



## Delta convergence of grills

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### ABSTRACT

In this paper, we explore different notions of sets namely  $\alpha$ -open, semi-open, pre-open, b-open and  $\beta$ -open to define the concept of delta-convergence and delta-adherence of grills in a topological space. Further, we discuss their characterizations

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### Keywords

G-grill

 $\delta$ -convergence, $\delta$ -adherence.

### Introduction

We assume basic knowledge of topology; open and closed sets, closure, interior for different family of sets namely  $\alpha$ -open sets, semi-open sets, pre-open sets, b-open sets and  $\beta$ -open sets. In metric spaces, properties such as continuity, closure and compactness can be stated completely in terms of sequences. This breaks down in general topological spaces, where sequences can't even tell whether a set is closed. In 1922, Moore and Smith discovered a generalization of sequences, called nets which are adequate to describe those things. Despite their sheer dissimilarity nets are equivalent to filters in terms of results (which include Tychonoff's theorem, purported to be one of the hardest in topology).

Choquet[8] introduced an attractive theory of Grills, a collection satisfies some condition in Topological space. The topology equipped with the grill collection is called Grill topology.

This topic has an excellent potential for application in other branches of mathematics like compactifications, Proximity spaces, different types of extension problems etc. This subject was continued to study by general topologists Roy and Mukherjee [18], [19] in recent years. It was probably the two articles of M.K. Singal and Asha Mathur [20],[21] that initiated the study of nearly Compact spaces in general topology.

The purpose of this paper is to define and discuss the concepts of  $\delta$ -convergence and  $\delta$ -adherence of grills in the different family sets of X.

### Preliminaries

#### Definition 2.1 :[19]

A collection G of nonempty subsets of a set X is called a grill if

1.  $A \in G$  and  $A \subseteq B \subseteq X$  implies that  $B \in G$ , and
2.  $A \cup B \in G$  ( $A, B \subseteq X$ ) implies that  $A \in G$  or  $B \in G$ .

#### Definition 2.2 :[19]

A grill G on a set X is said to be a  $\sigma$ -grill if for any countable collection  $\{A_n : n \in \mathbb{N}\}$  of subsets of X,  $A_n \in G$  whenever  $A = \bigcap_{n \in \mathbb{N}} A_n \in G$  or each  $n \in \mathbb{N}$ .

#### Definition 2.3 :[26]

A subset of a space is said to be regularly open, it is the interior of some closed set or equivalently, if it is the interior of its own closure. A set is said to be regularly closed if it is the

closure of some open set or equivalently, if it is the closure of its own interior.

#### Definition 2.4 :[24]

Let  $(X, \tau)$  be a space. Then the following hold.

(a) For any grill (resp. filter) G on a space X, sec G is a filter (resp. grill) on X.

(b) If F is a filter and G is a grill such that  $F \subset G$ , then there is an ultrafilter U on X such that  $F \subset U \subset G$ .

#### Definition 2.5 :[16]

A nonempty set A of a space  $(X, \tau)$  is said to be  $\mu$ -closed relative to X if for every cover  $\{V_\alpha : \alpha \in \Lambda\}$  of A by  $\mu$ -open sets, there exists a finite subset  $\Lambda_t$  of  $\Lambda$  such that  $A \subset \bigcup_{\alpha \in \Lambda_t} V_\alpha$ ;  $\alpha \in \Lambda_t$ . A space X is called  $\mu$ -closed if  $A = X$ .

### Convergence and adherence of grills

#### Definition 3.1

A grill G on X is said to be  $\delta$ -adheres at x in X if for each  $U \in \tau$  containing x and each  $G \in G$ ,  $\text{intl}(U) \cap G \neq \emptyset$ .

#### Definition 3.2

A grill G on X is said to be  $\delta$ -converges at x in X if for each  $U \in \tau$  containing x, there exists  $G \in G$  such that  $G \subset \text{intl}U$ . Clearly a grill G converges to x in X if and only if G contains the collection  $\{\text{intl} U_x \setminus U_x \in X\}$ .

#### Note 3.2.1

Let  $(X, \tau)$  be a space and  $P = \{\sigma, \pi, b, \beta\}$ . For  $\mu \in P$ , A grill G on X is said to be  $\delta_\mu$ -adheres at x in X if for each  $U \in \mu$  containing x and each  $G \in G$ ,  $\text{int}_\mu \text{cl}_\mu(U) \cap G \neq \emptyset$  is said to be  $\delta_\mu$ -converges to x in X if for each  $U \in \mu$  containing x, there exists  $G \subset \text{int}_\mu \text{cl}_\mu(U)$ . Clearly, a grill G is  $\delta_\mu$ -converges to x in X if and only if G contains the collection  $\{\text{int}_\mu \text{cl}_\mu(U) \setminus x \in A \in \mu\}$ .

#### Definition 3.3

A filter F on a topological space  $(X, \tau)$  is said to be  $\delta_\mu$ -adheres at x in X if for each  $U \in \mu$  containing x and each  $F \in F$ ,  $\text{int}_\mu \text{cl}_\mu(U) \cap F \neq \emptyset$ , F is said to be  $\delta_\mu$ -converges to x in X if for each  $U \in \mu$  containing x, there exists  $F \subset \text{int}_\mu \text{cl}_\mu(U)$ .

#### Definition 3.4

Let G be a grill on the topological space  $(X, \tau)$ . Then  $G(\delta_\mu, x) = \{A \subset X ; x \in c_{\delta_\mu}(A)\}$ .

**Definition 3.5**

Let  $(X, \tau, G)$  be a grill topological space, then  $\text{sec}G(\delta_\mu, x) = \{A \subset X; A \cap G \neq \emptyset \text{ for all } G \in G(\delta_\mu, x)\}$ .

**Theorem 3.6**

Let  $(X, \tau)$  be a space with a grill  $G$ . If  $G, \delta_\mu$  - adheres at some point  $x \in X$ , then  $G$  is  $\delta_\mu$  - converge to  $x$ .

**Proof**

Suppose  $G, \delta_\mu$  - adheres at  $x \in X$ . Then for each  $\mu$ -open set  $U$  containing  $x$  and each  $G \in G, \text{int}_\mu \text{cl}_\mu(U) \cap G \neq \emptyset \Rightarrow \text{int}_\mu \text{cl}_\mu(U) \in \text{sec}G$  for each  $\mu$ -open set  $U$  containing  $x$ . Therefore  $X - \text{int}_\mu \text{cl}_\mu(U) \notin G$  whenever  $x \in U \in \mu$ . Since  $X \in G$  and  $G$  is a grill on  $X, \text{int}_\mu \text{cl}_\mu(U) \in G$  for each  $\mu$ -open set containing  $x$ . Thus,  $G$  is  $\delta_\mu$ -convergent to  $x$ .

**Theorem 3.7**

Let  $G$  be a grill on a space  $(X, \tau)$ . Then the grill  $G, \delta_\mu$  - adheres at  $x \in X$  if and only if  $G \subset G(\delta_\mu, x)$ .

**Proof**

If  $G, \delta_\mu$  - adheres at  $x \in X$  implies that  $\text{int}_\mu \text{cl}_\mu(U) \cap G \neq \emptyset$  for all  $G \in G$  and for every  $\mu$ -open set  $U$  containing  $x$ . Therefore,  $x \in c_{\delta_\mu}(G)$  for all  $G \in G$  and so  $G \in G(\delta_\mu, x)$  for all  $G \in G. \Rightarrow G \subset G(\delta_\mu, x)$ . Conversely assume that  $G \subset G(\delta_\mu, x)$ . Then for every  $G \in G, G \in G(\delta_\mu, x)$  and so  $x \in c_{\delta_\mu}(G) \Rightarrow \text{int}_\mu \text{cl}_\mu(U) \cap G \neq \emptyset$  for every  $U \in \mu$  containing  $x$ . Hence  $G, \delta_\mu$  - adheres at the point  $x$ .

**Theorem 3.8**

Let  $G$  be a grill on a space  $(X, \tau)$ . Then  $G$  is  $\delta$  - convergent to  $x \in X$  if and only if  $\text{sec}G(\delta_\mu, x) \subset G$ .

**Proof**

Let  $G$  be a grill on  $X$  which  $\delta_\mu$  - converges to  $x \in X$ . Then for each  $\mu$ -open set  $U$  containing  $x$ , there exists a  $G \in G$  such that  $G \subset \text{int}_\mu \text{cl}_\mu(U)$ . Since  $G$  is a grill on  $X, \text{int}_\mu \text{cl}_\mu(U) \in G$ . Let  $A \in \text{sec}G(\delta_\mu, x)$ . Then  $A \cap G \neq \emptyset$  for all  $G \in G(\delta_\mu, x)$  and so  $X \setminus A \notin G(\delta_\mu, x) \Rightarrow x \notin c_{\delta_\mu}(X - A)$  and so  $\text{cl}_\mu(V) \cap (X - A) = \emptyset$  for some  $V \in \mu$  containing  $x. \Rightarrow \text{cl}_\mu(V) \subset A$  which implies that  $A \in G$ . Thus  $\text{sec}G(\delta_\mu, x) \subset G$ . Conversely let us assume that  $G$  does not  $\delta_\mu$  - converge to  $x \in X$ . Then there exists a  $\mu$ -open set  $U$  containing  $x$  such that  $\text{int}_\mu \text{cl}_\mu(U) \notin G$  and  $\text{int}_\mu \text{cl}_\mu(U) \notin \text{sec}G(\delta_\mu, x)$ . Since  $\text{int}_\mu \text{cl}_\mu(U) \notin \text{sec}G(\delta_\mu, x)$ , there exists  $G \in \text{sec}G(\delta_\mu, x)$ , such that  $\text{cl}_\mu(U) \cap G = \emptyset$ . Since  $G \in \text{sec}G(\delta_\mu, x), x \in c_{\delta_\mu}(G)$  and so  $\text{int}_\mu \text{cl}_\mu(U) \cap G \neq \emptyset$  for all  $\mu$ -open set  $U$  containing  $x$ , a contradiction. Hence  $G, \delta_\mu$  - converges to  $x$ .

**Definition 3.9**

A nonempty set  $A$  of a space  $(X, \tau)$  is said to be  $AS_\mu$ -set relative to  $X$  if for every cover  $\{V_\alpha; \alpha \in \Lambda\}$  of  $A$  by  $\mu$ -open sets of  $X$ , there exists a finite subfamily  $\Lambda_0$  of  $\Lambda$  such that  $A \subset \cup\{\text{int}_\mu \text{cl}_\mu(V_\alpha); \alpha \in \Lambda_0\}$ .

**Definition 3.10**

- (1)The set of all  $AS_\mu$  spaces is a subcollection of the family of  $\mu$  closed spaces discussed in [16].
- (2)A space  $X$  is  $AS_\mu$  space if  $A=X$ . If  $\mu = \tau$  then the family of all  $AS_\tau$  - spaces coincides with the family of all nearly compact spaces.
- (3)If  $\mu = \pi$  then the family of spaces are called  $AS_\pi$  space.
- (4)If  $\mu = \beta$  then the family of spaces are called  $AS_\beta$  space.
- (5)If  $\mu = b$  then the family of spaces are called  $AS_b$  space.
- (6)If  $\mu = \sigma$  then the family of spaces are called  $AS_\sigma$  space.

**Theorem 3.11**

Let  $(X, \tau)$  be a space and  $\mu \in P$ . Then the following are equivalent.

- (a) $X$  is  $AS_\mu$  space.
- (b) Every maximal filterbase  $\delta_\mu$  - converges to some point of  $X$ .
- (c)Every filterbase  $\delta_\mu$  - adheres at some point of  $X$ .
- (d) For every family  $\{V_\alpha; \alpha \in \Lambda\}$  of  $AS_\mu$  subsets such that  $\cap\{V_\alpha; \alpha \in \Lambda\} = \emptyset$ , there exists a finite subset  $\Lambda_0$  of  $\Lambda$  such that  $\cap\{\text{int}_\mu(V_\alpha); \alpha \in \Lambda_0\} = \emptyset$ .

**Proof**

(a) $\Rightarrow$ (b). Let  $F$  be a maximal filter base on  $X$ . Suppose that  $F$  does not  $\delta_\mu$  - converge to any point of  $X$ . For each  $x \in X$ , there exists  $F_x \in F$  and  $\mu$ - open set  $U_x$  containing  $x$  such that  $F_x \cap \text{int}_\mu \text{cl}_\mu(U_x) = \emptyset$ . Now  $\{U_x; x \in X\}$  is a  $\mu$ -open cover for  $X$ . By (a), there exists a finite subset  $\{x_1, x_2, \dots, x_n\}$  of  $X$  such that  $X = \cup\{\text{int}_\mu \text{cl}_\mu; 1 \leq i \leq n\}$ . Since  $F$  is a filter base, there exists  $F \in F$  such that  $F \subset \cap F_{x_i}$ .

Now,  $\cap F_{x_i} = (\cap F_{x_i}) \cap (\cup \text{int}_\mu \text{cl}_\mu(U_{x_i})) = \cup((\cap F_{x_i}) \cap \text{int}_\mu \text{cl}_\mu(U_{x_i})) = \emptyset$ . Hence  $F = \emptyset$ , a contradiction.

(b) $\Rightarrow$ (c). Let  $F$  be a filter base on  $X$ . Then there exists a maximal filter base  $F'$  such that  $F \subset F'$ . By (b),  $F, \delta_\mu$  - converges to some point  $x \in X$ . For every  $F \in F$  and every  $U \in \mu$  containing  $x$ , there exists  $F' \in F'$  such that  $F' \subset \text{int}_\mu \text{cl}_\mu(U)$ . Now  $F' \subset \text{int}_\mu \text{cl}_\mu(U)$  implies that  $F' \cap F \subset \text{int}_\mu \text{cl}_\mu(U) \cap F$ . Since  $F'$  is a filterbase,  $F \cap F' \neq \emptyset$  and so  $F \cap \text{int}_\mu \text{cl}_\mu(U) \neq \emptyset$ . Hence  $F, \delta_\mu$  - adheres at  $x$ .

(c) $\Rightarrow$ (d). Let  $\{V_\alpha; \alpha \in \Lambda\}$  be a family of  $AS_\mu$ - subsets of  $X$  such that  $\cap\{V_\alpha; \alpha \in \Lambda\} = \emptyset$ . Let  $I$  be a family of all finite subsets of  $\Lambda$ . Assume that  $A_I = \cap\{\text{int}_\mu(V_\alpha); \alpha \in I\} \neq \emptyset$  for every  $I \in I$ .

Then the family  $F = \{A_I; I \in I\}$  is a filter base on  $X$ . By hypothesis,  $F, \delta_\mu$ -adheres at some point  $x \in X$ . Since  $\{X - V_\alpha; \alpha \in \Lambda\}$  is a  $\mu$ -open cover of  $X, x \in X - V_\beta$  for some  $\beta \in \Lambda$ . Since  $\text{int}_\mu \text{cl}_\mu(X - V_\beta) \cap \text{int}_\mu(V_\beta) = \emptyset$ , we have a contradiction to the fact that  $F, \delta_\mu$  - adheres at  $x \in X$ . Thus  $\cap\{\text{int}_\mu(V_\alpha); \alpha \in I\} = \emptyset$  for some  $I \in I$ .

(d) $\Rightarrow$ (a). Let  $\{V_\alpha; \alpha \in \Lambda\}$  be a cover of  $X$  by  $\mu$ -open sets of  $X$ . Then  $\{X - V_\alpha; \alpha \in \Lambda\}$  is a family of  $\mu$ -closed subsets of  $X$  such that  $\cap\{X - V_\alpha; \alpha \in \Lambda\} = \emptyset$ . By (d), there exists a finite subfamily  $\Lambda_0$  of  $\Lambda$  such that  $\cap\{\text{int}_\mu(X - V_{\alpha_0})\} = \emptyset$ . Hence  $X - \cap\{\text{int}_\mu(X - V_\alpha); \alpha \in \Lambda_0\} = X$  which implies that  $\cup\{\text{int}_\mu \text{cl}_\mu(V_\alpha); \alpha \in \Lambda\} = X$ . Hence  $X$  is a  $AS_\mu$ -space.

**Theorem 3.12**

A space  $(X, \tau)$  is a  $AS_\mu$ -space if and only if every grill on  $X, \delta_\mu$  - converges in  $X$ .

**Proof**

Let  $G$  be a grill on a  $AS_\mu$ -space  $X$ . Then by the lemma 1.1,  $\text{sec}G$  is a filter on  $X$ . Let  $B \in \text{sec}G$ . Then  $B \cap G \neq \emptyset$  for every  $G \in G, X - B \notin G$ . Hence  $B \in G$ , since  $G$  is a grill. Thus  $\text{sec}G \subset G$ . By lemma 1.1, there is an ultra filter  $U$  on  $X$  such that  $\text{sec}G \subset U$ . Since  $X$  is  $AS_\mu$ -space, by theorem 2.11,  $U, \delta_\mu$  - converges to some point  $x \in X$ . Then for each  $\mu$ -open set  $U$  containing  $x$ , there exists some  $G \in U$  such that  $G \subset \text{int}_\mu \text{cl}_\mu(U)$ . Since  $\text{int}_\mu \text{cl}_\mu(U) \in U, \text{int}_\mu \text{cl}_\mu(U) \in G$  for every  $U \in \mu$  containing  $x$ . Hence  $G$  is  $\delta_\mu$  - adheres to  $x$ . Conversely, let  $U$  be an ultrafilter on  $X$ . Since every ultrafilter is a grill,  $U, \delta_\mu$  - converges to some point of  $X$ .

**Corollary 3.13**

A space  $(X, \tau)$  is nearly compact if and only if every grill  $G$  on  $X, \delta$  - converges in  $X$ .

**Theorem 3.14**

A subset  $A$  of a space  $(X, \tau)$  is  $AS_\mu$ -subset relative to  $X$  if and only if every grill on  $X$  with  $a \in G, \delta_\mu$  - converges to a point in  $A$ .

**Proof**

Let  $A$  be  $AS_\mu$ -subset relative to  $X$  and  $G$  be a grill on  $X$

such that  $A \in G$  and  $G$  does not  $\delta_\mu$ -converge to any point of  $A$ . Then, for each  $x \in A$ , there exists  $U_x \in \mu$  containing  $x$  such that  $\text{int}_\mu \text{cl}_\mu(U_x) \notin G$ . Now  $\{U_x; x \in A\}$  is a cover of  $A$  by  $\mu$ -open sets of  $X$ . Since  $A$  is  $AS_\mu$  subset relative to  $X$ , there exists a finite subset  $B$  of  $A$  such that  $A \subset \cup\{\text{int}_\mu \text{cl}_\mu(U_x); x \in B\}$ . Since  $G$  is a grill,  $\cup\{\text{int}_\mu \text{cl}_\mu(U_x); x \in B\} \notin G$ . Hence  $A \notin G$ , a contradiction. Hence  $G$   $\delta_\mu$ -converges to some point in  $A$ . Conversely let  $U = \{U_\alpha; \alpha \in \Lambda\}$  be a cover of  $A$  by  $\mu$ -open sets of  $X$ . If  $A$  is not  $AS_\mu$ -subset relative to  $X$ , then for every finite subset  $\lambda_0$  of  $\Lambda$ ,  $A - \cup\{\text{int}_\mu \text{cl}_\mu(U_\alpha); \alpha \in \lambda_0\} \neq \emptyset$ . Let  $F = \{A - \cup\text{int}_\mu \text{cl}_\mu(U_\alpha); \alpha \in \lambda_0, \lambda_0 \subset \Lambda \text{ is finite}\}$ . Then  $F$  is a filterbase on  $X$ . The family  $F$  can be extended to an ultrafilter  $M$  on  $X$ . Then  $M$  is a grill on  $X$ . Since for every  $F \in F, F \subset A$ , we have  $A \in M$ . But  $U$  is a cover of  $X$ , then for all  $x \in A$ , there exists  $\beta \in \Lambda$  such that  $x \in U_\beta$ . For any  $G \in M, G \cap (A - \text{int}_\mu \text{cl}_\mu(U_\beta)) \neq \emptyset$ . Hence  $G \not\subset \text{int}_\mu \text{cl}_\mu(U_\beta)$  for all  $G \in M$  and so  $M$  does not  $\delta_\mu$ -converge to any point  $A$ , a contradiction. Thus if every grill on  $X$  with a  $\delta_\mu$ -converges to a point in  $A$ , then the subset  $A$  of a space  $(X, \tau)$  is  $AS_\mu$ -subset relative to  $X$ . Hence the proof.

**Theorem 3.15**

Let  $(X, \tau)$  be space such that every grill  $G$  on  $X$  with the property  $\cap\{c_{\delta_\mu}(G_i); 1 \leq i \leq n\} \neq \emptyset$  for every finite subfamily  $\{G_1, G_2, \dots, G_n\}$  of  $G, \delta_\mu$ -adheres in  $X$ , Then  $X$  is a  $AS_\mu$ -space.

**Proof**

Let  $U$  be any ultrafilter on  $X$ . Then  $U$  is a grill on  $X$ . Also, for each finite subcollection  $\{U_1, U_2, \dots, U_n\}$  of  $U, \cap\{U_i; 1 \leq i \leq n\} \neq \emptyset$ . Since  $\cap\{U_i; 1 \leq i \leq n\} \subset \{c_{\delta_\mu}(U_i); 1 \leq i \leq n\}, \cap\{c_{\delta_\mu}(U_i); 1 \leq i \leq n\} \neq \emptyset$ . Therefore  $X$  is  $AS_\mu$ -space. Hence by hypothesis,  $U$   $\delta_\mu$ -adheres.

**Definition 3.16**

A grill  $G$  on the topological space  $(X, \tau)$  is said to be  $\delta_\mu$ -linked if for any two members  $A, B \in G, c_{\delta_\mu}(A) \cap c_{\delta_\mu}(B) \neq \emptyset$ .

**Definition 3.17**

A grill  $G$  on the topological space  $(X, \tau)$  is said to be  $\delta_\mu$ -conjoint if every finite subfamily  $\{A_1, A_2, \dots, A_n\}$  of  $G, \text{int}_\mu(\cap\{c_{\delta_\mu}(A_i); 1 \leq i \leq n\}) \neq \emptyset$ .

**Remark 3.18**

- (1) Every  $\delta_\mu$ -conjoint grill is  $\delta_\mu$ -linked.
- (2) If  $\mu = \pi$ , we have  $p(\delta)$ -linked grill and  $p(\delta)$ -conjoint grill.

**Theorem 3.19**

Let  $(X, \tau)$  be a  $AS_\mu$ -space. Then every  $\delta_\mu$ -conjoint grill  $\delta_\mu$ -adheres in  $X$ .

**Proof**

Let  $G$  be a  $\delta_\mu$ -conjoint grill on a  $AS_\mu$ -space  $X$ . Since  $c_{\delta_\mu}(A)$  is  $AS_\mu$ -space for every  $A \subset X, \{c_{\delta_\mu}(A); A \in G\}$  is a collection of  $AS_\mu$ -sets in  $X$ . But  $G$  is  $\delta_\mu$ -conjoint, for any finite subfamily  $\{A_1, A_2, \dots, A_n\}$  of  $G, \text{int}_\mu(\cap\{c_{\delta_\mu}(A_i); 1 \leq i \leq n\}) \neq \emptyset$  and so  $\cap\{\text{int}_\mu(c_{\delta_\mu}(A_i)); 1 \leq i \leq n\} \neq \emptyset$ . Hence  $\cap\{c_{\delta_\mu}(A); A \in G\} \neq \emptyset$  and so there exists  $x \in X$  such that  $x \in c_{\delta_\mu}(A)$  for every  $A \in G$  which implies that  $A \in G(\delta_\mu, x)$  for all  $A \in G$  which implies that  $G \subset G(\delta_\mu, x)$ . Therefore,  $G$   $\delta_\mu$ -adheres at  $x$  in  $X$ . Hence the proof.

**Theorem 3.20**

Let  $(X, \tau)$  be a  $AS_\mu$ -space where  $\mu \in \{\sigma, b, \beta\}$ . Then every grill  $G$  on  $X$ , with the property that  $\cap\{c_{\delta_\mu}(G_i); 1 \leq i \leq n\} \neq \emptyset$  for every finite subfamily  $\{G_1, G_2, \dots, G_n\}$  of  $G, \delta_\mu$ -adheres in  $X$ .

**Proof**

Let  $X$  be a  $AS_\mu$ -space.  $G = \{G_\alpha; \alpha \in \Lambda\}$  be a grill on  $X$  with the property  $\cap\{c_{\delta_\mu}(G_\alpha); \alpha \in \Lambda_0\} \neq \emptyset$  for every finite subset  $\Lambda_0$  of  $\Lambda$ . Consider the family  $F = \{\cap\{c_{\delta_\mu}(G_\alpha); \alpha \in \Lambda_0, \Lambda_0 \subset \Lambda\}\}$ . Then  $F$  is a filterbase on  $X$ . Since  $X$  is  $AS_\mu$ -space,  $F, \delta_\mu$ -adheres at some point  $x \in X$  implies that  $x \in c_{\delta_\mu}(c_{\delta_\mu}(G))$  for every  $G \in G$ . Then  $x \in c_{\delta_\mu}(G)$  for every  $G \in G$ . Hence  $G \subset G(\delta_\mu, x)$  and hence the proof.

**Corollary 3.21**

Let  $(X, \tau)$  be a space where  $\mu \in \{\sigma, b, \beta\}$ . Then the following are equivalent.

- (a) Every grill  $G$  on  $X$ , with the property that  $\cap\{c_{\delta_\mu}(G_i); 1 \leq i \leq n\} \neq \emptyset$  for every finite subfamily  $\{G_1, G_2, \dots, G_n\}$  of  $G, \delta_\mu$ -adheres in  $X$ .
- (b)  $X$  is  $AS_\mu$ -space.

**Proof**

Using the theorems 2.20 and 2.15, we can prove this theorem.

**Definitions 3.22**

- 1. A space  $X$  is  $\mu N C$  compact where  $\mu \in \{\tau, \alpha, \sigma, \pi, b, \beta\}$  if every cover  $U$  of  $X$  by  $\mu$ -open sets of  $X$  has finite sub-collection such that the interiors of closures of whose members cover the space  $X$ .
- 2. A grill  $G$  on  $X$  is said to be  $\mu D$ -converge to a point  $x \in X$  if  $\text{int}_\mu \text{cl}_\mu(\mu(x)) \subset G$  where  $\mu(x)$  denote the family of all  $\mu$ -open sets containing the point  $x$ .
- 3. A space  $X$  is  $\delta_\mu$ -regular if for every grill on  $X$  which  $\delta_\mu$ -converges must  $\mu D$ -converge.

**Theorem 3.23**

A space  $(X, \tau)$  is  $\mu$ -nearly compact if and only if every grill  $\mu D$ -converges.

**Proof**

Let  $G$  be a grill on a  $\mu$  nearly compact space  $X$  such that  $G$  does not  $\mu D$ -converge to any point  $x \in X$ . Then for each  $x \in X$ , there exists a  $\mu$ -open set  $U_x$  containing  $x$  with  $U_x \notin G$ . Since  $\{U_x; x \in X\}$  is a cover of the  $\mu$  nearly compact  $X$  by  $\mu$ -open sets, there exist finitely many points  $\{x_1, x_2, \dots, x_n\}$  in  $X$  such that  $X = \cup\{U_{x_i}; 1 \leq i \leq n\}$ . Since  $X \in G$ , there exists  $U_{x_i} \in G$  for  $1 \leq i \leq n$  a contradiction.

Conversely, let every grill on  $X$   $\mu D$ -converge. Suppose  $X$  is not  $\mu$  nearly compact. Then there exists a cover  $U$  of  $X$  by  $\mu$ -open sets of  $X$  having no subcover. Then  $F = \{X - \cup U_0; U_0 \text{ is a finite subcollection of } U\}$  is a filter base on  $X$ . Then  $F$  is contained in an ultrafilter  $G$  and so  $G$  is a grill on  $X$ . By hypothesis,  $G, \mu D$ -converges to some point  $x \in X$ . Then for some  $U \in U, x \in U$  and hence  $U \in G$ . But  $X - U \in F \subset G$ . Hence  $U$  and  $X - U$  both belongs to  $G$ , which is an ultrafilter, a contradiction. Therefore if every grill  $\mu D$ -converges then the space is  $\mu$ -nearly compact.

**Theorem 3.24**

A  $\mu$ -nearly compact space  $X$  is  $AS_\mu$ -space. The converse holds if  $X$  is  $\delta_\mu$ -regular.

**Proof**

Suppose  $G$  is a grill on a  $AS_\mu$ -space  $X$ . Then  $G$   $\delta_\mu$ -converges in  $X$ , by theorem 2.12. Since  $X$  is  $\delta_\mu$ -regular,  $G$   $\mu D$ -converges. By theorem 2.24,  $X$  is  $\mu$ -nearly compact.

**Theorem 3.25**

Every  $\mu$ -regular space is  $\delta_\mu$ -regular.

**Proof**

Let  $G$  be a grill on a  $\mu$ -regular space  $X$ . Suppose  $\delta_\mu$ -converges to a point  $x \in X$ . For each  $U \in \mu$  containing  $x$ , there exists a  $V \in \mu$  containing  $x$  such that  $\text{int}_\mu \text{cl}_\mu(V) \subset U$ . By

hypothesis,  $\text{int}_\mu \text{cl}_\mu U \in G$  and so  $U \in G$ . Hence  $G, \mu D -$  converges to  $x$ . Thus  $X$  is  $\delta_\mu -$  regular.

**Remark 3.26**

For a topological space  $X$  the following are equivalent.

- (1)  $X$  is  $\mu$ -nearly compact.
- (2)  $X$  is  $AS_\mu$ -space.
- (3) Every family of  $\delta_\mu$ -closure of sets having finite intersection property in grill  $\delta_\mu -$  adheres in  $X$ .
- (4) Every filter (filter-base) of  $\mu$ -open sets has  $\delta -$  adherent point.
- (5) Every ultrafilter (ultrafilter-base) of  $\mu$ -open sets is  $\delta -$  convergent.
- (6) Every grill of  $\mu$ -open sets in the topological space  $X$  is  $\delta -$  adherent in  $X$ .

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