-\psi_{A}}\right]e^{(-(\psi_v+\psi_P+\psi_A+\psi_W+\psi_M+\gamma_Q)t)}dt-K_2t$$

… (91)

Setting, $\psi_v=.005,\psi_P=.006,\psi_M=.007,\psi_{1W}=.007,\psi_{1A}=.009,\psi_W=.005,\psi_A=.007,\gamma_Q=.02$ K_1=20, K_2= 10 and varying t=0, 1 2 ….10 in equations (91), we can get the cost effectiveness of the system as given in Figure-13.

Result and discussion

In this study, we analyzed the reliability, availability, MTTF, sensitivity and cost effective of the complex system incorporating risk factor. To numerically examine the behaviour of reliability and availability of the system, the various parameters are fixed as $\psi_v=.005,\psi_P=.006,\psi_M=.007,\ \psi_{1W}=.007,\psi_{1A}=.009,\ \gamma_Q=.02$ and subsequent values obtained have been shown in Figure-3. One can easily conclude from Figure-3 that the reliability of the system decreases with passage of time when all failures follow exponential time distribution. Figure-4 represents the variation of availability of the system. Critical examination of Figure-4 yields that the values of the availability decreases approximately in a constant manner with the increment in time. Further, by fixing the values of the parameters and varying one parameter we get the change of MTTF with respect to that parameter as exhibited in Figure-5. Observation of this figure reveals that the MTTF decreases with the increment in Vendor failure rate, Paint Shop failure rate and machine failure. Also it is interesting to mention here that at some points MTTF with respect to different parameters are found to be same. Further, Figure-6 shows that the MTTF of the system decreases smoothly for increased values of risk rate. The sensitivities of the system reliability with respect to ψ_V ,ψ_{1W}, ψ_M and γ_Q are shown in Figures-7, 8, 9 and 10 respectively. It can easily be observed that all the system parameters have the biggest impact approximately at the same time. Furthermore, we also observed that ψ_V and γ_Q are prominent parameters and almost have the equal sensitive effect on the system reliability. Table-2 shows that the sensitivity with respect to ψ_V and ψ_p on the MTTF which increases rapidly from -493.82 to -355.98 and -416.49 to -307.77 as ψ_V and ψ_p varies from .001 to .01 respectively as shown in Figure-11. Figure-12 reveals the sensitivity of MTTF for various values of risk rate γ_Q. To examine the profit function, revenue cost per unit time is taken as 20 and service cost 10, the result obtained have been shown in Figure-13. From this observation, it is very clear that the profit decreases as the service cost increases.

Table 2. Sensitivity analysis of MTTF with respect to ψ_V and ψ_P

ψ_V/ψ_P	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
$\partial MTTF/\partial\psi_V$	-493.82	-472.58	-452.69	-434.02	-416.49	-400	-384.46	-369.82	-355.98
$\partial MTTF/\partial\psi_P$	-416.49	-400	-384.46	-369.82	-355.99	-342.93	-330.57	-318.87	-307.77

Conclusion

In this paper, the operational readiness of an automobile assembly system is discussed using mathematical modelling approach. Also, the present study discussed the transition state probabilities, steady state probabilities, reliability, availability, MTTF analysis for Vendor, Paint Shop, Machine, Risk failure rate, sensitivity analysis and variation of costs with respect to time. Supplementary variable technique is used to change a non markovian process into markovian process. Further, the copula approach allows us to incorporate two different distributions in repair simultaneously hence overcome some of the well known limitations of traditional methods. It also provides greater flexibility as well as allows us a much wider range of possible dependence structures. The proposed method has the advantages of modelling and analyzing system reliability in a more flexible and intelligent manner.

This analysis may help managerial staff in the following ways.

a. Managing resources, Vendors.

b. Taking timely decisions.

c. Planning preventive maintenance policies.

d. Planning strategies of production.

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