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Quality improved integrated inventory model with trade credit and preservation technology

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ABSTRACT

Chang et al proposed an integrated inventory model with an order-size-dependent trade credit. However, quality issues were not discussed in their model. It is unrealistic for a production system to produce 100 percent good products. This paper extends Chang et al. model and studies an integrated supplier-retailer inventory model where trade credit and defective items are considered. Therefore, to incorporate the concept of supplier – retailer integration and order-size-dependent trade credit, we present a stylized model to determine the optimal strategy for an integrated supplier -retailer inventory system under the condition of trade credit linked to the order quantity, where the demand rate is considered to be a decreasing function of the retail price. In this paper, we propose an integrated supplier–retailer inventory model in which both supplier and retailer have adopted trade credit policies, and the retailer receives an arriving lot containing some defective items. An attempt is made to characterize the preservation technology for deteriorating items to reduce the deterioration rate. Moreover, we consider the capital investment in quality improvement. The objective of our analysis is to determine the optimal ordering, shipping, and quality improvement policies to maximize joint total profit per unit time. Results have been validated with relevant examples.

Introduction

In most of the early literature dealing with inventory problems, emphases were placed on either the retailer's or the supplier's perspective to minimize the cost and/or to maximize the profit. Recently, the integrated inventory models have become more and more important, because the supplier and the retailer can increase their mutual benefit through strategic cooperation. Goyal [1] developed an integrated model for a single supplier-single retailer system to find the optimal order quantity of the retailer so that the total cost at the system is minimized. Monahan [2] examined the quantity discount problem from the supplier point of view and obtained the minimum cost of the entire supply chain. Banerjee [3] presented a joint economic-lot-size model where a supplier produces for a retailer to order on a lot-for-lot basis. Goyal [4] generalized Banerjee's [3] model by relaxing the assumption of the lot-for-lot policy of the supplier and illustrated that the inventory cost can be reduced significantly if the supplier's economic production quantity is a positive integer multiple of the retailer's purchase quantity. Lu [5] assumed that the supplier's production rate is greater than the demand rate, and the delivery starts as soon as the quantity ordered by the retailer is produced, and later on goods are delivered on a lot-for-lot basis. Goyal [6] relaxed the lot-for-lot policy and assumed that if the demand is constant, shipment sizes will increase according to the ratio of production rate and demand rate. Goyal and Nebebe [7] proposed the first shipment to be smaller and is followed by shipments of equal size. Recently, Ouyang et al. [8] proposed an integrated inventory model with quality improvement and lead time reduction. Other related studies of the integrated inventory model include Yang and Wee [9], Yang et al. [10], Wee and Chung [11], Teng et al. [12], and so on. In the traditional inventory models, the theme of defective items is always ignored. However, defective items can by caused by the incomplete production process and/or damage in transit. And the number of defective items will influence the on-hand level and the number of orders in the inventory system. In addition, if the retailer sells defective items without inspection, the customers will complain, return the goods, or even never come back. In all cases, substantial costs are incurred. Already there are some scholars who have studied and developed various analytical inventory models about defective items. Porteus [13] and Rosenblatt and Lee [14] are among the first ones who analyzed a significant relationship between quality imperfection and lot size. Next, Paknejad et al. [15] proposed a modified EOQ model with stochastic demand, and the model

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included the number of defective items in a lot as a random variable. In each delivery, the defective items will be found in each lot and sent back to the supplier in the delivery time of the next batch. Salameh and Jaber [16] presented an EPQ model with defective items, and they assumed that the production rate for the non-defective items is greater than the demand rate. Ouyang and Chang [17] presented an investment in quality improvement inventory model involving defective items production process with controllable lead time. There are more papers related to this issue of defective items such as Chung and Hou [18], Hou [19], Rahim and Al-Hajailan [20], Lin [21], Wee et al. [22], Sarkar [23],and Barzoki et al. [24], etc.Furthermore, in practical situations, in order to motivate retailer to increase order quantity and market share, the supplier often offers a trade credit to the retailer, that is, the retailer may receive goods or services without having to pay until sometime later. Haley and Higgins [25] first presented an inventory model with the permissible delay in payments. Ferris [26] derived a transactions theory of trade credit use from the motives of trading partners to economize on the joint costs of exchange. Kingsman [27] considered the effects of different ways of payment on ordering and stocking. Goyal [28] established an EOQ inventory model with interest earned and paid under the condition of permissible delay in payments. Aggarwal and Jaggi [29] extended Goyal's [28] model to include deteriorating items. Jamal et al. [30] further generalized this issue with allowable shortages. Buzacott and Zhang [31] proposed an inventory management to incorporate asset-based financing into production decisions. In their paper, the retailers buy a product from the suppliers and then sell it to the customers in which the retailers require asset-based financing by bank to purchase product from the suppliers. Among other relative inventory financing issues studies were Hill and Riener [32], Abad and Jaggi [33], Chen and Kang [34], Huang and Hsu [35], Ho et al.[36] and Thangam and Uthayakumar [37].Because of changing of the business environment, the delay payments of trade credit change with each passing day. There exist numerous interesting and relevant papers related to trade credits, but most assume that the supplier offers a trade credit to the retailer. However, the retailer wishes to motivate the customer's demand rate and to reduce the on-hand stock cost, and offers a trade credit to the customers. Huang [38] considered an EOQ inventory model in which both supplier and retailer have adopted trade credit policies. Su et al. [39] developed an integrated supplier-retailer inventory model in which the customer's demand for goods is positively correlated to the credit period offered by the retailer. They discussed how to obtain optimal order quantity, shipping, and inventory policy. In most of the inventory models in the literature, the rate of deterioration of goods is viewed as an exogenous variable, which is not subjected to control. Deteriorating inventory had been studied in the past decades and authors usually focused on constant or variable deterioration rate. Investing on preservation technology (PT) for reducing deterioration rate has received little attention in the past years. The consideration of PT is important due to rapid social changes and the fact that PT can reduce the deterioration rate significantly. Moreover, sales, inventory and order quantities are very sensitive to the rate of deterioration, especially for fast deteriorating products for example, fruits, flowers and sea foods. The effect of preservation technology is used to reduce the deterioration rate. This paper extends Chang et al model and studies an integrated supplier-retailer inventory model where trade credit and defective items are considered. Numerical example has been used to illustrate the results given in this paper.

Notations and assumptions

In this paper, the mathematical model is developed on the basis of the following notation and assumptions.

Notation

P The supplier's production rate, P > D

D The retailer's demand rate

K The supplier's setup cost per order

A The retailer's ordering cost per unit ordered

F Transportation cost per shipment

h_v The supplier's holding cost per item per unit time

h_{b1} The retailer's holding cost of non-defective items per unit time, excluding interest charges

h_{b2} The retailer's holding cost of defective items per unit time(including treatment cost), excluding interest charges , $h_{b2} \le h_{b1}$

s The retailer's unit inspection cost

x The retailer's inspection rate per order

θ_o The supplier's defective rate of production quantity before the capital investment , $0 < \theta_o < 1$

θ Defective rate of production quantity through the capital investment $0 \le \theta \le \theta_o$

$C(\theta)$ Capital investment required to reduce the defective rate from θ_o to θ.

δ Percentage decrease in θ per $ increase in investment $C(\theta)$

c The supplier's production cost per unit

v The retailer's unit purchasing price, $v > c$

p The retailer's unit selling price, $p > v$

w The supplier's unit treatment cost of defective items

$\propto$ The retailer's capital opportunity cost per $ per unit time

f_{vc} The supplier's interest earned per $ unit time when buyer pay earlier during [0, M]

I_{vp} The supplier's interest paid per $ unit time

I_{Be} The retailer's interest earned per $ unit time

I_{Bp} The retailer's interest paid per $ unit time

Q The retailer's order quantity (for non-defective items) per order

Q_d The threshold quantity set by supplier at which the full delay payment permitted

β Proportion of partial delay payment permitted by the supplier

q The quantity which the supplier transports to the retailer per shipment

n number of shipments from the supplier to the retailer per order, a positive integer

M length of delay payment

T length of replenishment cycle

T_d Time interval in which Q_d/n units are depleted to zero due to demand

C_P The supplier's prevention cost for per item

C_S The supplier's screening cost

t The supplier's transportation maintenance cost

ζ Preservation technology cost for reducing deterioration rate in order to preserve the product

Assumptions

1. Consider single-supplier single-retailer for a single item in infinite planning horizon.
2. To avoid the shortage, the production rate of non-defective items needs to greater than demand rate. That is, $(1 - \theta) P > D$.
3. The retailer's order quantity Q (for non-defective items) and requests the supplier to transport the order quantity in n equally sized shipments, where n is a positive integer.
4. The relationship between the supplier's production cost (c), the retailer's purchase cost (v) and retail price (p) is $p > v > c$.
5. The defective items found through the retailer's inspection process will return to the supplier in a batch at the next beginning of replenishment time. Therefore, the retailer received items from the supplier, in which the quantity of non-defective is $(1 - \theta)q$, the length of replenishment cycle $T = \dfrac{(1-\theta)q}{D}$, the quantity per shipment $q = \dfrac{Q}{[n(1 - \theta)]}$, and the order quantity Q is the sum of all non-defective in n times = $n(1 - \theta)q = nDT$.
6. If the retailer's order quantity reach the threshold quantity (i.e. $Q \ge Q_d$). The supplier provides full delay payment and the credit period is M. Otherwise the supplier provides partial delay payment with β proportion ($0 \le \beta < 1$) and the remaining balance $1 - \beta$

proportion should pay immediately when the goods arrived, therefore the supplier can use the balance to earn interest rate f_{vc} during the period [0, M].

7. The capital investment, $C(\theta)$, in improving process quality (reducing defective rate) is given by the logarithmic function $C(\theta) = \frac{1}{\delta}\ln\left(\frac{\theta_0}{\theta}\right),\ 0 < \theta \le \theta_0 < 1.$

Mathematical model:

The Supplier's total profit per unit time :

The supplier's total profit per unit time included the sales revenue, the interest earned, the setup cost, the holding cost, opportunity cost, the screening cost, the prevention cost an the transportation maintenance cost. These components are calculated as follows

Sales revenue $D(v - c)$

Set up cost $\left(\frac{K}{nT}\right)$.

Holding cost $\left(\frac{h_v DT}{1-\theta}\left[\frac{D}{P(1-\theta)} + \frac{n-1}{2} - \frac{nD}{2P(1-\theta)}\right]\right)$

Handling cost of defective items $\left(\frac{\omega\theta q}{T} = \frac{\omega\theta D}{1-\theta}\right)$

Opportunity cost of capital investment in quality $\left(\alpha C(\theta) = \frac{\alpha}{\delta}\ln\left(\frac{\theta_0}{\theta}\right)\right)$

The interest earned during [0, M] is

$$\frac{(1-\beta)vQf_{vc}M}{nT} = (1-\beta)Dvf_{vc}M \quad \text{when } Q < Q_d (T < T_d) \quad \left[Q\ \frac{Q}{nT} = D\right]$$

$$= 0 \text{ when } Q \ge Q_d (T \ge T_d)$$

The opportunity cost due to delay payment is

$$\frac{\beta v(1-\theta)qI_{Vp}M}{T} = \beta Dv(1-\theta)qI_{Vp}M \quad \text{when } Q < Q_d (T < T_d) \quad \left[Q\ T = \frac{(1-\theta)q}{D}\right]$$

$$\frac{v(1-\theta)qI_{Vp}M}{T} = DvI_{Vp}M \quad \text{when } Q \ge Q_d (T \ge T_d)$$

The screening cost is given by DC_s

The prevention cost is given by QC_p

The transportation maintenance cost is given by nt.

Hence the supplier's total profit per unit time denoted by $STP(\theta)$ can be expressed as follows

$$STP(\theta) = \begin{cases} STP_1(\theta) & \text{if } T < T_d \\ STP_2(\theta) & \text{if } T \ge T_d \end{cases}$$

where

$STP_1(\theta)$ = Sales revenue – setup cost – holding cost – handling cost of defective items – opportunity cost of capital investment – opportunity cost due to delay payment – screening Cost - prevention Cost- transportation maintenance cost + Interest.

$$= D(v-c) - \frac{K}{nT} - \frac{h_v DT}{1-\theta}\left[\frac{D}{P(1-\theta)} + \frac{n-1}{2} - \frac{nD}{2P(1-\theta)}\right] - \frac{w\theta D}{1-\theta} - \frac{\alpha}{\delta}\ln\left(\frac{\theta_0}{\theta}\right)$$

$$- \beta DvI_{Vp}M - DC_S - QC_P - nt + (1-\beta)Dvf_{Vc}M$$

$$STP_2(\theta) = D(v-c) - \frac{K}{nT} - \frac{h_v DT}{1-\theta}\left[\frac{D}{P(1-\theta)} + \frac{n-1}{2} - \frac{nD}{2P(1-\theta)}\right] - \frac{w\theta D}{1-\theta} - \frac{\alpha}{\delta}\ln\left(\frac{\theta_0}{\theta}\right)$$

$$- DvI_{Vp}M - Dc_s - Qc_p - nt.$$

The Retailer's total profit per unit time :

The Retailer's total profit per unit time is composed of

Sales revenue $D(p - v)$

Ordering cost $\left(\frac{A}{nT}\right)$

Fixed Transportation Cost $\left(\frac{F}{T}\right)$

Holding Cost $\left(\frac{h_{b1}DT}{2}\left[1+\frac{\theta D}{x(1-\theta)^2}\right] + \frac{h_{b2}\theta DT}{1-\theta}\left[1-\frac{D}{2x(1-\theta)}\right]\right)$

Inspection Cost $\left(\frac{sq}{T} - \frac{sD}{1-\theta}\right)\left[Q\ T = \frac{(1-\theta)q}{D}\right]$

Preservation technology cost $\left(\frac{\zeta}{T}\right)$

Interest earned during [0, M]

Case 1.1

$$\frac{pI_{Be}}{T}\left(\int_0^T Dt\,dt + DT(M-T)\right) = \frac{pI_{Be}}{T}\left(\frac{DT^2}{2} + DT(M-T)\right)$$

$$= \frac{pI_{Be}}{T}\left(DTM - \frac{DT^2}{2}\right)$$

$$= DpI_{Be}\left(M - \frac{T}{2}\right) \quad \text{when } T < T_d \le M$$

Case 1.2

$$\frac{pI_{Be}}{T}\left(\int_0^T Dt\,dt + DT(M-T)\right) = DpI_{Be}\left(M - \frac{T}{2}\right) \text{ when } T \le M \le T_d$$

Case 1.3

$$\frac{pI_{Be}}{T}\int_0^T Dt\,dt = \frac{pI_{Be}}{T}\left[\frac{DM^2}{2}\right] = DpI_{Be}\frac{M^2}{2T} \text{ when } M \le T < T_d$$

Case 2.1

$$\frac{pI_{Be}}{T}\left[\int_0^T Dt\,dt + DT(M - T)\right] = DpI_{Be}(M - T/2) \text{ when } T_d \le T \le M$$

Case 2.2

$$\frac{pI_{Be}}{T}\int_0^M Dt\,dt = \frac{DpI_{Be}M^2}{2T} \text{ when } T_d \le M \le T$$

Case 2.3

$$\frac{pI_{Be}}{T}\int_0^M Dt\,dt = \frac{DpI_{Be}M^2}{2T} \text{ when } M \le T_d \le T$$

Opportunity cost due to partial delay payment

Case 1.1

$$\frac{(1-\beta)QvI_{BP}M}{nT} = (1-\beta)DvI_{BP}M \text{ when } T < T_d \le M$$

Case 1.2

$$\frac{(1-\beta)QvI_{BP}M}{nT} = (1-\beta)DvI_{BP}M \text{ when } T \le M \le T_d$$

Case 1.3

$$\frac{(1-\beta)QvI_{BP}M}{nT} = (1-\beta)DvI_{BP}M \text{ when } M \le T < T_d$$

Case 2.1

$$\frac{(1-\beta)QvI_{BP}M}{nT} = 0 \text{ when } T_d \le T \le M$$

Case 2.2

$$\frac{(1-\beta)QvI_{BP}M}{nT} = 0 \text{ when } T_d \le M \le T$$

Case 2.3

$$\frac{(1-\beta)QvI_{BP}M}{nT} = 0 \text{ when } M \le T_d \le T$$

Opportunity cost for the items still on hand

Case 1.1 $\quad 0 \quad$ when $T < T_d \le M$

Case 1.2 $\quad 0 \quad$ when $T \le M \le T_d$

Case 1.3

$$\frac{vI_{BP}}{T}\int_M^T D(T-t)\,dt = \frac{DvI_{BP}}{2T}(T-M)^2 \text{ when } M \le T < T_d$$

Case 2.1 $\quad 0 \quad$ when $T_d \le T \le M$

Case 2.2

$$\frac{vI_{BP}}{T}\int_M^T D(T-t)\,dt = \frac{DvI_{BP}}{2T}(T-M)^2 \text{ when } T_d \le M \le T$$

Case 2.3

$$\frac{vI_{BP}}{T}\int_M^T D(T-t)\,dt = \frac{DvI_{BP}}{2T}(T-M)^2 \text{ when } M \le T_d \le T$$

$\therefore$ The retailer's total profit per unit time (denote by RTP(n, T) can be expressed as follows

$$RTP(n,T) = \begin{cases} RTP_1(n,T) & \text{if } T < T_d \\ RTP_2(n,T) & \text{if } T \ge T_d \end{cases}$$

where

$$RTP_1(n,T) = \begin{cases} RTP_{11}(n,T) & \text{if } T < T_d \le M \\ RTP_{12}(n,T) & \text{if } T \le M \le T_d \\ RTP_{13}(n,T) & \text{if } M \le T < T_d \end{cases}$$

$$RTP_2(n,T) = \begin{cases} RTP_{21}(n,T) & \text{if } T_d \le T \le M \\ RTP_{22}(n,T) & \text{if } T_d \le M \le T \\ RTP_{23}(n,T) & \text{if } M \le T_d \le T \end{cases}$$

and

$RTP_{11}(n, T)$ = Sales revenue – ordering cost - fixed transportation cost - holding cost – inspection cost - preservation technology cost-opportunity cost due to delay payment – opportunity cost for the items still on hand + interest

$$= D(p-v) - \frac{A}{nT} - \frac{F}{T} - \frac{h_{b1}DT}{2}\left[1+\frac{\theta D}{x(1-\theta)^2}\right] - \frac{h_{b2}\theta DT}{1-\theta}\left[1-\frac{D}{2x(1-\theta)}\right] - \frac{sD}{1-\theta} - \frac{\zeta}{T} - (1-\beta)DvI_{BP}M + DpI_{Be}\left(\frac{M-T}{2}\right)$$

$RTP_{12}(n, T) = RTP_{11}(n, T)$

$$RTP_{13}(n,T) = D(p-v) - \frac{A}{nT} - \frac{F}{T} - \frac{h_{b1}DT}{2}\left[1+\frac{\theta D}{x(1-\theta)^2}\right] - \frac{h_{b2}\theta DT}{1-\theta}\left[1-\frac{D}{2x(1-\theta)}\right] - \frac{sD}{1-\theta} - \frac{\zeta}{T} - (1-\beta)DvI_{BP}M - \frac{DvI_{BP}(T-M)^2}{2T} + \frac{DpI_{Be}M^2}{2T}$$

$$RTP_{21}(n,T) = D(p-v) - \frac{A}{nT} - \frac{F}{T} - \frac{h_{b1}DT}{2}\left[1+\frac{\theta D}{x(1-\theta)^2}\right] - \frac{h_{b2}\theta DT}{1-\theta}\left[1-\frac{D}{2x(1-\theta)}\right] - \frac{sD}{1-\theta} - \frac{\zeta}{T} + DpI_{Be}\left(M - \frac{T}{2}\right)$$

$$RTP_{22}(n,T) = D(p-v) - \frac{A}{nT} - \frac{F}{T} - \frac{h_{b1}DT}{2}\left[1+\frac{\theta D}{x(1-\theta)^2}\right] - \frac{h_{b2}\theta DT}{1-\theta}\left[1-\frac{D}{2x(1-\theta)}\right] - \frac{sD}{1-\theta} - \frac{\zeta}{T} - \frac{DvI_{BP}(T-M)^2}{2T} + \frac{DpI_{Be}M^2}{2T}$$

$RTP_{23}(n, T) = RTP_{22}(n, T)$

The joint total profit per unit time :

This integrated inventory system is made up through the cooperation between the supplier and the retailer. Therefore the joint total profit per unit time (denoted by $JTP(n, T, \theta)$) can be expressed as :

$$JTP(n, T, \theta) = \begin{cases} JTP_1(n, T, \theta) \text{ if } T < T_d \\ JTP_2(n, T, \theta) \text{ if } T \geq T_d \end{cases}$$

where

$$JTP_1(n, T, \theta) = \begin{cases} JTP_{11}(n, T, \theta) = STP_1(\theta) + RTP_{11}(n, T), \text{ if } T < T_d \leq M \\ JTP_{12}(n, T, \theta) = STP_1(\theta) + RTP_{12}(n, T), \text{ if } T < M \leq T_d \\ JTP_{13}(n, T, \theta) = STP_1(\theta) + RTP_{13}(n, T), \text{ if } M \leq T < T_d \end{cases}$$

$$JTP_2(n, T, \theta) = \begin{cases} JTP_{21}(n, T, \theta) = STP_2(\theta) + RTP_{21}(n, T), \text{ if } T_d \leq T \leq M \\ JTP_{22}(n, T, \theta) = STP_2(\theta) + RTP_{22}(n, T), \text{ if } T_d \leq M \leq T \\ JTP_{23}(n, T, \theta) = STP_2(\theta) + RTP_{23}(n, T), \text{ if } M \leq T_d \leq T_d \end{cases}$$

Solution Procedure:

First to find the optimum value of n for given T and θ. Taking the first order partial derivative of $JTP_1(n, T, \theta)$ and $JTP_2(n, T, \theta)$ with respect to n for given T and θ, respectively we get

$$\frac{\partial}{\partial n} JTP_i(n, T, \theta) = \frac{K}{n^2T} - \frac{h_vDT}{2(1-\theta)} + \frac{h_vD^2T}{2P(1-\theta)^2} - t + \frac{A}{n^2T}, \text{ i= 1,2}$$

The second order partial derivative with respect to n for given T and θ is given by

$$\frac{\partial^2}{\partial n^2} JTP_i(n, T, \theta) = \frac{-2K}{n^3T} - \frac{-2A}{n^3T} = \frac{-2(A+K)}{n^3T} < 0, i = 1, 2$$

Equating the first order partial derivative to zero, we get

$$\frac{K+A}{n^2T} - \frac{h_vDT}{2(1-\theta)} + \frac{h_vD^2T}{2P(1-\theta)^2} - t = 0$$

$$\Rightarrow \frac{K+A}{n^2T} = t + \frac{h_vDT}{2(1-\theta)} - \frac{h_vD^2T}{2P(1-\theta)^2}$$

$$n = \sqrt{\frac{K+A}{T\left(t + \frac{h_vDT}{2(1-\theta)} - \frac{h_vD^2T}{2P(1-\theta)^2}\right)}}$$

To solve T and θ that makes the joint total profit maximum for given n. It is discussed in 6 cases as follows.

Case 1.1 : $T < T_d \leq M$

For any given n and θ, taking the first order partial derivative of $JTP_{11}(n, T, \theta)$ with respect to T we obtain

$$\frac{\partial}{\partial T} JTP_{11}(n, T, \theta) = \frac{K}{nT^2} - \frac{h_vD}{1-\theta}\left(\frac{D}{P(1-\theta)} + \frac{n-1}{2} - \frac{nD}{2P(1-\theta)}\right) + \frac{A}{nT^2} + \frac{F}{T^2} + \frac{\zeta}{T^2}$$
$$- \frac{h_{b1}D}{2}\left(1 + \frac{\theta D}{x(1-\theta)^2}\right) - \frac{h_{b2}\theta D}{1-\theta}\left(1 - \frac{D}{2x(1-\theta)}\right) + DpI_{Be}\left(\frac{-1}{2}\right)$$

$$= \frac{K+A+Fn+\zeta n}{nT^2} - \frac{DpI_{Be}}{2} - \frac{D^2}{(1-\theta)^2}\left(\frac{h_v}{P} - \frac{nh_v}{2P} + \frac{\theta h_{b1}}{2x} - \frac{h_{b2}\theta}{2x}\right)$$

$$- D\left[\frac{h_v(n-1)}{2(1-\theta)} + \frac{h_{b1}}{1-\theta} + \frac{h_{b2}\theta}{1-\theta}\right]$$

$$= \frac{K+A+Fn+\zeta n}{nT^2} - \frac{DpI_{Be}}{2} - \frac{D^2}{(1-\theta)^2}\left(\frac{h_v}{P}\left(1-\frac{n}{2}\right) + \frac{\theta}{2x}\left(h_{b1} - h_{b2}\right)\right)$$

$$- D\left[\frac{h_v(n-1)}{2(1-\theta)} + \frac{h_{b1}}{1-\theta} + \frac{h_{b2}\theta}{1-\theta}\right]$$

$$\frac{\partial}{\partial T} JTP_{11}(n, T, \theta) = \frac{S_n}{nT^2} - \frac{DpI_{Be}}{2} - \phi_{n\theta} = 0$$

where $S_n = K+A + Fn + \zeta n > 0$ and

$$\phi_{n\theta} = \frac{D^2}{(1-\theta)^2}\left[\frac{h_v(1-n/2)}{P} + \frac{\theta}{2x}\left[h_{b1} - h_{b2}\right]\right] - D\left[\frac{h_v(n-1)}{2(1-\theta)} + \frac{h_{b1}}{2} + \frac{h_{b2}\theta}{1-\theta}\right] > 0$$

Then, taking the second order partial derivative of $JTP_{11}(n, T, \theta)$ with respect to T, we get $\frac{\partial^2}{\partial T^2} JTP_{11}(n, T, \theta) = \frac{-2S_n}{nT^3} < 0$

Equating the first order partial derivative to zero, we get

$$\frac{S_n}{nT^2} - \frac{DpI_{Be}}{2} - \phi_{n\theta} = 0$$

$$\frac{S_n}{nT^2} = \frac{DpI_{Be}}{2} + \phi_{n\theta}$$

$$T = T_{11}(n,\theta) = \sqrt{\frac{S_n}{n(\phi_{n\theta} + \frac{DpI_{Be}}{2})}}$$

Hence $JTP_{11}(n, T, \theta)$ has maximum value at $T = T_{11}(n, \theta)$ for given n and θ.

Case 1.2 : $T \le M < T_d$

$$\frac{\partial}{\partial T} JTP_{12}(n, T, \theta) = \frac{\partial}{\partial T} JTP_{11}(n, T, \theta)$$ and

$$\frac{\partial^2}{\partial T^2} JTP_{12}(n, T, \theta) = \frac{-2S_n}{nT^3} < 0$$

$$T = T_{12}(n,\theta) = \sqrt{\frac{S_n}{n(\phi_{n\theta} + \frac{DpI_{Be}}{2})}}$$

Hence $JTP_{12}(n, T, \theta)$ has maximum value at $T = T_{12}(n, \theta)$ for given n and θ.

Case 1.3 : $M \le T < T_d$

$$\frac{\partial}{\partial T} JTP_{13}(n, T, \theta) = \frac{K}{nT^2} - \frac{h_v D^2}{P(1-\theta)^2} - \frac{h_v D(n-1)}{2(1-\theta)} + \frac{h_v n D^2}{2P(1-\theta)^2} + \frac{A}{nT^2} + \frac{F}{T^2} - \frac{h_{b1} D}{2} - \frac{h_{b1}\theta D^2}{2x(1-\theta)^2}$$

$$-\frac{h_{b2}\theta D}{1-\theta}+\frac{h_{b2}\theta D^2}{2x(1-\theta)^2}-\frac{DpI_{Be}M^2}{2T^2}-\frac{DVI_{BP}(T-M)}{T}+\frac{DVI_{BP}(T-M)^2}{2T^2}+\frac{\zeta}{T^2}$$

$$=\frac{K+A+F_n+\zeta n}{nT^2}-\frac{D^2}{(1-\theta)^2}\left[\frac{h_v(1-n/2)}{P}+\frac{\theta}{2x}\left(h_{b1}-h_{b2}\right)\right]$$

$$-D\left[\frac{h_v(n-1)}{2(1-\theta)}+\frac{h_{b1}}{2}+\frac{h_{b2}\theta}{1-\theta}\right]-\frac{DpI_{Be}M^2}{2T^2}$$

$$-\frac{DVI_{BP}}{2}+\frac{DVI_{BP}M^2}{2T^2}$$

$$=\frac{S_n}{nT^2}-\phi_{n\theta}-\frac{DVI_{BP}}{2}+\frac{DVI_{BP}M^2}{2T^2}-\frac{DpI_{Be}M^2}{2T^2}\quad\frac{S_n}{nT^2}-\phi_{n\theta}-\frac{DVI_{BP}}{2}+\frac{DVI_{BP}M^2}{2T^2}-\frac{DpI_{Be}M^2}{2T^2}$$

The second order partial derivative

$$\frac{\partial^2}{\partial T^2}JTP_{13}(n, T, \theta)=\frac{-2S_n}{nT^3}-\frac{DVI_{BP}M^2}{T^3}+\frac{DpI_{Be}M^2}{T^3}$$

Equating the first order partial derivative to zero , we get

$$\frac{S_n}{nT^2}-\phi_{n\theta}-\frac{DVI_{BP}}{2}+\frac{DVI_{BP}M^2}{2T^2}-\frac{DpI_{Be}M^2}{2T^2}=0$$

$$\Rightarrow\frac{2S_n+DVI_{BP}M^2.\,n-nDpI_{Be}M^2}{2nT^2}=\phi_{n\theta}+\frac{DVI_{BP}}{2}$$

$$\Rightarrow\frac{2S_n-nDM^2(pI_{Be}-VI_{BP})}{nT^2}=2\phi_{n\theta}+DVI_{BP}$$

$$\Rightarrow\frac{2S_n-nDM^2(pI_{Be}-VI_{BP})}{n\left[2\phi_{n\theta}+DVI_{BP}\right]}=T^2$$

$$\Rightarrow T=T_{13}(n,\theta)=\sqrt{\frac{2S_n-nDM^2(pI_{Be}-VI_{BP})}{n\left[2\phi_{n\theta}+DVI_{BP}\right]}}$$

Hence $JTP_{13}(n, T, \theta)$ has maximum value at $T=T_{13}(n, \theta)$ for given n and θ.

Case 2.1 : $T_d \le T \le M$

$$\frac{\partial}{\partial T}JTP_{22}(n, T, \theta)=\frac{K}{nT^2}-\frac{h_vD}{1-\theta}\left(\frac{D}{P(1-\theta)}+\frac{n-1}{2}-\frac{nD}{2P(1-\theta)}\right)+\frac{A}{nT^2}+\frac{F}{T^2}$$

$$-\frac{h_{b1}D}{2}\left(1+\frac{\theta D}{x(1-\theta)^2}\right)-\frac{h_{b2}\theta D}{1-\theta}\left(1-\frac{D}{2x(1-\theta)}\right)+DpI_{Be}\left(\frac{-1}{2}\right)+\frac{\zeta}{T^2}$$

$$=\frac{K+A+Fn+\zeta n}{nT^2}-\frac{DpI_{Be}}{2}-\frac{D^2}{(1-\theta)^2}\left(\frac{h_v}{P}\left(1-\frac{n}{2}\right)+\frac{\theta}{2x}\left(h_{b1}-h_{b2}\right)\right)$$

$$-D\left[\frac{h_v(n-1)}{2(1-\theta)}+\frac{h_{b1}}{1-\theta}+\frac{h_{b2}\theta}{1-\theta}\right]$$

$$\frac{\partial}{\partial T}JTP_{21}(n, T, \theta)=\frac{S_n}{nT^2}-\frac{DpI_{Be}}{2}-\phi_{n\theta}=0\qquad\frac{\partial^2}{\partial T^2}JTP_{21}(n, T, \theta)=\frac{-2S_n}{nT^3}<0$$

Equating the first order partial derivative to zero, we get

$$\frac{S_n}{nT^2} - \frac{DpI_{Be}}{2} - \phi_{n\theta} = 0$$

$$\frac{S_n}{nT^2} = \frac{DpI_{Be}}{2} - \phi_{n\theta}$$

$$T = T_{21}(n,\theta) = \sqrt{\frac{S_n}{n(\phi_{n\theta} + \frac{DpI_{Be}}{2})}}$$

Hence $JTP_{21}(n, T, \theta)$ has maximum value at $T = T_{21}(n, \theta)$ for given n and θ.

Case 2.2 : $T_d \le M \le T$

$$\frac{\partial}{\partial T} JTP_{22}(n, T, \theta) = \frac{K}{nT^2} - \frac{h_v D^2}{P(1-\theta)^2} - \frac{h_v D(n-1)}{2(1-\theta)} + \frac{h_v n D^2}{2P(1-\theta)^2} + \frac{A}{nT^2} + \frac{F}{T^2} - \frac{h_{b1} D}{2} - \frac{h_{b1}\theta D^2}{2x(1-\theta)^2}$$

$$-\frac{h_{b2}\theta D}{1-\theta} + \frac{h_{b2}\theta D^2}{2x(1-\theta)^2} - \frac{DpI_{Be}M^2}{2T^2} - \frac{DVI_{BP}(T-M)}{T} + \frac{DVI_{BP}(T-M)^2}{2T^2} + \frac{\zeta}{T^2}$$

$$= \frac{S_n}{nT^2} - \phi_{n\theta} - \frac{DVI_{BP}}{2} + \frac{DVI_{BP}M^2}{2T^2} - \frac{DpI_{Be}M^2}{2T^2}$$

The second order partial derivative

$$\frac{\partial^2}{\partial T^2} JTP_{22}(n, T, \theta) = \frac{-2S_n}{nT^3} - \frac{DVI_{BP}M^2}{T^3} + \frac{DpI_{Be}M^2}{T^3} < 0$$

Equating the first order partial derivative to zero , we get

$$\frac{S_n}{nT^2} - \phi_{n\theta} - \frac{DVI_{BP}}{2} + \frac{DVI_{BP}M^2}{2T^2} - \frac{DpI_{Be}M^2}{2T^2} = 0$$

$$\Rightarrow \frac{2S_n + DVI_{BP}M^2.\, n - nDpI_{Be}M^2}{2nT^2} = \phi_{n\theta} + \frac{DVI_{BP}}{2}$$

$$\Rightarrow \frac{2S_n - nDM^2(pI_{Be} - VI_{BP})}{nT^2} = 2\phi_{n\theta} + DVI_{BP}$$

$$\Rightarrow \frac{2S_n - nDM^2(pI_{Be} - VI_{BP})}{n\left[2\phi_{n\theta} + DVI_{BP}\right]} = T^2$$

$$\Rightarrow T = T_{22}(n, \theta) = \sqrt{\frac{2S_n - nDM^2(pI_{Be} - VI_{BP})}{n\left[2\phi_{n\theta} + DVI_{BP}\right]}}$$

Hence $JTP_{22}(n, T, \theta)$ has maximum value at $T = T_{22}(n, \theta)$ for given n and θ.

Case 2.3 : $M \le T_d \le T$

$$\frac{\partial}{\partial T} JTP_{23}(n, T, \theta) = \frac{\partial}{\partial T} JTP_{22}(n, T, \theta)$$

and

$$\frac{\partial^2}{\partial T^2} JTP_{23}(n, T, \theta) = \frac{\partial^2}{\partial T^2} JTP_{22}(n, T, \theta) < 0$$

Equating the first order partial derivative to zero ,we get

$$T = T_{23}(n, \theta) = \sqrt{\frac{2S_n - nDM^2(pI_{Be} - VI_{BP})}{n\left[2\phi_{n\theta} + DVI_{BP}\right]}}$$

Hence $JTP_{23}(n, T, \theta)$ has maximum value at $T = T_{23}(n, \theta)$ for given n and θ.

Next , taking the first order partial derivative of JTP(n, T, θ) with respect to θ for given n nd T is given by

$$\frac{\partial}{\partial\theta}JTP(n, T, \theta) = \frac{2D^2h_v(1\text{-}n/2)}{P(1\text{-}\theta)^3} + \frac{D^2(h_{b1} - h_{b2})(1+\theta)}{2x(1-\theta)^3} + \frac{D}{(1-\theta)^2}\left[\frac{h_v(n\text{-}1)}{2} + h_{b2}\right] - \frac{D(s+w)}{(1-\theta)^2} + \frac{\alpha}{\delta\theta}$$

Taking the second order partial derivative

$$\frac{\partial^2}{\partial\theta^2}JTP(n, T, \theta) = -T\left\{\frac{6D^2h_v(1\text{-}n/2)}{P(1\text{-}\theta)^4} + \frac{D^2(h_{b1} - h_{b2})(2+\theta)}{x(1-\theta)^4} + \frac{D}{(1-\theta)^3}\left[h_v(n\text{-}1) + 2h_{b2}\right]\right\} - \frac{D(s+w)}{(1-\theta)^3} - \frac{\alpha}{\delta\theta^2} < 0$$

Hence θ is the optimal solution for given n and T.

Therefore the optimal shipment times n, replenishment cycle T and defective rate θ that makes the joint total profit JTP(n, T, θ) maximum.

Numerical Examples

Example 1: To illustrate the above solution procedure, we consider an inventory system with the following data: D=10,000 units/year, P=30,000 units/year, A=\$50 /order, K=\$120 /setup, F=\$25 /shipment, c=\$11 /unit, v=\$20 /unit, p=\$25 /unit, h_v=\$0.2 /unit/year, h_{b1}=\$0.2 /unit/year, h_{b2}=\$0.1 /unit/year, w=\$5 /unit, s=\$0.5 /unit, x=175,200 units/year, M=30 days(=0.0822 year), f_{Vc}=0.02, I_{Vp}=0.05, I_{Be}=0.025, I_{Bp}=0.035, $\delta = 0.001,\ \alpha = 0.2,\ \ \beta = 0.3,\ \theta_0 = 0.02$ Q= 2466units; $Q_d = 3000 units; T_d = 0.1$; $n = 3; C(\theta) = \$1715.48;\ C_p = \$0.02$; $C_s = \$0.01;\ t = \$3; \zeta = \$75$

Following the proposed models, we can obtain the optimal replenishment cycle T*=0.0822<0.15=Td (i.e., the optimal order quantity Q*=2466<3000=Q_d). Hence, the vendor only offers partial delay payment and pays capital investment amount C(θ)=\$1,715.48 which improves the defective rate from 0.02 to 0.00359756. As a result, the total profit per year of vendor is \$88,245.24 buyer is \$44,091.74, and joint is \$132336.98.

Example 2: In this example, we consider an inventory system with the following data: D=5,000 units/year, P=15,000 units/year, A=\$30 /order, K=\$100 /setup, F=\$15 /shipment, c=\$8 /unit, v=\$13 /unit, p=\$15 /unit, h_v=\$0.2 /unit/year, h_{b1}=\$0.2 /unit/year, h_{b2}=\$0.1 /unit/year, w=\$4 /unit, s=\$1 /unit, x=1,00,000units/year, M=15 days(=0.0412 year), f_{Vc}=0.01,I_{Vp}=0.03,I_{Be}=0.015, I_{Bp}=0.013, $\delta = 0.003,\ \alpha = 0.1,\ \ \beta = 0,\ \theta_0 = 0.01;$ Q= 3000units; $Q_d = 2466 units; T_d = 0.3$; $n = 4; C(\theta) = \$2000; C_p = \$0.03;$ $C_s = \$0.005;\ t = \$5; \zeta = \$50$

Following the proposed models, we can obtain the optimal replenishment cycle T*=0.0412<0.3=Td (i.e., the optimal order quantity Q*3000>2466=Q_d). Hence, the vendor only offers partial delay payment and pays capital investment amount C(θ)=\$2,000 . As a result, the total profit per year of vendor is \$24,327.48, buyer is \$4,462.04, and joint is \$28,789.52.

Conclusion

This paper finds that the supplier should set the proportion of partial delay payment and the threshold quantity more careful, therefore the supplier can avoid the more loss in profit and to attract the sales more effective. And the more quantity the retailer orders, the more capital investment the supplier has to pay. This study highlights the concept of preservation technology when demand depends on credit period and selling price. Numerical example is presented to illustrate the theoretical results.

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