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Seismic attenuation (Q) Estimation from VSP Data using Kolsky's attenuation and dispersion model

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ABSTRACT

P- and S-wave attenuations are studied using vertical and horizontal vibrator sources and zero offset VSP data from the Ross Lake heavy oilfield, Saskatchewan. We find that the S-wave shows a larger amplitude loss and phase change than the P-wave over the same depths. This suggests that we will need to pay attention to attenuation in matching the phase of PP and PS images. A new approach to spectral ratio method has been developed to calculate a robust continuous interval Q factor from zero-offset VSP data. We also establish an estimate quality indicator (QQI) curve to highlight where we can obtain a reasonable Q factor. Poor Q estimates may arise from casing-bond problems, multiple casing areas, or source inconsistencies. Our VSP-derived Qp curve shows an inverse linear relationship with the VSP-derived Vp/Vs curve. Finally, the bulk value of Qp, Vp/Vs and Vp are estimated for three main geological formations in this oilfield.

Introduction

The spectral ratio method is widely used to determine an attenuation or Q factor from VSP data (Tonn, 1991). For two receivers at depths d1 and d2:Two fundamental properties associated with seismic wave propagation through subsurface materials are the energy dissipation of plane waves with high frequencies, and the velocity dispersion causing high-frequency plane waves to travel faster than low-frequency waves. Mathematically, these dissipation and dispersion effects of the viscoacoustic media may be represented by a specified quality factor, *Q:*

$$Q(w) = \frac{|w|}{2\alpha(w)c(w)} \qquad (1.1)$$

where w is the angular frequency, *c(w)* is the phase velocity and *α(w)* is the attenuation coefficient in units of inverse length. Equation (1.1) is a key formula for the earth *Q* model (Kolsky, 1953; Mason, 1958; Futterman, 1962) and is valid under a low-loss assumption, *Q* >>1. Such a small dissipation assumption is valid under most conditions of interest in geophysics. In geophysics literature, different earth *Q* models are expressed in terms of different definitions of the attenuation coefficient *a(w)* and different definitions of the phase velocity c(w). Before we start discussing the design and application of an inverse *Q* filter, we need first to specify a mathematical *Q* model.

Kolsky's attenuation-dispersion model

The Kolsky model (Kolsky, 1953) assumes the attenuation α(w) to be strictly linear with frequency over the range of measurement:

$$\alpha(w) = \frac{|w|}{2c_r Q_r} \qquad (1.2)$$

And defines the phase velocity as (Kolsky 1956):

$$\frac{1}{c(w)} = \frac{1}{c_r}\left(1 - \frac{1}{\pi Q_r}\ln\left|\frac{w}{w_r}\right|\right) \qquad (1.3)$$

Where c_r and Q_r are the phase velocity and the Q value at a reference frequency w_r. Kolsky's model was derived from and fitted well with experimental observations. However, the basic Kolsky model does not satisfy the minimum delay condition in dispersive earth media. It is therefore necessary to modify it by the Kramers-Krônig dispersion relation to fully correct the velocity dispersion:

$$\frac{w}{c(w)} - \frac{w}{c_\infty} = H\{a(w)\} \qquad (1.4)$$

where H denotes the Hilbert transform and c_∞ is the limit of c(w) as w approach infinity.This procedure is very well documented in Wang's book. Briefly I can state that we take the attenuation coefficient (1.2) and compare it with phase velocity (1.3) using Kramers-Krônig dispersion relation. In general we can state that if we define an attenuation coefficient α , we can use the Kramers

-Krônig dispersion relation and get a phase velocity that will give a damping-model that has a causal solution of the wave equation by introducing the Hilbert transformation in (1.4).

Modification to the Kolsky model

The phase velocity formula given in the basic Kolsky model is now modified by setting an appropriate reference frequency, to minimize the phase errors in inverse *Q* filtering for seismic data processing (Wang and Quo, 2004b). Recall that in the Kolsky model, given the attenuation coefficient (1.2), the phase velocity is defined by equation (1.3). This expression, however, is merely an asymptotic formula for $w >> w_r$ (Futterman, 1962). For exploration seismic data, which have relative low frequencies (say, < 500 Hz), we propose to modify the preceding expressions (1.3) and (1.4) as follows:

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$$\frac{1}{c(w)}=\frac{1}{c_r}\left(1-\frac{1}{\pi Q_r}\ln\left|\frac{w}{w_r}\right|\right)\approx\frac{1}{c_r}\left|\frac{w}{w_h}\right|^{-\gamma} \quad (1.5)$$

where w_h is a new, undetermined, tuning parameter. This tuning parameter still has units of frequency, but is no longer the smallest frequency of the seismic band. We now show that for seismic exploration, the tuning constant w_h is the highest possible seismic frequency. The exact mechanism of seismic dispersion through the earth is unknown. If possible a generic model should be considered when designing an inverse *Q* filter. One such generic condition is the minimum delay described by the Kramers-Kronig dispersion relation. Given the attenuation coefficient (1.12), we now compare the phase velocity (1.15) in the modified Kolsky model with the phase velocity estimated using the Kramers-Kronig dispersion relation (1.14).

Mathematical definition of *Q-models*

When a plane wave propagates through a homogeneous viscoelastic medium, the effects of amplitude attenuation and velocity dispersion may be combined conveniently into a single dimensionless parameter, *Q*, the medium-quality factor. The *Q* parameter can be expressed as follows:

$$Q(w)=\frac{1}{2}\left(\frac{|w|}{\alpha(w)c(w)}-\frac{\alpha(w)c(w)}{|w|}\right) \quad (3.1)$$

As the *Q* value must be positive, the above equation leads to Assuming

$$\frac{\alpha(w)c(w)}{|w|}<<1 \quad (3.2)$$

the Q expression (3.1) is approximated to

$$Q(w)=\frac{|w|}{2\alpha(w)c(w)} \quad (3.3)$$

The above approximate expression has been presented earlier (equation 1.1) as the definition of *Q*. Considering this definition, the assumption (3.2) is equivalent to $Q^{-1}(w)$. With this small-dissipation assumption, $Q^{-1}(w)$ is often known as the dissipation factor. Such a small-dissipation assumption is valid under most conditions of interest in geophysics; for instance, Kolsky (1953, p. 106), Mason (1958, p.214) and Futterman (1962, eq.20). To intuitively understand the condition $Q^{-1}(w)<<1$ let us see two simple examples. For $Q\approx 5$, the accurate solution is Q = 0.5x (10-0.1) = 0.495. That is, for $Q^{-1}(w)\leq 0.2$, the maximum error for the *Q* value is 1%. For $Q\approx 10$, the accurate solution is Q = 9.975. That is, the maximum error is only 0.25% for $Q^{-1}(w)\leq 0.1$.

We now show the derivation of equation (3.1) and approximation (3.3). We start from the stress-strain relation: ∑(w) = M(w) E(w) where *∑(w)* and *∑(w)* are the stress and the strain components at frequency w, and are related by an appropriate elastic modulus *M(w)*. Given a sinusoidally varying stress, the strain response will also be sinusoidal. The phase lag of strain behind the stress may define the dissipation factor (White, 1965)

$$\xi(w)=\frac{1}{Q(w)}=\frac{M_{Im}(w)}{M_{Re}(w)} \quad (3.5)$$

where $M_{Re}(w)$ and $M_{Im}(w)$ are the real and imaginary part of the complex modulus *M(co)*. The derivation of $M_{Rt}(oo)$ and $M\backslash_m(a>)$ is given as follows. A plane wave propagation may be expressed as

$U(x,w) = U_0(w)exp[i(wt-kx)]$ (3.6)

where $U_0(w)$ is the Fourier transform of the propagating pulse, and *k* is the wavenumber, and *x* and *t* are the travel distance and time, respectively. In viscoelastic media, wavenumber *k* becomes a complex function, defined as

$$\kappa(w)=\frac{w}{c(w)}=\frac{w}{c\ (w)}-i\alpha(w) \quad (3.7)$$

where *c(w)* is the complex velocity:

$$c(w)=\sqrt{\frac{M(w)}{\rho}} \quad (3.8)$$

and ρ is the density. Then the wave-propagation expression (3.6) becomes

$U(x,w) = U_0(w)exp[-\alpha(w)x]exp[iw(t-x/c(w))]$ (3.9)

This shows that $\alpha(w)$ is related to the amplitude attenuation, and $w/c(w)$ is related to the phase change along the propagation distance *x*. Both $\alpha(w)$ and *c(w)* are real, positive and even functions of frequency *w*. From equations (3.7) and (3.8), we have the equation

$$\left(\frac{1}{c(w)}-i\frac{\alpha(w)}{w}\right)^2=\frac{\rho}{|M(w)|^2}\left(M_{Re}(w)-iM_{Im}(w)\right)$$

This leads to

$$M_{Re}(w)=\left(\frac{1}{c^2(w)}-\frac{\alpha^2(w)}{w^2}\right)\frac{|M(w)|^2}{\rho}$$ and

$$M_{Im}(w)=\left(\frac{2\alpha w)}{wc(w)}-\right)\frac{|M(w)|^2}{\rho}$$

Substitution of equations (3.11) and (3.12) into equation (3.5) will result in the *Q* definition of equation (3.1) straightforwardly. Recall that in equation (3.1), the assumption of positive *Q* values leads to the relation $[\alpha(w)c(w)/w]^2<1$. Therefore, $M_{Re}(w)$ in equation (3.11) is a positive and even function of w. The imaginary part $M_{Im}(w)$ in equation (3.12) is an odd function of *co* and is positive for w> 0. Note also that a natural limitation on $\alpha(w)$ is implied by the Wiener criterion:

$$\lim_{w\to\infty}\frac{\alpha(w)}{w}=0 \quad (3.13)$$

From equations (3.11) and (3.12), we obtain the attenuation coefficient (Aki and Richards, 1980)

$$\alpha(w)=\frac{|w|M_{Im}(w)}{|M(w)|}\sqrt{\frac{\rho}{2[M_{Re}(w)+|M(w)|]}}=|w|\sqrt{\frac{\rho}{M_{Re}(w)}\frac{\xi}{\sqrt{2(1+\xi^2)(1+\sqrt{1+\xi^2})}}}$$

(3.14) and the phase velocity

$$\frac{1}{c(w)}=\frac{1}{|M(w)|}\sqrt{\frac{\rho[M_{Re}(w)+|M(w)|]}{2}}=\sqrt{\frac{\rho}{M_{Re}(w)}}\sqrt{\frac{(1+\sqrt{1+\xi^2})}{2(1+\xi^2)}}$$

(3.15)

where ξ is the dissipation factor defined by equation (3.5). For $\xi=Q^{-1}<<1$equations (3.14) and (3.15) can be approximated to

$$\alpha(w)=|w|\sqrt{\frac{\rho}{2M_{Re}(w)}\frac{\xi}{2}} \quad (3.16)$$

and

$$\frac{1}{c(w)}=\sqrt{\frac{\rho}{M_{Re}(w)}} \quad (3.17)$$

Combining these two approximations will also lead to the approximate formula (3.3).

or the transmission response of the medium to be causal, the attenuation and the phase velocity must satisfy the Kramers-Kronig relations. Dispersion relations of the Kramers-Kronig type (Kronig, 1926; Kramers, 1927), well known in electric circuit theory, determine, for example, the real part of the propagation constant from the values of the imaginary part summed over the entire range of frequencies for wave motions that are linear. The Kramers-Kronig dispersion relations for the attenuation and phase velocity are given by

$$\frac{w}{c(w)} - \frac{w}{c_\infty} = H\{\alpha(w)\} \quad \text{And} \quad \frac{\alpha(w) - \alpha(0)}{w} = -H\{\frac{1}{c(w)} - \frac{1}{c_\infty}\} \qquad (3.18)$$

where *H {.}* denotes the Hilbert transform, c_∞ is the limit of c*(w)* as w -> ∞,

c_∞= lim c*(w)*$_{w->\infty}$ = *lim*$_{w->\infty}$ $\sqrt{\frac{M_{Re}(w)}{\rho}}$ is the limit of *a(co)* as *a)—>0*, and α(0) = lim c*(w)*$_{w->\infty}$ =0

If the relations (3.18) and (3.19) are fulfilled, then the transmission response is minimum delay with respect to the first arrival time (e.g. Aki and Richards, 1980).

For the application of inverse *Q* filtering to real seismic traces, we henceforth consider the complex

Kolsky's *Q* model and the complex wave number

The Kolsky model is used extensively in seismic inverse *Q* filtering because the parameters involved are relatively easy to estimate from a seismic data t itself or some other seismic measurements. For the Kolsky model, given i attenuation coefficient (2.2) and the phase velocity (2.3), we may use ation (3.1) to derive the quality factor *Q* as

$$Q(w) = Q_r - \frac{1}{4Q_r} - \frac{1}{\pi}\left(1 + \frac{1}{4Q_r^{\ 2}}\right)\ln\left|\frac{w}{w_r}\right|$$

or approximations

$$Q(w) = Q_r\left(1 - \frac{1}{\pi Q_r}\ln\left|\frac{w}{w_r}\right|\right) \approx Q_r\left|\frac{w}{w_r}\right|^{-\gamma}$$

Where $\gamma = (\pi Q_r)^{-1}$ (3.24)

In the previous chapter, we discussed that it is necessary to modify the .sic Kolsky model so as to accurately represent the velocity dispersion effect. Given the modified phase velocity (2.7), the modified Kolsky *Q* model is expressed as

$$Q(w) = Q_r\left(1 - \frac{1}{\pi Q_r}\ln\left|\frac{w}{w_r}\right|\right) \approx Q_r\left|\frac{w}{w_r}\right|^{-\gamma} \qquad (3.25)$$

Using this expression, we may rewrite the attenuation coefficient as

$$\alpha(w) = \frac{|w|}{2c_r Q_r} \approx \frac{|w|}{2c_r Q(w)}\left|\frac{w}{w_h}\right|^{-\gamma} \qquad (3.26)$$

Substituting this expression and the modified phase velocity (2.7) into equation (3.7), we may obtain the complex wavenumber as

$$k(w) \approx \left(1 - \frac{i}{2Q(w)}\right)\frac{|w|}{c_r}\left|\frac{w}{w_h}\right|^{-\gamma}$$

This definition of complex wavenumber &(<y) is the basis for designing a inverse *Q* filter throughout the rest of this book.

Comparison with different *Q* models

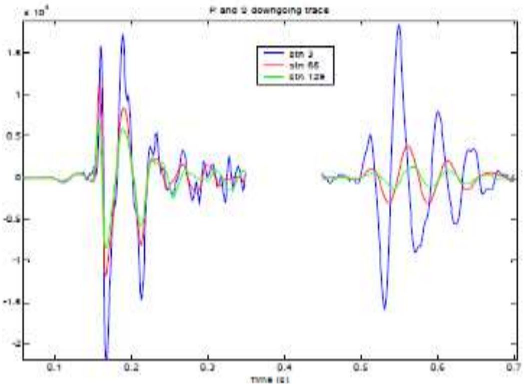
Although the Kolsky viscoelastic model is widely used in seismic data processing, for its simplicity, the basic Kolsky model does not rigorously satisfy the Kramers-Kronig dispersion relation. Several other expressions that satisfy the dispersion relationship can also be found in the literature. In this section, some of these models are stated briefly with explicit expressions for both the attenuation and the phase velocity functions. Each of these models is compared with the modified Kolsky model to find out how each model can be used interchangeably in an inverse *Q* filter.

This section compares a total of eight earth *Q* models (including the modified Kolsky model) to reveal similarities and differences between them. They are:

1) the modified Kolsky model (linear attenuation);
2) the Strick-Azimi model (power-law attenuation);
3) the Kjartansson model (constant *Q);*
4) Azimi's second and third models (non-linear attenuation);
5) Müller's model (power-law 0;
6) the Zener model (the standard linear solid);
7) the Cole-Cole model (a general linear-solid); and
8) a new general linear model.

These eight models may be classified into two groups. The first group consists of models 1-5, and the other group includes models 6-8. The main difference between these two groups is the behaviour of the phase velocity when the frequency approaches zero, where the first group has a zero-valued phase velocity, and the second group has a finite, nonzero phase velocity. One exception is the Cole-Cole model, which can give either zero or nonzero phase velocity at zero frequency, depending upon an exponent that is chosen.The linear solid model, including the standard and general versions, may be preferred in finite-difference modelling algorithms because it gives additional differential equations that can be approximated by finite differences. One of the features of linear solid models is that they have finite phase velocity and attenuation coefficient at infinite frequencies. Conversely, the Kolsky model and Kjartansson's constant-Q model, for instance, assume $c_\infty = \infty$ and $\alpha_\infty = \infty$ Throughout the comparison the parameters used for the calculation of the modified Kolsky model (are Q_r=50, c_r=2500ms and w_h= 1000π i.e. the highest possible seismic frequency is about 500 Hz.

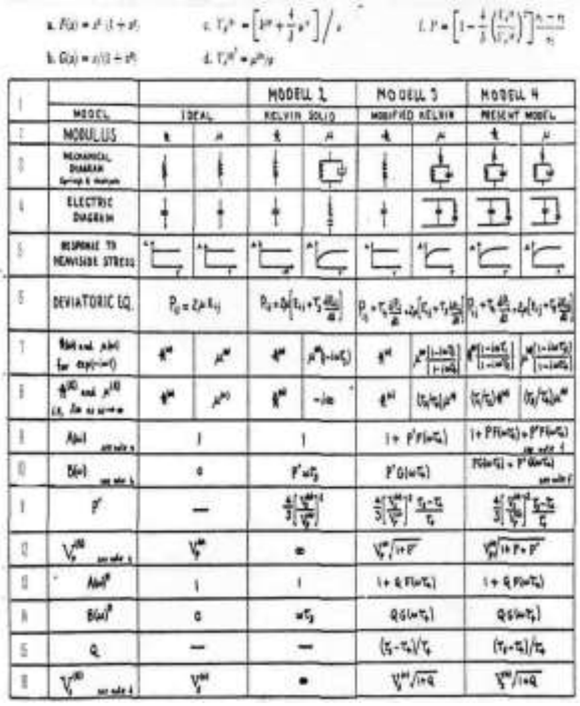
We will not discuss models 1-5. They are investigated by Wang and since our first attempt to introduce damping models where the Kelvin model, we will go further with the models introduced by Horton that is the standard linear solid model according to Wang. The basic idea with these models is that stress and strain are linked in the wave equation.



Zener or standard linear solid model

The Zener (1948) or standard linear solid model is the most general linear equation that links the stress and strain as we have seen in (1.?). (Ben-Menahem and Singh, 1981). It defines the attenuation coefficient as:

$$\alpha(w) = \frac{w^2\tau_c}{c_0 Q_c(1 + w^2\tau_c^{\ 2})} \qquad (3.3)$$



and the phase velocity as

$$\frac{1}{c(w)} \approx \frac{1}{c_0}\left(1-\frac{w^2\tau_c{}^2}{Q_c(1+w^2\tau_c{}^2)}\right) \qquad (3.4)$$

where c_0 is the phase velocity c(w) for w -» 0, and τ_c and Q_c are two constant parameters describing the attenuation property in the standard linear solid model: the attenuation at the peak of the attenuation function with respect to the frequency is $Q^{-1}{}_c$, and the (angular) frequency at this location is $\tau_c{}^{-1}$

Mathematical outline is as following:

$$M(w) = M_R\,\frac{1+iw\tau_3}{1+iw\tau_4} \qquad (3.50)$$

where τ_3 is the strain relaxation time, τ_3 is the stress relaxation time and M_R is a deformation modulus with sub-index $_R$ denoting the relaxed modulus.

The mechanical 'relaxation' means that the strain produced by the sudden application of a fixed stress to a material increases asymptotically with time. Similarly, the stress produced when the material is suddenly strained relaxes asymptotically (Kolsky, 1953). It is found that stress waves whose periods are close to the 'relaxation times' of such a medium are severely attenuated when passing through it.

Physically, the 'relaxed' elastic modulus M_R is the final value of the ratio of stress to strain after relaxation has taken place, whereas a so-called 'unrelaxed' elastic modulus, M_v, is the initial value of the ratio of stress to strain, before relaxation has time to occur. Mathematically, the relaxed modulus M_R is obtained from *M(ca)* for <a->0 (see equation 3.50).

he unrelaxed modulus can be given by

M_U = lim $M(w)$ = M_R τ_3 / τ_3 (3.51)

Thus, M_R and M_U are also called the low- and the high-frequency moduli, respectively.A special case of the standard linear solid model is the Kelvin-Voigt viscoelastic model obtained when $\tau_4 = 0$, so that

$M(w) = M_R\,(1 + iw\,\tau_3)$ (3.52)

The real part of the complex modulus *M(w)* in equation (3.50) may be written as

$$M_{Re}(w) = M_R\,\frac{1+iw^2\tau_3\tau_4}{1+iw^2\tau^2{}_4} \qquad (3.53)$$

and the attenuation Q^{-1} is given by

$$Q^{-1}(w) = \xi(w) = \frac{|w|(\tau_3-\tau_4)}{1+iw^2\tau^2{}_4} \qquad (3.54)$$

which measures the lag of the strain behind the stress. We assume here $\tau_3 > \tau_4$. Substituting them into equations (3.16) and (3.17), we may obtain the attenuation

$$\alpha(w) = \frac{w^2(\tau_3-\tau_4)}{2c_0(1+iw^2\tau^2{}_4)} \qquad (3.55)$$

and the phase velocity

$$\frac{1}{c(w)} \approx \frac{1}{c_0}\left(1-\frac{w^2\tau_4(\tau_3-\tau_4)}{2(1+iw^2\tau_3\tau_4)}\right) \qquad (3.56)$$

where c_0 is the phase velocity for w -> 0,

$$c_0 = \sqrt{\frac{M_{Re}(w)}{\rho}} = lim_{w\to\infty}\sqrt{\frac{M_{Re}(w)}{\rho}} \qquad (3.57)$$

Attempting to use a single relaxation time, we may define two parameters as follows:

$$Q^{-1}{}_c = \frac{(\tau_3-\tau_4)}{2\sqrt{\tau_3\tau_4}} \quad \text{and} \quad \tau_c = \sqrt{\tau_3\tau_4} \qquad (3.58)$$

For the standard linear solid model, considering equation (3.51), this can be written as $Q\sim^{l}$ = *jAM,* where AM is the modulus defect

$$\nabla M = \frac{M_U - M_R}{\sqrt{M_U M_R}} \qquad (3.59)$$

that is, the normalized difference between the unrelaxed and relaxed moduli. For the standard linear solid model, the attenuation at the peak of the attenuation function with respect to frequency is ~ AM and the (angular) frequency at this location is l/τ_c.

The parameters τ_3 and τ_4 in (3.55) and (3.56) may be expressed as (Casula and Carcione, 1992):

and

$$\tau_3 = \tau_c\left(\sqrt{1+\frac{1}{Q_c{}^2}}+\frac{1}{Q_c}\right) \qquad (3.60a)$$

And

$$\tau_4 = \tau_c\left(\sqrt{1+\frac{1}{Q_c{}^2}}-\frac{1}{Q_c}\right) \qquad (3.60b)$$

obtained from the Q_c and r_c definition in (3.58). Therefore, we may rewrite expressions (3.55) and (3.56) as (2.32) for the attenuation and as (2.33) for the phase velocity. With these two expressions, Zener's *Q* model may be expressed as

$$Q^{-1}(w) \approx \frac{2|w|\tau_c}{Q_c(1+w^2\tau^2{}_c)} \qquad (3.61)$$

To compare this standard linear solid model with the Kolsky model, as shown in Figure 2.10, we may first set

$$\tau_c = \frac{1}{w_h} \qquad (3.6)$$

since the highest frequency of the seismic band has the strongest attenuation.We then use the following two approximations:

$$\frac{1}{c_0}=\frac{1}{c_r}\left(\frac{1}{2Q_r}\left|\frac{w}{w_h}\right|+\left|\frac{w}{w_h}\right|^{-\frac{1}{\pi Q_r}}\right)$$

Right side of equation 3.6 can be written:

$$\frac{1}{c_r}\left(1+\frac{1}{2Q_r}\right) \qquad (3.7)$$

and

$$Q_c=\left(\frac{1+2Q_r\left|\frac{w}{w_h}\right|^{-\left(1+\frac{1}{\pi Q_r}\right)}}{1+\left|\frac{w}{w_h}\right|^{-2}}\right) \qquad (3.8)$$

The Cole-Cole model

To generalize the standard linear solid model, the Cole and Cole (1941) model proposed for dielectrics can be used. In the Cole-Cole model, the attenuation coefficient may be expressed as

$$\alpha(w)\approx\frac{\gamma\,|w\ \tau_c|^{1+\gamma}\sin(\frac{\pi}{2}\gamma)}{c_0\tau_c Q_c[1+2\,|w\tau_c|^{\gamma})\cos(\frac{\pi}{2}\gamma)+|w\tau_c|^{2\gamma}]} \qquad (3.9)$$

and the phase velocity may be expressed as

$$\frac{1}{c(w)}\approx\frac{1}{c_0}\left(1-\frac{\gamma\,|w\ \tau_c|^{\gamma}\,[\cos(\frac{\pi}{2}\gamma)+|w\tau_c|^{\gamma}]}{Q_c[1+2\,|w\tau_c|^{\gamma})\cos(\frac{\pi}{2}\gamma)+|w\tau_c|^{2\gamma}]}\right) \qquad (3.10)$$

To generalize the standard linear solid model, we may use the model proposed for dielectrics by Cole and Cole (1941). Jones (1986) extended the Cole-Cole model to viscoelastic media for fitting laboratory data involving frequency-dependent absorption and dispersion, and found that it was more satisfying than the previously used dispersion-absorption relationships because of its generality and because its relaxation time was related to the petrophysical property of pore geometry (viscosity). In the Cole-Cole model, the complex modulus may be written as

$$M(w)=M_R\,\frac{1+(iw\tau_3)^{\gamma}}{1+(iw\tau_4)^{\gamma}} \qquad (3.62)$$

When *y = 1*, this is the standard linear solid model. The unrelaxed modulus is given by

M_U = lim $M(w)$ = M_R $(\tau_3/\tau_4)^{\gamma}$ (3.63)

Therefore equation (3.62) may also be expressed as

$$M(w)\approx M_U+\frac{M_R-M_U}{1+(iw\tau_4)^{\gamma}} \qquad (3.64)$$

which is the original formula presented in Cole and Cole (1941, eq.5). The real part of the complex modulus is

$$M_{Re}(w)=M_R\,\frac{1+(w^2\tau_3\tau_4)^{\gamma}+(|w\tau_3|^{\gamma}+|w\tau_4|^{\gamma})\cos(\frac{\pi}{2}\gamma)}{1+|iw\tau_4|^{2\gamma}+2\,|iw\tau_4|^{\gamma}\cos(\frac{\pi}{2}\gamma)}$$

and the inverse *Q* function is given by

$$Q^{-1}(w)=\frac{(|w\tau_3|^{\gamma}-|w\tau_4|^{\gamma})\sin(\frac{\pi}{2}\gamma)}{1+|w^2\tau_4\tau_3|^{\gamma}+|w\tau_3|^{\gamma}+|w\tau_4|^{\gamma}\cos(\frac{\pi}{2}\gamma)} \qquad (3.65)$$

From these two expressions we use expression (3.16) to compute the attenuation coefficient,

$$\alpha(w)=\frac{w(|w\tau_3|^{\gamma}-|w\tau_4|^{\gamma})\sin(\frac{\pi}{2}\gamma)}{2c_0[1+|w^2\tau_4\tau_3|^{\gamma}+|w\tau_3|^{\gamma}+|w\tau_4|^{\gamma}\cos(\frac{\pi}{2}\gamma)]} \qquad (3.67)$$

and expression (3.17) for the phase velocity,

$$\frac{1}{c(w)}=\frac{1}{c_0}\left(1-\frac{(|w\tau_3|^{\gamma}-|w\tau_4|^{\gamma})[|w\tau_4|^{\gamma}+\cos(\frac{\pi}{2}\gamma)]}{2[1+|w\tau_4|^{2\gamma}+2\,|w\tau_4|^{\gamma}\cos(\frac{\pi}{2}\gamma)]}\right)$$

Based on the definition of τ_3 and τ_4 in equation (3.58), we have the following two approximate expressions (Ursin and Toverud, 2002):

$$\tau_3^{\ \gamma}\approx\tau_c^{\ \gamma}\left(1+\frac{\gamma}{Q_c}+\frac{\gamma^2}{2Q_c^{\ 2}}\right) \qquad (3.69a)$$

And

$$\tau_4^{\ \gamma}\approx\tau_c^{\ \gamma}\left(1-\frac{\gamma}{Q_c}+\frac{\gamma^2}{2Q_c^{\ 2}}\right) \qquad (3.69b)$$

The attenuation coefficient (3.67) and the phase velocity (3.68) can be i approximated to expressions (2.37) and (2.38), respectively. Finally, the earth *Q* model may be expressed as

$$Q^{-1}(w)=\frac{2\gamma\,|w\tau_c|^{\gamma}\sin(\frac{\pi}{2}\gamma)}{Q_c[1+|w^2\tau_c|^{2\gamma}+2\,|w\tau_c|^{\gamma}\cos(\frac{\pi}{2}\gamma)}$$

To compare the Cole-Cole model with the Kolsky model, we set $\tau_c = w_h^{-1}$, as we did in the Zener model. Meanwhile, we set

$Q_c=\frac{\gamma^2}{4}\pi Q_r$ (3.11) And $\frac{1}{c_0}=\frac{1}{c_r}\left(1+\frac{2}{\gamma\pi Q_r}\right)$ (3.12)

Figure 2.11 shows the attenuation and the phase velocity in the Cole-Cole model with (a) *y* = 1.0 and (b) *y* = 0.4. Compared with those in the Kolsky model, the Cole-Cole model with *y* = 0.4 has a better fit than using *y* = 1.0, which is the standard linear solid or Zener model.

A general linear model

Finally, an alternative to the Cole-Cole model is introduced to generalize the linear solid model (Wang and Guo, 2004b).

Mathematical outline follows here:

An alternative to the Cole-Cole model to generalize the Zener or standard linear solid model can be established using products and/or sums of different attenuation-dispersion models to obtain a more complicated behaviour as a function of frequency (Ben-Menahem and Singh, 1981). For instance, Liu *et al.* (1976) used a sum of standard linear solids to obtain a model with constant *Q* over three orders of magnitude of frequency. I now present an example of a general linear model, in which the complex velocity *c(a>)* is defined as (Wang and Guo, 2004b):

$$\frac{1}{c(w)}=\frac{1}{c_\infty}\left(1+\frac{a}{\sqrt{1+iw\tau}}+\frac{b}{1+iw\tau}\right) \qquad (3.71)$$

where r denotes a relaxation time constant. Considering *c(co) =)l p*, equation (3.71) is a standard linear solid model in parallel with a non-rational function involving a square root, which also shows some similarity to the Cole-Cole model.

This model is similar to Hanyga and Seredynska's (1999a, 1999b) model, which they claimed was appropriate for wave-propagation modelling in poro- and viscoelastic media. However, their model was without the factor l/v_x. Here I introduce this 1/v^ factor so that both sides of the equation have physical units of inverse velocity. When *ca* — > 0 and *a*> — > oo, the phase velocities can be expressed as

$$\mathrm{Re}\left\{\frac{1}{c_0}\right\} = \frac{1}{c_\infty}(1+a+b) \quad (3.72)$$

And

$$\mathrm{Re}\left\{\frac{1}{c_0}\right\} = \frac{1}{c_\infty} \quad (3.73)$$

respectively. The general linear model (3.71) defines the attenuation coefficient as

$$\alpha(w) \equiv -\mathrm{Im}\left\{\frac{w}{c(w)}\right\} \neq \frac{1}{c_\infty}\left(\frac{a}{2}(1+\frac{1}{2}w^2\tau^2)+b\right)\frac{w^2\tau}{1+w^2\tau^2} \quad (3.13)$$

and the phase velocity is defined as

$$\frac{1}{c(w)} \equiv \mathrm{Re}\left\{\frac{1}{c(w)}\right\} \approx \frac{1}{c_\infty}\left\{1+[a(1+\frac{5}{8}w^2\tau^2)+b]\frac{1}{1+w^2\tau^2}\right\} \quad (3.14)$$

In the previous, we have shown that this general linear model is able to match the modified Kolsky model reasonably well (Figure 2.12). Conversely, Ursin and Toverud (2002) mentioned Hanyga and Seredynska's (1999a, 1999b) model as an example of a general linear solid model but did not compare it with other models, either analytically or numerically. For this new general linear model, whuch was first presented in Wang and Guo (2004b), the earth Q modl may be expressed as

$$Q^{-1}(w) \approx \frac{|w\tau|(a+2b+\frac{1}{2}aw^2\tau^2)}{1+a+b+(1+\frac{5}{8}a)w^2\tau^2}$$

In this general linear model, the attenuation coefficient is defined as

$$\alpha(w) \approx \frac{1}{c_\infty}\left(\frac{a}{2}(1+\frac{1}{2}w^2\tau^2)+b\right)\frac{w^2\tau}{1+w^2\tau^2} \quad (3.13)$$

and the phase velocity is defined as

$$\frac{1}{c(w)} \approx \frac{1}{c_\infty}\left(1+a(1+\frac{5}{8}w^2\tau^2)+b\right)\frac{w^2\tau}{1+w^2\tau^2} \quad (3.14)$$

where r is a constant denoting the relaxation time, and *a* and *b* are two constant coefficients defining the general linear solid model.

To match this general linear model to the Kolsky model, we set

$_\infty = c_r$ a= - $(8/7Q_r)$ and *b =13/7* Q_r (3.15)

The basic Kolsky model is suitable for high-frequency waves. It is modified here by introducing a tuning parameter, which can be referred to as the highest possible frequency of the exploration and production seismic band, so that the model has an accurate representation about the velocity dispersion within the seismic frequency band. The investigation reveals that the modified Kolsky model can be fitted fairly well by any other model with a set of parameters derived analytically. Such matching exercises provide us with good confidence that when the Kolsky model is chosen for designing the inverse *Q* filter, the result should in principle be comparable with those using other different *Q* models.

Results and Discussion

The QQI_P curve shows a negative slope from 200m to 400m, which means that the amplitudes of high frequency components are increasing with depth. The possible reasons for this unphysical phenomen might be poor coupling between the casing and cement or between the cement and formation. The double-casing interval is a formidable complication. Therefore in this case, the FIRST trustable Qave is about 40 at about 445m depth. The Qave ~ 18 at about 200m may not be reliable. As the VSP is acquired from the bottom of the well up, the surface condition at the source location may be changing as the vibrator continues to shake and enhance its frequency contents. This, of course, violates the assumption of a constant source. It would be useful to have a monitor geophone. Confidently estimating Qs proved elusive in this data set. Looking at Figure 6, we can pick some good points between 200m to 750m and get a partial set of Qs. values. Below 750m, it’s hard to follow a positive slope.

Conclusion

We use the spectral ratio method to calculate Q values. A reliable continuous interval Qp curve from about 450m to 1050m in well 11-25 of Husky’s Ross Lake oilfield has been derived from a zero-offset VSP by this approach. Meanwhile, a quality indicator for Q factor estimation (QQI) has been established. This QQI curve reveals where the normal spectra ratio method gives us unstable Q values. The VSP-derived Qp curve demonstrates an inverse linear relationship with the VSP-derived Vp/Vs curve. Finally, the bulk value of Qp, Vp/Vs and Vp are estimated for three main geological formations in this oilfield.

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