Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 72 (2014) 25487-25500

ISSN: 2229-712X

Dispersion analysis of non-homogeneous transversely isotropic electro-magneto-elastic plate of arbitrary cross-section

P. Ponnusamy and A. Amuthalakshmi

Department of Mathematics, Government Arts College, Coimbatore – 641 018, Tamil nadu, India.

ARTICLE INFO

Article history:
Received: 2 May 2014;
Received in revised form:
19 June 2014;
Accepted: 2 July 2014;

Keywords
Vibrations of electro-magneto-elastic plates/cylinders,
Mechanical vibrations,
Stress-strain relations,
Electro-magneto-elastic materials,
Piezoelectric materials.

ABSTRACT

This paper analyzes the vibration of non-homogeneous transversely isotropic electro-magneto-elastic plate of arbitrary cross-section using the Fourier expansion collocation method. The frequency equations are derived for the arbitrary cross-sectional boundary conditions, since the boundary is irregular in shape; it is difficult to satisfy the boundary along the surface of the plate directly. Hence, the Fourier expansion collocation method is applied along the boundary to satisfy the boundary conditions. The secant method is applied to determine the roots of the frequency equation. The non-dimensional frequencies are computed numerically and are plotted in the form of dispersion curves and further their characteristics are discussed. This problem may be extended to any kinds of cross-sections using the proper geometrical relations.

Introduction

The vibration of non-homogeneous transversely isotropic electro-magneto-elastic plate has been of great importance due to its property of coupling effect between electric and magnetic fields which are applied in smart material structures. These materials have the ability to transfer energy namely magnetic, electric and mechanical from one form to another. The composite consisting of piezoelectric and piezomagnetic components have found increasing application in engineering structures, particularly in smart/intelligent structure system. The electro-magneto-elastic materials are used as magnetic field probes, electric packing, acoustic, hydrophones, medical, ultrasonic, image processing, sensor and actuators with the responsibility of electro-magnetic-mechanical energy conversion.

Wave propagation in arbitrary cross-sectional plates and cylinders were analyzed and to find out the phase velocities in different modes of vibration namely longitudinal, torsional and flexural by constructing frequency equation was derived by Nagaya [1-3]. He formulated the Fourier expansion collocation method for this purpose and the same method is used in this problem. Wang and Shen [4] discussed the two-dimensional problem of inclusion of arbitrary shape in magneto-electro-elastic composites. Buchanan [5] investigated free vibration of an infinite magneto-electro-elastic cylinder. Recently Abd-Alla et al. [6] studied the effect of magnetic field and non-homogeneous in various elastic media. Abd-Alla and Mahmoud [7, 8] investigated magneto-thermo elastic problems in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model and the effect of the rotation on propagation of thermoelastic waves in non-homogeneous infinite cylinder of isotropic material. Chen and Chen [9] investigated the Love wave behavior in magneto-electro-elastic multilayered structures by the propagation matrix method. Using the propagator matrix and state-over approaches, an analytical treatment is presented for the propagation of harmonic waves in magneto-electro-elastic multilayered plates by Chen et al. [10]. Chen et al. [11, 12] studied the free vibration and general solution of non-homogeneous transversely isotropic magneto-electro-elastic hollow cylinder. Chen et al. [13] showed theoretically that there actually exists a class of vibration of which the frequencies depend on the elastic property only. Wei and Su [14] studied the wave propagation and energy transportation along cylindrical piezoelectric piezomagnetic material. Hou and Leung [15] obtained the analytical solution for the axisymmetric plane strain magneto-electro-elastic dynamics of hollow cylinder for axisymmetric flexural wave in piezoelectric – piezomagnetic cylinders. Later Hou et al. [16] discussed the transient response of non-homogenous plane strain problem.

Tele:
E-mail addresses: ponnusamypp@yahoo.com

Pan [17] derived an exact three-dimensional solution for a simply supported multilayered orthotropic magneto-electro-elastic plate. Pan and Heyliger [18] investigated the free vibration of piezoelectric – magnetostrictive plate. Chen et al. [19] derived the general solution for transversely isotropic magneto-electro-elastic-thermo-elasticity.Ponnusamy [20-22] investigated the vibration in a generalized thermo elastic solid cylinder of arbitrary cross-section and plate of polygonal cross-section using Fourier expansion collocation method and studied the wave propagation of piezoelectric solid bar of circular cross-section immersed in fluid using secant method. Late the same author [23] discussed the wave propagation in electro-magneto-elastic solid plate of polygonal cross-section using the Fourier expansion collocation method.

In this article, the wave propagation of transversely isotropic non-homogeneous electro-magneto-elastic plate of arbitrary cross-section is studied using the theory of elasticity. For arbitrary cross-sections the boundary is irregular, therefore Fourier collocation technique is applied to obtain the frequency equations. The roots of the frequency equations are obtained using the secant method applicable for complex roots. The computed non-dimensional frequencies are plotted in the form of dispersion curves and the behaviour of the curves is discussed.

Formulation of the Problem

We consider a transversely isotropic non-homogeneous electro-magneto-elastic plate of arbitrary cross-sections. The system displacements and stresses are defined by the polar coordinates r and θ in an arbitrary point inside the plate and denote the displacements u_r in the direction of r and u_θ in the tangential direction θ. The in-plane vibration and displacement of arbitrary cross-sectional plate is obtained by assuming that there is no vibration and displacements along the $z-$axis in the cylindrical coordinate system (r,θ,z). The two-dimensional stress equations of motion, electric and magnetic conduction equation in the absence of body forces are

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{1}{r}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)=\rho\frac{\partial^2 u_r}{\partial t^2},$$

$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{2}{r}\sigma_{r\theta}=\rho\frac{\partial^2 u_\theta}{\partial t^2} \tag{1}$$

The electric conduction equation is

$$\frac{\partial D_r}{\partial r}+\frac{1}{r}D_r+\frac{1}{r}\frac{\partial D_\theta}{\partial\theta}=0, \tag{2}$$

The magnetic conduction equation is

$$\frac{\partial B_r}{\partial r}+\frac{1}{r}B_r+\frac{1}{r}\frac{\partial B_\theta}{\partial\theta}=0, \tag{3}$$

Where,

$$\sigma_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta},$$

$$\sigma_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta},$$

$$\sigma_{r\theta}=2c_{66}e_{r\theta}, \tag{4}$$

$$D_r=\varepsilon_{11}E_r+m_{11}H_r,$$

$$D_\theta=\varepsilon_{11}E_\theta+m_{11}H_\theta, \tag{5}$$

$$B_r=m_{11}E_r+\mu_{11}H_r,$$

$$B_\theta=m_{11}E_\theta+\mu_{11}H_\theta, \tag{6}$$

Where $\sigma_{rr},\sigma_{\theta\theta},\sigma_{r\theta}$ are the stress components, c_{11},c_{12},c_{66} are elastic constants, ε_{11} is the dielectric constants, μ_{11} is the magnetic permeability coefficients, m_{11} is the electro-magneto material coefficients, ρ is the mass density of the material, D_r,D_θ are the electric displacements, B_r,B_θ are the magnetic displacement components.

The strain e_{ij} related to the displacements corresponding to the polar coordinates (r,θ) are given by

$$e_{rr}=\frac{\partial u_r}{\partial r},\ e_{\theta\theta}=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r},\ e_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right), \tag{7}$$

Where u_r, u_θ are the mechanical displacements along the radial, circumferential directions respectively.

The electric field vector $E_i,\ (i=r,\theta)$ is related to the electric potential E as

$$E_r=-\frac{\partial E}{\partial r},\ \ E_\theta=-\frac{1}{r}\frac{\partial E}{\partial\theta}\cdot \tag{8}$$

Similarly, the magnetic field vector $H_i,\ (i=r,\theta)$ is related to the magnetic potential H as

$$H_r=-\frac{\partial H}{\partial r},\ H_\theta=-\frac{1}{r}\frac{\partial H}{\partial\theta}. \tag{9}$$

Substituting Eqs. (7) – (9) to the Eqs. (1) – (6), we obtain

$$\sigma_{rr}=c_{11}\frac{\partial u_r}{\partial r}+c_{12}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}\right),$$

$$\sigma_{\theta\theta}=c_{12}\frac{\partial u_r}{\partial r}+c_{11}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}\right),$$

$$\sigma_{r\theta}=c_{66}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right), \tag{10}$$

and

$$D_r=-\varepsilon_{11}\frac{\partial E}{\partial r}-m_{11}\frac{\partial H}{\partial r},$$

$$D_\theta=-\frac{\varepsilon_{11}}{r}\frac{\partial E}{\partial\theta}-\frac{m_{11}}{r}\frac{\partial H}{\partial\theta},$$

$$B_r=-m_{11}\frac{\partial E}{\partial r}-\mu_{11}\frac{\partial H}{\partial\theta},$$

$$B_\theta=-\frac{m_{11}}{r}\frac{\partial E}{\partial\theta}-\frac{\mu_{11}}{r}\frac{\partial H}{\partial\theta}\cdot \tag{11}$$

The elastic constants c_{11},c_{12},c_{66}, magnetic permeability coefficient μ_{11}, dielectric constants ε_{11}, electromagnetic material coefficients m_{11}, density ρ are expressed as functions of the radial coordinates are

$$c_{11}=(L+V)r^{2m},\ c_{12}=Lr^{2m},\ c_{66}=\frac{Vr^{2m}}{2},\ \mu_{11}=V'r^{2m},\ m_{11}=m_{11}'r^{2m},\ \varepsilon_{11}=\varepsilon_{11}'r^{2m},\ \rho=\rho_0 r^{2m}, \tag{12}$$

Where L,V,V' and ρ_0 are constants, m is the rational number, substituting Eq. (12) in Eqs. (10) – (11), we obtain the stress-displacement equation for non-homogeneous materials

$$\sigma_{rr}=r^{2m}\left[(L+V)\frac{\partial u_r}{\partial r}+L\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}\right)\right],$$

$$\sigma_{\theta\theta}=r^{2m}\left[L\frac{\partial u_r}{\partial r}+(L+V)\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}\right)\right],$$

$$\sigma_{r\theta}=\frac{V}{2}r^{2m}\left(\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}+\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right), \tag{13}$$

and

$$D_r = r^{2m}\left(-\varepsilon_{11}{}'\frac{\partial E}{\partial r} - m_{11}{}'\frac{\partial H}{\partial r}\right),$$

$$D_\theta = r^{2m}\left(-\frac{\varepsilon_{11}{}'}{r}\frac{\partial E}{\partial \theta} - \frac{m_{11}{}'}{r}\frac{\partial H}{\partial \theta}\right),$$

$$B_r = r^{2m}\left(-m_{11}{}'\frac{\partial E}{\partial r} - V'\frac{\partial H}{\partial r}\right),$$

$$B_\theta = r^{2m}\left(-\frac{m_{11}{}'}{r}\frac{\partial E}{\partial \theta} - \frac{V'}{r}\frac{\partial H}{\partial \theta}\right), \tag{14}$$

Substituting Eqs. (13) – (14) into Eqs. (1) – (3), we obtain the set of displacement equations as follows

$$\left(L+V\right)\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2}\right) + \frac{V}{2r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\left(2L+V\right)}{2r}\frac{\partial^2 u_\theta}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\left(2L+3V\right)}{2}\frac{\partial u_\theta}{\partial \theta}$$
$$+\frac{2m}{r}\left(\left(L+V\right)\frac{\partial u}{\partial r} + L\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right)\right) = \rho_0\frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{V}{2}\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2}\right) + \frac{\left(2L+V\right)}{r}\frac{\partial^2 u_r}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\left(2L+3V\right)}{2}\frac{\partial u_r}{\partial \theta}$$
$$+\frac{\left(L+V\right)}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{Vm}{r}\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) = \rho_0\frac{\partial^2 u_\theta}{\partial t^2}$$

$$\varepsilon_{11}{}'\left(\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E}{\partial \theta^2}\right) + m_{11}{}'\left(\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H}{\partial \theta^2}\right) + \frac{2m}{r}\left(\varepsilon_{11}{}'\frac{\partial E}{\partial r} + m_{11}{}'\frac{\partial H}{\partial r}\right) = 0,$$

$$m_{11}{}'\left(\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E}{\partial \theta^2}\right) + V'\left(\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H}{\partial \theta^2}\right) + \frac{2m}{r}\left(m_{11}{}'\frac{\partial E}{\partial r} + V'\frac{\partial H}{\partial r}\right) = 0, \tag{15}$$

The Eq. (15) is a coupled partial differential equation of two displacements, the electric potentials and magnetic potential components.

Solutions of the Problem

To uncouple Eq. (15), we seek the solutions in the following form

$$u_r\left(r,\theta,t\right) = \sum_{n=0}^{\infty}\varepsilon_n\left(r^{-1}\psi_{n,\theta} - \phi_{n,r} + r^{-1}\overline{\psi}_{n,\theta} - \overline{\phi}_{n,r}\right)$$

$$u_\theta\left(r,\theta,t\right) = \sum_{n=0}^{\infty}\varepsilon_n\left(r^{-1}\phi_{n,\theta} - \psi_{n,r} - r^{-1}\overline{\phi}_{n,\theta} - \overline{\psi}_{n,r}\right)$$

$$E\left(r,\theta,t\right) = \sum_{n=0}^{\infty}\varepsilon_n\left(E_n + \overline{E}_n\right)$$

$$H\left(r,\theta,t\right) = \sum_{n=0}^{\infty}\varepsilon_n\left(H_n + \overline{H}_n\right) \tag{16}$$

Where $\varepsilon_n = 1/2$ for $n = 0$, $\varepsilon_n = 1$ for $n \geq 1$, $\phi_n\left(r,\theta\right), \psi_n\left(r,\theta\right), E_n\left(r,\theta\right), H_n\left(r,\theta\right)$ are the displacement potentials for the symmetric mode and $\overline{\phi}_n\left(r,\theta\right), \overline{\psi}_n\left(r,\theta\right), \overline{E}_n\left(r,\theta\right)$ and $\overline{H}_n\left(r,\theta\right)$ are the displacement potentials for the antisymmetric modes of vibrations.

Substituting Eq. (16) in Eq. (15), we get

$$\left(L+V\right)\nabla_1^2\phi_n + 2m\left(\left(\frac{L+V}{r}\right)\frac{\partial \phi_n}{\partial r} - \frac{L}{r^2}\phi_n\right) - \rho_0\frac{\partial^2 \phi_n}{\partial t^2} = 0, \tag{17a}$$

$$\varepsilon_{11}'\nabla_1^2 E_n + m_{11}'\nabla_1^2 H_n + \frac{2m}{r}\left(\varepsilon_{11}'\frac{\partial E_n}{\partial r} + m_{11}'\frac{\partial H_n}{\partial r}\right) = 0, \tag{17b}$$

$$m_{11}'\nabla_1^2 E_n + V'\nabla_1^2 H_n + \frac{2m}{r}\left(m_{11}'\frac{\partial E_n}{\partial r} + V'\frac{\partial H_n}{\partial r}\right) = 0, \tag{17c}$$

and

$$\frac{V}{2}\nabla_1^2\psi_n + Vm\left(\frac{1}{r}\frac{\partial\psi_n}{\partial r} - \frac{\psi_n}{r}\right) = 0, \tag{18}$$

Where $\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\cdot$

We consider the free vibration of non-homogeneous transversely isotropic plate, so we assume the solutions as follows

$$\phi_n(r,\theta,t) = r^{-m}\phi_n(r)\cos n\theta e^{i\omega t},$$

$$E_n(r,\theta,t) = r^{-m}E_n(r)\cos n\theta e^{i\omega t},$$

$$H_n(r,\theta,t) = r^{-m}H_n(r)\cos n\theta e^{i\omega t}, \tag{19}$$

and

$$\psi_n(r,\theta,t) = r^{-2m}\psi_n(r)\cos n\theta e^{i\omega t}\cdot \tag{20}$$

Substituting Eqs. (19)-(20) in the Eqs.(17) and (18), we obtain

$$\phi_n''(r) + \frac{1}{r}\phi_n'(r) + \left(\frac{\rho_0\omega^2 a^2}{(L+V)} - \frac{1}{r^2}\frac{\left((m^2+n^2)(L+V)+2mL\right)}{(L+V)}\right)\phi_n(r) = 0,$$

(i.e.) $$\phi_n''(r) + \frac{1}{r}\phi_n'(r) + \left(\alpha^2 r^2 - \beta^2\right)\phi_n(r) = 0, \tag{21}$$

Where $\alpha^2 = \frac{\rho_0 a^2\omega^2}{(L+V)}$, $\beta^2 = \frac{\left[(m^2+n^2)(L+V)+2mL\right]}{(L+V)}\cdot$

Eq. (21) is a Bessel equation of order β, its solution is

$$\phi_n(r) = \left[A_{1n}J_\beta(\alpha r) + A_{1n}'Y_\beta(\alpha r)\right]\cos n\theta, \tag{22}$$

Where A_{1n} and A_{1n}' are the arbitrary constants, $J_\beta(\alpha r)$ and $Y_\beta(\alpha r)$ denote the Bessel functions of the first and second kind of order β, respectively.

Substitute Eq. (20) into the Eq. (18), we get

$$\psi_n''(r) + \frac{1}{r}\psi_n'(r) + \left(\frac{2\rho_0\omega^2 a^2}{V} - \frac{1}{r^2}\left(4m^2+4m+n^2\right)\right)\psi_n(r) = 0,$$

(i.e.) $$\psi_n''(r) + \frac{1}{r}\psi_n'(r) + \left(k^2 r^2 - \delta^2\right)\psi_n = 0, \tag{23}$$

Eq. (23) is a Bessel equation of order δ, its solution is

$$\psi_n(r) = \left(A_{4n}J_\delta(kr) + A_{4n}'Y_\delta(kr)\right)\sin n\theta, \tag{24}$$

Where A_{4n} and A_{4n}' are arbitrary constants and $J_\delta(kr)$ and $Y_\delta(kr)$ denote the Bessel function of the first and second kind of order δ respectively.

Substituting Eq. (19) into the Eqs. (17b) and (17c), we obtain

$$\left(\varepsilon_{11}{}'\frac{\partial^2 E_n}{\partial r^2}+\frac{\varepsilon_{11}{}'}{r}(2m+1)\frac{\partial E_n}{\partial r}+\frac{\varepsilon_{11}{}'}{r^2}\frac{\partial^2 E_n}{\partial\theta^2}\right)+\left(m_{11}{}'\frac{\partial^2 H_n}{\partial r^2}+\frac{m_{11}{}'}{r}(2m+1)\frac{\partial H_n}{\partial r}+\frac{m_{11}{}'}{r^2}\frac{\partial^2 H_n}{\partial\theta^2}\right)=0,$$

(i.e.)
$$\varepsilon_{11}{}'\left(E_n''(r)+\frac{1}{r}E_n'(r)-\frac{p^2}{r^2}E_n(r)\right)+m_{11}{}'\left(H_n''(r)+\frac{1}{r}H_n'(r)-\frac{p^2}{r^2}H_n(r)\right)=0, \tag{25}$$

and

$$m_{11}{}'\left(E_n''(r)+\frac{1}{r}E_n'(r)-\frac{(m^2+n^2)}{r^2}E_n(r)\right)+V'\left(H_n''(r)+\frac{1}{r}H_n'(r)-\frac{(m^2+n^2)}{r^2}H_n(r)\right)=0,$$

(i.e.)
$$m_{11}{}'\left(E_n''(r)+\frac{1}{r}E_n'(r)-\frac{p^2}{r^2}E_n(r)\right)+V'\left(H_n''(r)+\frac{1}{r}H_n'(r)-\frac{p^2}{r^2}H_n(r)\right)=0, \tag{26}$$

Where $p^2=m^2+n^2\cdot$

Solving Eqs. (25) and (26), we get

$$E_n''(r)+\frac{1}{r}E_n'(r)-\frac{p^2}{r^2}E_n(r)=0, \tag{27}$$

$$H_n''(r)+\frac{1}{r}H_n'(r)-\frac{p^2}{r^2}H_n(r)=0, \tag{28}$$

The general solutions to the Eqs. (27) and (28) are

$$E_n(r,\theta,t)=\left(A_{2n}r^p+A_{2n}{}'r^{-p}\right)\cos n\theta e^{i\omega t},$$

$$H_n(r,\theta,t)=\left(A_{3n}r^p+A_{3n}{}'r^{-p}\right)\cos n\theta e^{i\omega t}, \tag{29}$$

Where $A_{2n}, A_{2n}{}', A_{3n}, A_{3n}{}'$ are the arbitrary constants.

The general solutions to the solid plate of arbitrary cross-sections are considered as

$$\phi_n(r,\theta,t)=A_{1n}J_\beta(\alpha r)\cos n\theta, \tag{30a}$$

$$E_n(r,\theta,t)=A_{2n}r^p\cos n\theta, \tag{30b}$$

$$H_n(r,\theta,t)=A_{3n}r^p\cos n\theta, \tag{30c}$$

and

$$\psi_n(r,\theta,t)=A_{4n}J_\delta(kr)\sin n\theta\cdot \tag{30d}$$

Boundary conditions and frequency equations

In this problem, the free vibration of non-homogeneous transversely isotropic electro-magneto-elastic plate of arbitrary cross-section is considered. Since the boundary is irregular in shape, it is difficult to satisfy the boundary conditions along the surface of the plate directly. Hence, the Fourier expansion collocation method is applied to satisfy the boundary conditions. For the plate, the normal stress σ_{xx} and shearing stresses σ_{xy}, the electric field D_x and the magnetic field B_x is equal to zero for the stress free boundary. Thus, the following types of boundary conditions are assumed for the plate of arbitrary cross-section is

$$(\sigma_{xx})_i=(\sigma_{xy})_i=(D_x)_i=(B_x)_i=0, \tag{31}$$

Where $(\ )_i$ is the value at the boundary Γ_i. Since the vibration displacements are expressed in terms of the coordinates r and θ, it is convenient to treat the boundary conditions when the derivatives in the equations of the stresses are transformed in terms of the coordinates r and θ instead of the coordinates x_i and y_i.

The relations between the displacements for the $i-$th segment of straight line boundaries are

$$u_r = u_r \cos(\theta - \gamma_i) - u_\theta \sin(\theta - \gamma_i),$$
$$u_\theta = u_\theta \cos(\theta - \gamma_i) + u_r \sin(\theta - \gamma_i), \tag{32}$$

Since the angle γ_i between the reference axis and normal of the $i-$th boundary has a constant value in a segment Γ_i, we obtain

$$\frac{\partial r}{\partial x_i} = \cos(\theta - \gamma_i),\ \ \frac{\partial \theta}{\partial x_i} = \frac{1}{r}\sin(\theta - \gamma_i)$$
$$\frac{\partial r}{\partial y_i} = \sin(\theta - \gamma_i),\ \ \frac{\partial \theta}{\partial y_i} = \frac{1}{r}\cos(\theta - \gamma_i). \tag{33}$$

Using the Eqs. (32) and (33), the normal and shearing stresses are transformed as

$$\sigma_{xx} = \left(c_{11}\cos^2(\theta-\gamma_i) + c_{12}\sin^2(\theta-\gamma_i)\right)\frac{\partial u_r}{\partial r} + \frac{1}{r}\left(c_{11}\sin^2(\theta-\gamma_i) + c_{12}\cos^2(\theta-\gamma_i)\right)\left(u_r + \frac{\partial u_\theta}{\partial \theta}\right) +$$
$$c_{66}\left(\frac{u_\theta}{r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{\partial u_\theta}{\partial r}\right)\sin 2(\theta-\gamma_i) = 0,$$
$$\sigma_{xy} = c_{66}\left(\left(\frac{\partial u_r}{\partial r} - \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r}\right)\sin 2(\theta-\gamma_i) + \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)\cos 2(\theta-\gamma_i)\right) = 0,$$
$$D_x = -\varepsilon_{11}\frac{\partial E_r}{\partial r} - m_{11}\frac{\partial H_r}{\partial r} = 0,$$
$$B_x = -m_{11}\frac{\partial E}{\partial r} - \mu_{11}\frac{\partial H}{\partial r} = 0. \tag{34}$$

Applying non-homogeneity to the Eq. (34), we get

$$\sigma_{xx} = \left((L+V)\cos^2(\theta-\gamma_i) + L\sin^2(\theta-\gamma_i)\right)\frac{\partial u_r}{\partial r} + \frac{1}{r}\left((L+V)\sin^2(\theta-\gamma_i) + L\cos^2(\theta-\gamma_i)\right)\left(u_r + \frac{\partial u_\theta}{\partial \theta}\right)$$
$$+\frac{u_\theta}{2}\left(\frac{u_\theta}{r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{\partial u_\theta}{\partial r}\right)\sin 2(\theta-\gamma_i) = 0,$$
$$\sigma_{xy} = \frac{V}{2}\left(\left(\frac{\partial u_r}{\partial r} - \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r}\right)\sin 2(\theta-\gamma_i) + \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)\cos 2(\theta-\gamma_i)\right) = 0,$$
$$D_x = -\varepsilon_{11}\frac{\partial E}{\partial r} - m_{11}\frac{\partial H}{\partial r} = 0,$$
$$B_x = -m_{11}\frac{\partial E}{\partial r} - \mu_{11}\frac{\partial H}{\partial r} = 0. \tag{35}$$

Substituting Eqs. (30a) and (30d) in Eq. (31), the boundary conditions are transformed for stress free arbitrary cross-sectional plate as follows:

$$\left[(S_{xx})_i + (\bar{S}_{xx})_i\right]e^{i\Omega T_a} = 0,$$
$$\left[(S_{xy})_i + (\bar{S}_{xy})_i\right]e^{i\Omega T_a} = 0,$$
$$\left[(E_x)_i + (\bar{E}_x)_i\right]e^{i\Omega T_a} = 0,$$
$$\left[(H_x)_i + (\bar{H}_x)_i\right]e^{i\Omega T_a} = 0,$$

Where

$$S_{xx} = 0.5\left(A_{10}e_0^1 + A_{20}e_0^2 + A_{30}e_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n}e_n^1 + A_{2n}e_n^2 + A_{3n}e_n^3 + A_{4n}e_n^4\right),$$

$$S_{xy} = 0.5\left(A_{10} f_0^1 + A_{20} f_0^2 + A_{30} f_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n} f_n^1 + A_{2n} f_n^2 + A_{3n} f_n^3 + A_{4n} f_n^4\right),$$

$$E_x = 0.5\left(A_{10} g_0^1 + A_{20} g_0^2 + A_{30} g_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n} g_n^1 + A_{2n} g_n^2 + A_{3n} g_n^3 + A_{4n} g_n^4\right),$$

$$H_x = 0.5\left(A_{10} h_0^1 + A_{20} h_0^2 + A_{30} h_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n} g_n^1 + A_{2n} g_n^2 + A_{3n} g_n^3 + A_{4n} g_n^4\right), \tag{36}$$

$$\overline{S}_{xx} = 0.5\overline{e}_0^4 \overline{A}_{40} + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{e}_n^1 + \overline{A}_{2n}\overline{e}_n^2 + \overline{A}_{3n}\overline{e}_n^3 + \overline{A}_{4n}\overline{e}_n^4\right),$$

$$\overline{S}_{xy} = 0.5\overline{f}_0^4 \overline{A}_{40} + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{f}_n^1 + \overline{A}_{2n}\overline{f}_n^2 + \overline{A}_{3n}\overline{f}_n^3 + \overline{A}_{4n}\overline{f}_n^4\right),$$

$$\overline{E}_{x} = 0.5\overline{g}_0^4 \overline{A}_{40} + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{g}_n^1 + \overline{A}_{2n}\overline{g}_n^2 + \overline{A}_{3n}\overline{g}_n^3 + \overline{A}_{4n}\overline{g}_n^4\right),$$

$$\overline{H}_{x} = 0.5\overline{h}_0^4 \overline{A}_{40} + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{h}_n^1 + \overline{A}_{2n}\overline{h}_n^2 + \overline{A}_{3n}\overline{h}_n^3 + \overline{A}_{4n}\overline{h}_n^4\right), \tag{37}$$

The coefficients $e_n^i - \overline{h}_n^i$ are given in the Appendix A.

Performing the Fourier series expansion to the Eq. (31) along the boundary, the boundary conditions along the boundary of the surface are expanded in the form of double Fourier series. When the plate is symmetric about more than one axis, the boundary conditions in the case of symmetric mode can be written in the form of a matrix as follows:

$$\begin{bmatrix}
E_{00}^1 & E_{00}^2 & E_{00}^3 & 0 & E_{01}^1 & \cdots & E_{0N}^1 & E_{01}^2 & \cdots & E_{0N}^2 & E_{00}^3 & \cdots & E_{0N}^3 & E_{01}^4 & \cdots & E_{0N}^4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
E_{N0}^1 & E_{N0}^2 & E_{N0}^3 & 0 & E_{N1}^1 & \cdots & E_{NN}^1 & E_{N1}^2 & \cdots & E_{NN}^2 & E_{N1}^3 & \cdots & E_{NN}^3 & E_{N1}^4 & \cdots & E_{NN}^4 \\
F_{00}^1 & F_{00}^2 & F_{00}^3 & 0 & F_{01}^1 & \cdots & F_{0N}^1 & F_{01}^2 & \cdots & F_{0N}^2 & F_{00}^3 & \cdots & F_{0N}^3 & F_{01}^4 & \cdots & F_{0N}^4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
F_{N0}^1 & F_{N0}^2 & F_{N0}^3 & 0 & F_{N1}^1 & \cdots & F_{NN}^1 & F_{N1}^2 & \cdots & F_{NN}^2 & F_{N1}^3 & \cdots & F_{NN}^3 & F_{N1}^4 & \cdots & F_{NN}^4 \\
G_{00}^1 & G_{00}^2 & G_{00}^3 & 0 & G_{01}^1 & \cdots & G_{0N}^1 & G_{01}^2 & \cdots & G_{0N}^2 & G_{00}^3 & \cdots & G_{0N}^3 & G_{01}^4 & \cdots & G_{0N}^4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
G_{N0}^1 & G_{N0}^2 & G_{N0}^3 & 0 & G_{N1}^1 & \cdots & G_{NN}^1 & G_{N1}^2 & \cdots & G_{NN}^2 & G_{N1}^3 & \cdots & G_{NN}^3 & G_{N1}^4 & \cdots & G_{NN}^4 \\
H_{00}^1 & H_{00}^2 & H_{00}^3 & 0 & H_{01}^1 & \cdots & H_{0N}^1 & H_{01}^2 & \cdots & H_{0N}^2 & H_{00}^3 & \cdots & H_{0N}^3 & H_{01}^4 & \cdots & H_{0N}^4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
H_{N0}^1 & H_{N0}^2 & H_{N0}^3 & 0 & H_{N1}^1 & \cdots & H_{NN}^1 & H_{N1}^2 & \cdots & H_{NN}^2 & H_{N1}^3 & \cdots & H_{NN}^3 & H_{N1}^4 & \cdots & H_{NN}^4
\end{bmatrix}
\begin{bmatrix}
A_{10} \\ \vdots \\ A_{40} \\ A_{11} \\ \vdots \\ A_{1N} \\ A_{21} \\ \vdots \\ A_{2N} \\ A_{31} \\ \vdots \\ A_{4N}
\end{bmatrix} = 0 \tag{38}$$

Where

$$E_{mn}^j = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} e_n^j\left(R_i, \theta\right)\cos m\theta d\theta,$$

$$F_{mn}^j = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} f_n^j\left(R_i, \theta\right)\cos m\theta d\theta,$$

$$G_{mn}^j = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} g_n^j\left(R_i, \theta\right)\cos m\theta d\theta,$$

$$H_{mn}^j = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} h_n^j\left(R_i, \theta\right)\cos m\theta d\theta, \tag{39}$$

Similarly the matrix for the antisymmetric mode is obtained as

$$\begin{bmatrix}
\bar{E}^4_{10} & \bar{E}^1_{11} & \cdots & \bar{E}^1_{1N} & \bar{E}^2_{11} & \cdots & \bar{E}^2_{1N} & \bar{E}^3_{11} & \cdots & \bar{E}^3_{1N} & \bar{E}^4_{11} & \cdots & \bar{E}^4_{1N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{E}^4_{N0} & \bar{E}^1_{N1} & \cdots & \bar{E}^1_{NN} & \bar{E}^2_{N1} & \cdots & \bar{E}^2_{NN} & \bar{E}^3_{N1} & \cdots & \bar{E}^3_{NN} & \bar{E}^4_{N1} & \cdots & \bar{E}^4_{NN} \\
\bar{F}^4_{10} & \bar{F}^1_{11} & \cdots & \bar{F}^1_{1N} & \bar{F}^2_{11} & \cdots & \bar{F}^2_{1N} & \bar{F}^3_{11} & \cdots & \bar{F}^3_{1N} & \bar{F}^4_{11} & \cdots & \bar{F}^4_{1N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{F}^4_{N0} & \bar{F}^1_{N1} & \cdots & \bar{F}^1_{NN} & \bar{F}^2_{N1} & \cdots & \bar{F}^2_{NN} & \bar{F}^3_{N1} & \cdots & \bar{F}^3_{NN} & \bar{F}^4_{N1} & \cdots & \bar{F}^4_{NN} \\
\bar{G}^4_{10} & \bar{G}^1_{11} & \cdots & \bar{G}^1_{1N} & \bar{G}^2_{11} & \cdots & \bar{G}^2_{1N} & \bar{G}^3_{11} & \cdots & \bar{G}^3_{1N} & \bar{G}^4_{11} & \cdots & \bar{G}^4_{1N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{G}^4_{N0} & \bar{G}^1_{N1} & \cdots & \bar{G}^1_{NN} & \bar{G}^2_{N1} & \cdots & \bar{G}^2_{NN} & \bar{G}^3_{N1} & \cdots & \bar{G}^3_{NN} & \bar{G}^4_{N1} & \cdots & \bar{G}^4_{NN} \\
\bar{H}^4_{10} & \bar{H}^1_{11} & \cdots & \bar{H}^1_{1N} & \bar{H}^2_{11} & \cdots & \bar{H}^2_{1N} & \bar{H}^3_{11} & \cdots & \bar{H}^3_{1N} & \bar{H}^4_{11} & \cdots & \bar{H}^4_{1N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{H}^4_{N0} & \bar{H}^1_{N1} & \cdots & \bar{H}^1_{NN} & \bar{H}^2_{N1} & \cdots & \bar{H}^2_{NN} & \bar{H}^3_{N1} & \cdots & \bar{H}^3_{NN} & \bar{H}^4_{N1} & \cdots & \bar{H}^4_{NN}
\end{bmatrix}
\begin{bmatrix}
\bar{A}_{10} \\ \bar{A}_{11} \\ \vdots \\ \bar{A}_{1N} \\ \bar{A}_{21} \\ \vdots \\ \bar{A}_{2N} \\ \bar{A}_{31} \\ \vdots \\ \bar{A}_{3N} \\ \vdots \\ \bar{A}_{4N}
\end{bmatrix} = 0, \tag{40}$$

Where

$$\bar{E}^j_{mn} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} \bar{e}^j_n\left(R_i,\theta\right)\sin m\theta d\theta,$$

$$\bar{F}^j_{mn} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} \bar{f}^j_n\left(R_i,\theta\right)\sin m\theta d\theta,$$

$$\bar{G}^j_{mn} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} \bar{g}^j_n\left(R_i,\theta\right)\sin m\theta d\theta,$$

$$\bar{H}^j_{mn} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I} \bar{h}^j_n\left(R_i,\theta\right)\sin m\theta d\theta. \tag{41}$$

Solid circular plate

The frequency equation for solid circular plate can be written in the form

$$|A| = 0, \tag{42}$$

where A is the 4×4 matrix with elements $a_{ij}\left(i, j=1,2,3,4\right)$ are given by

$$a_{1i} = -n\left(n-1\right)J_\beta\left(\alpha r\right)+\left(\alpha r\right)J_{\beta+1}\left(\alpha r\right)+\left(1+\bar{L}\right)\left(\alpha r\right)^2 J_\beta\left(\alpha r\right),\ i=1,2,3$$

$$a_{14} = n\left\{\left(n-1\right)J_\delta\left(kr\right)-\left(kr\right)J_{\delta+1}\left(kr\right)\right\},$$

$$a_{2i} = 2n\left\{\left(n-1\right)J_\beta\left(\alpha r\right)-\left(\alpha r\right)J_{\beta+1}\left(\alpha r\right)\right\}, i=1,2,3$$

$$a_{24} = -2n\left\{n\left(n-1\right)J_\delta\left(kr\right)+\left(kr\right)J_{\delta+1}\left(kr\right)\right\}+\left(kr\right)^2 J_\delta\left(kr\right),$$

Numerical results and discussions

The numerical analysis of the frequency equation is carried out for non-homogeneous transversely isotropic electro-magneto-elastic plate of elliptic and cardioids cross-section. The electro-magnetic material constants based on graphical result of Aboudi [24] used for numerical calculations. The material constants are $c_{11} = 218\times10^9\,N/m^2$, $c_{12} = 120\times10^9\,N/m^2$, $c_{66} = 49\times10^9\,N/m^2$, $\varepsilon_{11} = 0.4\times10^{-9}\,C/Vm$, $\mu_{11} = -200\times10^{-6}\,Ns^2/c^2$ and $m_{11} = 0.0074\times10^{-9}\,Ns/VC$. Substituting R_i and the angle γ_i, between the reference axis and the normal to the $i-$th boundary line, the integrations of the Fourier coefficients $e^i_n, f^i_n, g^i_n, h^i_n, \bar{e}^i_n, \bar{f}^i_n, \bar{g}^i_n$ and $\bar{h}^i_n$ can be expressed in terms of the angle θ. Using the coefficients into Eqs. (39) and (41), the frequencies are obtained for non-homogeneous transversely isotropic electro-magneto-elastic plates of arbitrary cross-sectional plate.

In the present problem, there are three kinds of basic independent modes of wave propagation have been considered namely longitudinal and two flexural (symmetric and antisymmetric) modes for geometries having more than one symmetry. For geometries having only one symmetry, two modes of wave propagation are studied since the two flexural (symmetric and antisymmetric) modes are coupled in this case.

Longitudinal mode

The geometrical relations for the elliptic cross-section given in Eq. (43) are used directly for the numerical calculations and three kinds of basic independent modes of wave propagation are studied. In case of longitudinal modes of elliptical cross-section, the cross-section vibrates along the axis of the plate, so that the vibration and displacements in the cross-section is symmetrical about both major and minor axis. Hence, the frequency equation is obtained by choosing both terms of n and m as 0,2,4,6,… in Eq. (38) for the numerical calculations. Since the boundary of the cross-sections namely, elliptic and cardioids are irregular in shape, it is difficult to satisfy the boundary conditions along the curved surface in the range $\theta = 0$ and $\theta = \pi$ is divided into 15 segments, such that the distance between any two segments is negligible and the integration is performed for each segment numerically by using the Gauss five-point formula. The non-dimensional frequencies are computed for $0 < \Omega \le 1.0$ using the secant method (applicable for the complex roots Antia [25]).

Flexural mode

In the case of flexural mode of elliptical cross-section, the vibration and displacements are antisymmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equations are obtained from Eq. (40) by choosing $n,m = 1,3,5,\ldots$. Two kinds of flexural (symmetric and antisymmetric) modes are considered. The computed non-dimensional frequencies are presented in the form of dispersion curves.

The same problem is solved for cardioidal cross-sectional plate using the geometrical relations given in Eq. (44) and (45), which are functions depending on a parameter S. Since a cardioid is symmetrical about only one axis, the longitudinal and flexural symmetric modes are carried out by choosing $n,m = 0,1,2,3,\ldots$ in Eq. (38) and flexural (antisymmetric) modes are obtained by choosing $n,m = 1,2,3,\ldots$ in Eq. (40).

Elliptic cross-section

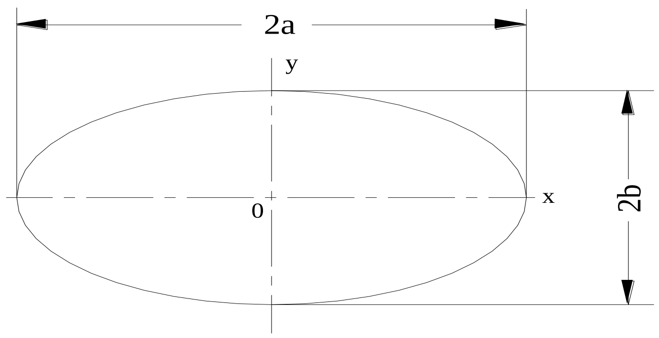

Fig 1. Elliptic cross section

The elliptic cross-section of the plate is shown in Fig. 2, and its geometric relations used for the numerical calculations given below are taken from Eq. (10) of Nagaya [2] as:

$$R_i/b = \frac{a/b}{\left(\cos^2\theta + (a/b)^2 \sin^2\theta\right)^{1/2}},$$

$$\gamma_i = \frac{\pi}{2 - \tan^{-1}}\left(\frac{(b/a)^2}{\tan\theta_i^*}\right), \text{ for } \theta_i^* < \pi/2,$$

$$\gamma_i = \frac{\pi}{2}, \text{ for } \theta_i^* = \frac{\pi}{2},$$

$$\gamma_i = \frac{\pi}{2 + \tan^{-1}\left(\frac{(b/a)^2}{\left|\tan\theta_i^*\right|}\right)}, \text{ for } \theta_i^* > \frac{\pi}{2}, \tag{43}$$

Where a is the semi major axis and b is the semi minor axis of the elliptic plate and $\theta_i^* = (\theta_i - \theta_{i-1})/2$, R_i is the coordinate r at the boundary, γ_i is the angle between the normal to the segment and the reference axis at the i – th boundary.

Cardioid cross-section

The cardioid cross-section of the plate is shown in Fig. 3, and its geometrical relations used for the numerical calculations are given in the Eqs. (44) and (45) as follows;

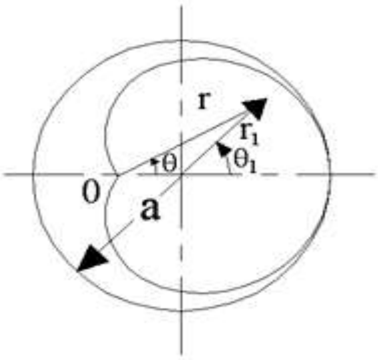

Fig. 2 Cardioid cross section

$$R_i/a = \frac{1 + s^2 + 2s\cos\theta_1}{1+s},$$

$$\theta = \cos^{-1}\frac{\cos\theta_1 + s\cos 2\theta_1}{\left(1 + s^2 + 2s\cos\theta_1\right)^{1/2}}, \tag{44}$$

Where a is the radius of the circumscribing circle and

$$G(\theta_1) = \frac{\cos\theta_1 + 2s\cos 2\theta_1}{-\sin\theta_1 - 2\sin 2\theta_1},$$

$$\gamma_i = \frac{\pi}{2}, \text{ for } G(\theta_i^*) = 0,$$

$$\gamma_i = \frac{\pi}{2} - \tan^{-1}\left(-G(\theta_i^*)\right), \text{ for } G(\theta_i^*) < 0, \tag{45}$$

$$\gamma_i = \frac{\pi}{2} + \tan^{-1}\left(G(\theta_i^*)\right), \text{ for } G(\theta_i^*) > 0,$$

Where $\theta_i^* = (\theta_i - \theta_{i-1})/2$, is the mean angle of the segment i and R_i is the coordinate r at the boundary, γ_i is the angle between the normal to the segment and the reference axis at the i – th boundary. The parameter s represents a circle when $s = 0$ and represents a cardioids when $s = 0.5$.

Dispersion curves

The results of longitudinal modes of vibrations are plotted in the form of dispersion curves, the notations Lm denotes longitudinal mode in all the graphs. The 1 refers the first mode and 2 the second and so on. From the graphs obtained, it can be noticed that the dispersion for the plates in the fundamental mode is high. But in higher modes, the dispersive curves are almost straight, along the direction of propagation. Hence it may be concluded it has a non-dispersive behaviors. It is also to be mentioned that the cross over points in various curves of different modes indicate that for a particular frequency of vibration, the mechanical energy is communicative between its directions of wave propagation in the respective mode. A graph is drawn between aspect ratio $a/b = 1.5$ and non-dimensional frequency Ω of elliptical cross-section of longitudinal modes of vibration and is shown in Fig. 3. From Fig. 3, it is observed that the non-dimensional frequency increases as aspect ratio increases. Also it is observed that the non-dimensional frequency increases as modes of vibration increases. Graphs are drawn between the mode and non-dimensional frequency Ω of longitudinal modes of elliptical and cardioid cross-sectional plate and are shown in Figs. 4 and 5. From Figs. 4 and 5, it is observed that the non-dimensional frequency increases as modes of vibration increases. Further as the aspect ratio increases the dimensionless frequency also increases. The cross-over points represent the transfer of energy between the modes of vibration.

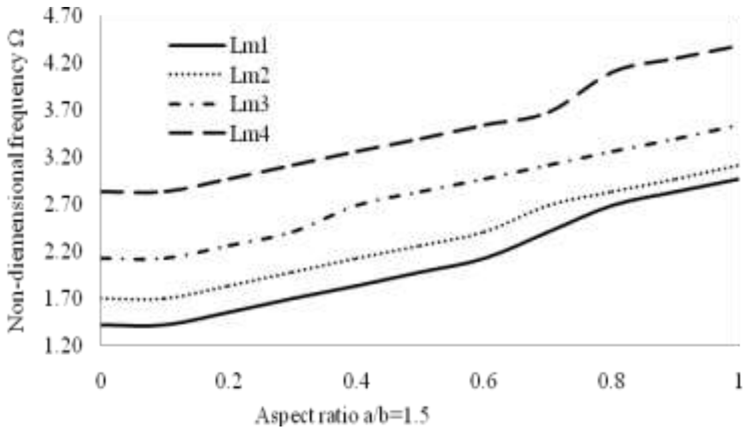

Fig 3. Non-dimensional aspect ratio a/b=1.5 versus dimensionless frequency of elliptical cross-section of longitudinal modes of vibration

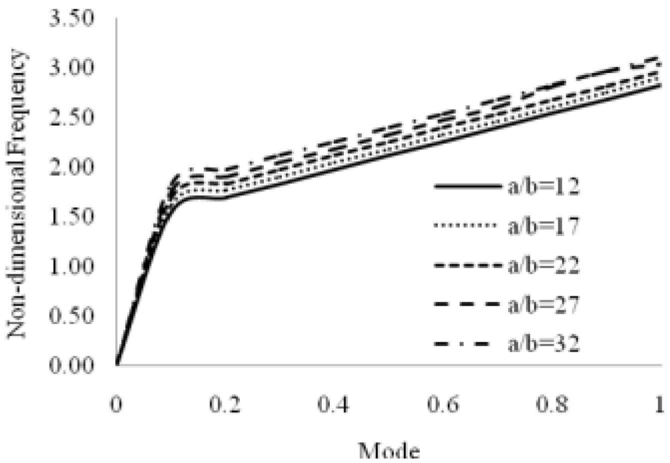

Fig 4. Mode versus non-dimensional frequency Ω of elliptical cross-section of longitudinal modes of vibration

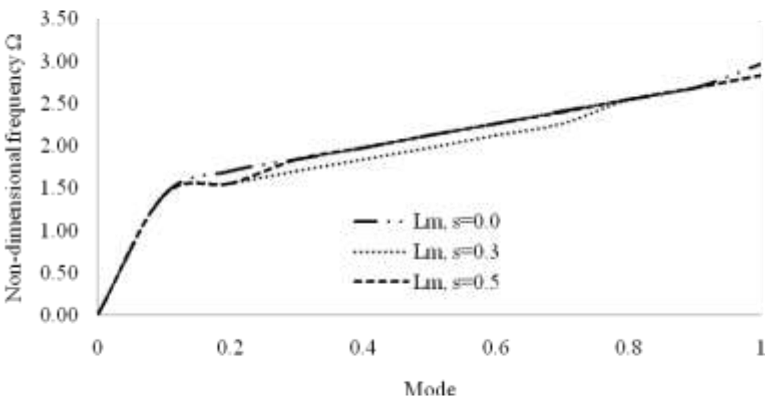

Fig 5. Mode versus non-dimensional frequency Ω for longitudinal modes of cardioid cross-sections

Conclusions

The vibration of non-homogeneous transversely isotropic electro-magneto-elastic plate of arbitrary cross-section is studied using the Fourier expansion collocation method. The frequency equations are obtained from the arbitrary cross-sectional boundary conditions, since the boundary is irregular in shape; it is difficult to satisfy the boundary along the surface of the plate directly. Hence, the Fourier expansion collocation method is applied along the boundary to satisfy the boundary conditions. The roots of the frequency equations are obtained by using the secant method applicable for complex roots. The computed non-dimensional frequencies are plotted in the form of dispersion curves and their characteristics are discussed.

Acknowledgement

Dr. P. Ponnusamy is thankful to University Grants Commission, New Delhi, for funding to undertake this research work, **Ref. F. No. 39-46 / 2010 (SR),** and the Directorate of Collegiate Education, Tamil Nadu, for the permission rendered towards the same. His gratitude also extends to Govt. Arts College (Autonomous), Coimbatore-18, for providing with the facilities to take up this work.

References

[1] K. Nagaya, “Method for solving vibration problems of a plate with arbitrary shape”, J. Acoust. Soc. Am, vol. 67 no. 6, pp. 2029-2033, 1980.

[2] K. Nagaya, “Direct method on the determination of Eigen frequencies of arbitrary shaped plates”, J. Trans. ASME, vol. 105, pp. 132-136, 1983.

[3] K. Nagaya, “Vibration of a thick walled pipe or ring of arbitrary shape in its plate”, J. Applied Mechanics, vol. 50, pp. 757-764, 1983.

[4] X. Wang, and Y. P. Shen, “Inclusion of arbitrary shape in magneto-electro-elastic composite materials”, Int. J. Engg. Sci., vol. 41, pp. 85-102, 2003.

[5] G. R. Buchanan, “Free vibration of an infinite magneto-electro-elastic cylinder”, J. Sound and Vibration, vol. 268, pp. 413-426, 2003.

[6] A. M. Abd-Alla, S. R. Mahmoud, and N. A. AL-Shehri, “Effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material”, Applied Mathematics and Computation, vol. 217, no. 22, pp. 8914-8922, 2011.

[7] A. M. Abd-Alla, and S. R. Mahmoud, “Effect of the rotation on propagation of thermoelastic waves in a non-homogeneous infinite cylinder of isotropic material”, Int. J. Mathematical Analysis, vol. 4, no. 42, pp. 2051-2064, 2010.

[8] A. M. Abd-Alla, and S. R. Mahmoud, “Magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model”, Meccanica, vol. 45, no. 4, pp. 451-462, 2010.

[9] J. Y. Chen, and H. L. Chen, “Love wave propagation in magneto-electro-elastic multilayered structures”, Acta Materiae Compositae Sincia, vol. 23, pp. 181-184, 2006. (Chinese)

[10] J. Y. Chen, E. Pan, and H. L. Chen, “Wave propagation in magneto-electro-elastic multilayered plates”, Int. J. Solid and Structures, vol. 44, pp. 1073-1085, 2007.

[11] W. Q. Chen, K. Y. Lee, and H. J. Ding, “On free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plates”, J. Sound and Vibration, vol. 279, pp. 237-251, 2005.

[12] W. Q. Chen, K. Y. Lee, and H. J. Ding, “General solution for transversely isotropic magneto-electro-thermo-elastic and potential theory method”, Int. J. Engg. Sci., vol. 42, pp. 1361-1379, 2004.

[13] W. Q. Chen, Y. L. Kang, and H. J. Ding, “On free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plates”, J. Sound and Vibration, vol. 279, pp. 237-251, 2005.

[14] J. Wei, and X. Y. Su, “Wave propagation and energy transportation along cylindrical piezoelectric piezomagnetic material”, Acla Scienliarum Naluralium Lniversi LAlis Pekinensis, vol. 42, pp. 310-314, 2006. (Chinese)

[15] P. F. Hou and A. Y. T. Leung, “The transient response of magneto-electro-elastic hollow cylinders”, Smart Material Structure, vol. 13, pp. 762-776, 2004.

[16] P. F. Hou, H. J. Ding, and A. Y. T. Leung, “The transient responses of a special non-homogeneous axisymmetric plane strain problem”, J. Sound and Vibration, vol. 291, pp. 19-47, 2006.

[17] E. Pan, “Exact solution for simply supported and multilayered magneto-electro-elastic plates”, J. Applied Mechanics, vol. 68, pp. 608-618, 2011.

[18] E. Pan, and P. R. Heyliger, “Free vibrations of simply supported and multilayered magneto-electro-elastic plates”, J. Sound and Vibration, vol. 252, pp. 429-442, 2002.

[19] W. Q. Chen, Y. L. Kang, and H. J. Ding, “General solution for transversely isotropic magneto-electro-thermo-elasticity and the potential theory method”, Int. J. Engg. Sci., vol. 42, pp. 1361-1379, 2004.

[20] P. Ponnusamy, “Wave propagation in a generalized thermo elastic solid cylinder of arbitrary cross-section”, Int. J. Solid and Struct., vol. 44, pp. 5336-5348, 2007.

[21] P. Ponnusamy, “Dispersion analysis of generalized thermo elastic plate of polygonal cross-sections”, Applied Mathematical Modelling, vol. 36, pp. 3343-3358, 2012.

[22] P. Ponnusamy, “Wave propagation in a piezoelectric solid bar of circular cross-section immersed in fluid”, Int. J. Pressure Vessels and Piping, vol. 105-106, pp. 12-18, 2013.

[23] P. Ponnusamy, “Wave propagation in electro-magneto-elastic solid plate of polygonal cross-sections”, vol. 9, no. 1, pp. 23-48, 2012.

[24] J. Aboudi, “Micromechanical analysis of fully coupled electro-magneto-thermo-elastic multiphase composites”, Smart material and structure, vol. 10, pp. 867-877, 2001.

[25] H. M. Antia, “Numerical Methods for scientists and engineers”, Hindustan Book Agency, New Delhi, 2002.

Appendix A

$$e_n^1 = \Big[\{\beta(\beta-1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\}\big(\bar{L}+\sin^2(\theta-\gamma_i)\big)-\{\beta(\beta+1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\}\big(\bar{L}+\cos^2(\theta-\gamma_i)\big)$$
$$+(\alpha r)^2\{(1+\bar{L})\cos^2(\theta-\gamma_i)+\bar{L}\sin^2(\theta-\gamma_i)\}J_\beta(\alpha r)\Big]\cos n\theta - n\{(\beta-1)J_\beta(\alpha r)-(\alpha r)J_{\beta+1}(\alpha r)\}\sin 2(\theta-\gamma_i)\sin n\theta$$

$$e_n^2 = 0$$

$$e_n^3 = 0$$

$$e_n^4 = \{n(\delta-1)J_\delta(kr)-(kr)J_{\delta+1}(kr)\}\cos 2(\theta-\gamma_i)\cos n\theta$$
$$-\left\{\left(\delta\left(\frac{\delta}{2}+1\right)+\left(\frac{n^2-(kr)^2}{2}\right)\right)J_\delta(kr)-(kr)J_{\delta+1}(kr)\right\}\sin n\theta\sin 2(\theta-\gamma_i),$$

$$f_n^1 = \Big[2\{\beta J_\beta(\alpha r)-(\alpha r)J_{\beta+1}(\alpha r)\}+\big((\alpha r)^2-\beta^2-n^2\big)J_\beta(\alpha r)\Big]\cos n\theta\sin 2(\theta-\gamma_i)$$
$$+2n\{(\beta-1)J_\beta(\alpha r)-(\alpha r)J_{\beta+1}(\alpha r)\}\sin n\theta\cos 2(\theta-\gamma_i),$$

$$f_n^2 = 0$$

$$f_n^3 = 0$$

$$f_n^4 = 2n\{\delta J_\delta(kr)-(kr)J_{\delta+1}(kr)\}\cos n\theta\sin 2(\theta-\gamma_i)+$$
$$\Big[2\{\delta J_\delta(kr)-(kr)J_{\delta+1}(kr)\}+\big((kr)^2-\delta^2-n^2\big)J_\delta(kr)\Big]\sin n\theta\cos 2(\theta-\gamma_i)$$