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A Family of Estimators for Population Variance Using Two Auxiliary Variables

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ABSTRACT
This paper deals with the problem of estimating the population variance when some information on two auxiliary variables is available. It is shown that the proposed estimator is more efficient than the usual mean estimator and other existing estimators. The study is also extended to two-phase sampling. Theoretical results are supported by an empirical study.

Introduction

In sample surveys, it is well known that the use of the auxiliary information increases the efficiency of the estimators. We can use the knowledge of the auxiliary variable x for estimating the study variable y. When the correlation between the study variate and the auxiliary variate is positive ratio method of estimation is most effective and when this correlation is negative product method of estimation can be employed effectively. Let x and z denote the study variates taking the values (x_i,z_i).

In order to have an estimate of the study variable y. Isaki (1983) and Singh et al. (2007) proposed following estimators

$$t_1 = s_y^2\left(\frac{S_x^2}{s_x^2}\right) \tag{1.1}$$

$$t_2 = s_y^2\left(\frac{S_z^2}{s_z^2}\right) \tag{1.2}$$

$$t_3 = s_y^2\exp\left(\frac{S_x^2 - s_x^2}{S_x^2 - s_x^2}\right) \tag{1.3}$$

$$t_4 = s_y^2\exp\left(\frac{s_z^2 - S_z^2}{s_z^2 - S_z^2}\right) \tag{1.4}$$

The Bias and MSE expression's of the estimator's t_i (i=1, 2, 3, 4) up to the first order of approximation are, respectively, given by

$$B(t_1) = \frac{S_y^2}{n}\left[\delta_2 - \delta_3\right] \tag{1.5}$$

$$B(t_2) = \frac{S_y^2}{n}\left[\delta_4 - \delta_5\right] \tag{1.6}$$

$$B(t_3) = \frac{S_y^2}{n}\left[\frac{\delta_2}{8} - \frac{\delta_3}{2} + \frac{3}{8}\right] \tag{1.7}$$

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$$B(t_4)=\frac{S_y^2}{n}\left[\frac{\delta_4}{8}-\frac{\delta_5}{2}-\frac{5}{8}\right] \tag{1.8}$$

$$MSE(t_1)=\frac{S_y^4}{n}\left[\delta_1+\delta_2-2\delta_3\right] \tag{1.9}$$

$$MSE(t_2)=\frac{S_y^4}{n}\left[\delta_1+\delta_4-2\delta_5\right] \tag{1.10}$$

$$MSE(t_3)=\frac{S_y^4}{n}\left[\delta_1+\frac{\delta_2}{4}-\delta_3-\frac{1}{4}\right] \tag{1.11}$$

$$MSE(t_4)=\frac{S_y^4}{n}\left[\delta_1+\frac{\delta_4}{4}+\delta_5-\frac{9}{4}\right] \tag{1.12}$$

Following Naik and Gupta (1996) and Singh et al. (2007), we propose the estimators t_5 and t_6 as

$$t_5=s_y^2\left(\frac{S_x^2}{s_x^2}\right)^{\alpha_1}\left(\frac{S_z^2}{s_z^2}\right)^{\alpha_2} \tag{1.13}$$

$$t_6=s_y^2\exp\left(\frac{S_x^2-s_x^2}{S_x^2-s_x^2}\right)^{\beta_1}\exp\left(\frac{s_z^2-S_z^2}{s_z^2+S_z^2}\right)^{\beta_2} \tag{1.14}$$

where $\alpha_1,\alpha_2,\beta_1$ and β_2 are real constants.

BIAS AND MSE of t_5 and t_6

To obtain the bias and MSE of t_5 and t_6 to the first degree of approximation, we define

$$s_y^2=S_y^2(1+e_0), s_x^2=S_x^2(1+e_1), s_z^2=S_z^2(1+e_2)$$

Such that, $E(e_i)=0;\ \ i=0,1,2.$

Also,

$$E(e_0^2)=\frac{(\delta_1-1)}{n},\ E(e_1^2)=\frac{(\delta_2-1)}{n},\ E(e_2^2)=\frac{(\delta_4-1)}{n}$$

$$E(e_0e_1)=\frac{(\delta_3-1)}{n},\ E(e_0e_2)=\frac{(\delta_5-1)}{n},\ \ E(e_1e_2)=\frac{(\delta_6-1)}{n},$$

Expressing equation (1.13) in terms of e's, we have

$$t_5=S_y^2\left[(1+e_0)(1+e_1)^{-\alpha_1}(1+e_2)^{-\alpha_2}\right]\ =S_y^2\left[1+e_0-\alpha_1e_1-\alpha_2e_2+\frac{\alpha_1(\alpha_1+1)}{2}e_1^2+\frac{\alpha_2(\alpha_2+1)}{2}e_2^2+\alpha_1\alpha_2e_1e_2-\alpha_1e_0e_1-\alpha_2e_0e_2\right] \tag{2.1}$$

Subtracting $\overline{Y}$ from both the sides of equation (2.1) and then taking expectation of both sides, we get the bias of the estimator t_5, up to the first order of approximation, as

$$B(t_5)=\frac{S_y^2}{n}\left[\frac{\alpha_1(\alpha_1+1)}{2}(\delta_2-1)+\frac{\alpha_2(\alpha_2+1)}{2}(\delta_4-1)+\alpha_1\alpha_2(\delta_6-1)-\alpha_1(\delta_3-1)-\alpha_2(\delta_5-1)\right] \tag{2.2}$$

From (2.1), we have

$$(t_5-\overline{Y})=\overline{Y}\left[e_0-\alpha_1e_1-\alpha_2e_2\right] \tag{2.3}$$

Squaring both sides of (2.3) and then taking expectation, we get, MSE of the estimator t_5 , up to the first order of approximation, as

$$MSE(t_5)=\frac{S_y^4}{n}\left[(\delta_1-1)+\alpha_1^2(\delta_2-1)+\alpha_2^2(\delta_4-1)+2\alpha_1\alpha_2(\delta_6-1)-2\alpha_1(\delta_3-1)-2\alpha_2(\delta_5-1)\right] \quad (2.4)$$

To obtain the bias and MSE of t_6, to the first order of approximation, we express equation (1.14) in term of e's, as

$$t_6=\overline{Y}\left(1+e_0+\exp\left(\frac{-e_1}{2+e_1}\right)^{\beta_1}\exp\left(\frac{e_2}{2+e_2}\right)^{\beta_2}\right)$$

$$=\overline{Y}\left[1+e_0-\frac{\beta_1 e_1}{2}+\frac{\beta_1^2 e_1^2}{4}+\frac{\beta_2 e_2}{2}-\frac{\beta_1\beta_2 e_1 e_2}{4}+\frac{\beta_2^2 e_2^2}{4}+\frac{\beta_2 e_0 e_2}{2}-\frac{\beta_1 e_0 e_1}{2}\right] \quad (2.5)$$

Subtracting $\overline{Y}$ from both sides of equation (2.5) and then taking expectation of both sides, we get the bias of the estimator t_6 up to the first order of approximation, as

$$B(t_6)=\frac{S_y^2}{n}\left[\frac{\beta_1^2}{4}(\delta_2-1)+\frac{\beta_2^2}{4}(\delta_4-1)+\frac{\beta_1}{2}(\delta_3-1)-\frac{\beta_1\beta_2}{2}(\delta_6-1)+\frac{\beta_2}{2}(\delta_5-1)\right] \quad (2.6)$$

From (2.5), we have

$$(t_6-\overline{Y})=\overline{Y}\left[e_0-\frac{\beta_1 e_1}{2}+\frac{\beta_2 e_2}{2}\right] \quad (2.7)$$

Squaring both sides of (2.7) and then taking expectation, we get the MSE of the estimator t_6 up to the first order of approximation, as

$$MSE(t_6)=\frac{S_y^4}{n}\left[(\delta_1-1)+\frac{\beta_1^2}{4}(\delta_2-1)+\frac{\beta_2^2}{4}(\delta_4-1)+\frac{\beta_1}{2}(\delta_3-1)-\frac{\beta_1\beta_2}{2}(\delta_6-1)+\frac{\beta_2}{2}(\delta_5-1)\right] \quad (2.8)$$

3. ANOTHER ESTIMATOR

Following Naik and Gupta (1996) and Singh et al. (2007), we propose another improved estimator t_p as

$$t_p=w_0 s_y^2+w_1 s_y^2\left(\frac{S_x^2}{s_x^2}\right)^{\alpha_1}\left(\frac{S_z^2}{s_z^2}\right)^{\alpha_2}+w_2 s_y^2\exp\left(\frac{S_x^2-s_x^2}{S_x^2-s_x^2}\right)^{\beta_1}\exp\left(\frac{s_z^2-S_z^2}{s_z^2+S_z^2}\right)^{\beta_2} \quad (3.1)$$

where α_1,α_2, β_1 and β_2 are real constants and $w_i\,(i=0,1,2)$ are suitably chosen constants whose values are to be determined later.

Expressing (3.1) in terms of e's, we have

$$t_p=\overline{Y}(1+e_0)\left[w_0+w_1(1+e_1)^{-\alpha_1}(1+e_2)^{-\alpha_2}+w_2\exp\left(\frac{-\beta_1 e_1}{2}\right)\exp\left(\frac{\beta_2 e_2}{2}\right)\right] \quad (3.2)$$

Expanding the right hand side of equation (3.2) and retaining terms, up to second power of e's, we have

$$t_p=\overline{Y}\left[1+e_0+w_1\left(\frac{\alpha_1(1+\alpha_1)}{2}e_1^2+\frac{\alpha_2(1+\alpha_2)}{2}e_2^2-\alpha_1 e_1-\alpha_2 e_2+\alpha_1\alpha_2 e_1 e_2-\alpha_1 e_0 e_1-\alpha_2 e_0 e_2\right)\right.$$

$$\left.+w_2\left(\frac{\beta_1^2 e_1^2}{4}-\frac{\beta_1 e_1}{2}-\frac{\beta_1 e_0 e_1}{2}-\frac{\beta_1\beta_2 e_1 e_2}{4}+\frac{\beta_2^2 e_2^2}{4}+\frac{\beta_2 e_2}{2}+\frac{\beta_2 e_0 e_2}{2}\right)\right]. \quad (3.3)$$

subtracting $\overline{Y}$ from both sides of (3.3) and then taking expectation of both sides, we get the bias of the estimator t_p, up to the first order of approximation as

$$B(t_p)=w_1\frac{S_y^2}{n}\left[\frac{\alpha_1(1+\alpha_1)}{2}(\delta_2-1)+\frac{\alpha_2(1+\alpha_2)}{2}(\delta_4-1)+\frac{\alpha_1}{2}(\delta_6-1)-\frac{\alpha_1\alpha_2}{2}(\delta_6-1)+\frac{\alpha_2}{2}(\delta_5-1)\right]$$

$$+w_2\frac{S_y^2}{n}\left[\frac{\beta_1^2}{4}(\delta_2-1)+\frac{\beta_2^2}{4}(\delta_4-1)+\frac{\beta_1}{2}(\delta_6-1)-\frac{\beta_1\beta_2}{2}(\delta_6-1)+\frac{\beta_2}{2}(\delta_5-1)\right]. \quad (3.4)$$

from (3.3), we have

$$(t_p-\overline{Y})=\overline{Y}\left[e_0+w_1(-\alpha_1e_1-\alpha_2e_2)+w_2\left(-\frac{\beta_1e_1}{2}+\frac{\beta_2e_2}{2}\right)\right] \quad (3.5)$$

Squaring both sides of (3.5) and then taking expectation, we get MSE of the estimator t_p up to the first order of approximation, as

$$MSE(t_p)=\frac{S_y^4}{n}\left[(\delta_1-1)+w_1^2P_1+w_2^2P_2-2w_1P_3-w_2P_4+w_1w_2P_5\right] \quad (3.6)$$

Where,

$$\left.\begin{aligned} w_1&=\frac{4P_2P_3-P_4P_5}{4P_1P_2-P_5^2}\\ w_2&=\frac{2P_1P_4-2P_3P_5}{4P_1P_2-P_5^2}\end{aligned}\right\} \quad (3.7)$$

and

$$\left.\begin{aligned} P_1&=\alpha_1^2(\delta_2-1)+\alpha_2^2(\delta_4-1)+2\alpha_1\alpha_2(\delta_6-1)\\ P_2&=\frac{1}{4}\left[\beta_1^2(\delta_2-1)+\beta_2^2(\delta_4-1)-2\beta_1\beta_2(\delta_6-1)\right]\\ P_3&=\alpha_1(\delta_3-1)+\alpha_2(\delta_5-1)\\ P_4&=\beta_1(\delta_3-1)-\beta_2(\delta_5-1)\\ P_5&=\alpha_1\beta_1(\delta_2-1)-\alpha_2\beta_2(\delta_4-1)+\alpha_2\beta_1(\delta_6-1)-\alpha_1\beta_2(\delta_6-1)\end{aligned}\right\} \quad (3.8)$$

Double Sampling

In certain practical situations when S_x^2 is not known a priori, the technique of two-phase sampling is used. This scheme requires collection of information on x and z the first phase sample of size n' (n' <N) and on y for the second phase sample of size n (n< n') from the first phase sample.

The estimator's t_1, t_2, t_3 and t_4 in two-phase sampling take the following form

$$t_{d1}=s_y^2\left(\frac{s_x'^2}{s_x^2}\right) \quad (4.1)$$

$$t_{d2}=s_y^2\left(\frac{s_z'^2}{s_z^2}\right) \quad (4.2)$$

$$t_{d3}=s_y^2\exp\left(\frac{s_x'^2-s_x^2}{s_x'^2-s_x^2}\right) \quad (4.3)$$

$$t_{d4} = s_y^2 \exp\left(\frac{s'^2_z - S_z^2}{s'^2_z - S_z^2}\right) \quad (4.4)$$

The bias and MSE expressions of the estimators t_{d1}, t_{d2}, t_{d3} and t_{d4} up to first order of approximation, are respectively given as

$$B(t_{d1}) = S_y^2[f_1\delta_2 + f_2\delta_3] \quad (4.5)$$

$$B(t_{d2}) = \frac{S_y^2}{n'}[\delta_4 - \delta_5] \quad (4.6)$$

$$B(t_{d3}) = S_y^2\left[f_1\frac{\delta_2}{8} + f_2\frac{\delta_3}{2}\right] \quad (4.7)$$

$$B(t_{d4}) = S_y^2\left[f_1\frac{\delta_4}{8} + f_2\frac{\delta_5}{2}\right] \quad (4.8)$$

$$MSE(t_{d1}) = S_y^4\left[\frac{\delta_1 - 1}{n} + f_1(\delta_2 - 1) + 2f_2(\delta_3 - 1)\right] \quad (4.9)$$

$$MSE(t_{d2}) = S_y^4\left[\frac{\delta_1 - 1}{n} + \frac{\delta_4 - 2\delta_5 + 1}{n'}\right] \quad (4.10)$$

$$MSE(t_{d3}) = S_y^4\left[\frac{\delta_1 - 1}{n} + \frac{f_1}{4}(\delta_2 - 1) + f_2(\delta_3 - 1)\right] \quad (4.11)$$

$$MSE(t_{d4}) = S_y^4\left[\frac{\delta_1 - 1}{n} + \frac{f_1}{4}(\delta_4 - 1) - f_2(\delta_5 - 1)\right] \quad (4.12)$$

where,

$$S_z^2 = \frac{1}{n'-1}\sum_{i=1}^{n}\left(z_i - \bar{z}'\right)^2, \quad S'^2_x = \frac{1}{n'-1}\sum_{i=1}^{n'}\left(x_i - \bar{x}'\right)^2,$$

$$f_1 = \frac{1}{n} - \frac{1}{n'}, \quad f_2 = \frac{1}{n'} - \frac{1}{n}.$$

The estimator's t_5 and t_6, in two phase sampling, takes the following form

$$t_{d5} = s_y^2\left(\frac{s'^2_x}{s_x^2}\right)^{m_1}\left(\frac{S_z^2}{s'^2_z}\right)^{m_2} \quad (4.13)$$

$$t_{d6} = s_y^2 \exp\left(\frac{s'^2_x - s_x^2}{s'^2_x - s_x^2}\right)^{n_1} \exp\left(\frac{s'^2_z - S_z^2}{s'^2_z - S_z^2}\right)^{n_2} \quad (4.14)$$

where m_1, m_2, n_1 and n_2 are real constants.

BIAS AND MSE OF t_{d5} and t_{d6}

To obtain the bias and MSE of t_5 and t_6 to the first degree of approximation, we define

$$s_y^2 = S_y^2(1 + e_0), s'^2_x = S_x^2(1 + e'_1), s'^2_z = S_z^2(1 + e'_2)$$

Such that, $E(e'_i) = 0; \quad i = 0, 1, 2.$

Also,

$$E(e_0^2)=\frac{(\delta_1-1)}{n},\ E(e'^2_1)=\frac{(\delta_2-1)}{n'},\ E(e'^2_2)=\frac{(\delta_4-1)}{n'}$$

$$E(e_0e'_1)=\frac{(\delta_3-1)}{n'},\ E(e_0e'_2)=\frac{(\delta_5-1)}{n'},\quad E(e'_1e'_2)=\frac{(\delta_6-1)}{n'},$$

Expressing equation (4.13) in terms of e's, we have

$$t_{d5}=\overline{Y}(1+e_o)(1+e'_1)^{m_1}(1+e_1)^{-m_1}(1+e'_2)^{-m_2}$$

Expanding the right hand side of above equation and retaining terms up to second power of e's, we have

$$t_{d5}=S_y^2\Big[1+e_o-m_1e_1+\frac{m_1(m_1+1)}{2}e_1^2-m_1^2e_1e'_2+\frac{m_1(m_1-1)}{2}e'^2_1 \tag{5.1}$$
$$+m_1e'_1-m_2e'_2+\frac{m_2(m_2+1)}{2}e'^2_2-m_1e_1e_0+m_1e_0e'_1-m_2e_0e'_2\Big]$$

Subtracting $\overline{Y}$ from both sides of (5.1) and then taking expectation, we get the bias of the estimator t_{d5}, up to the first order of approximation, as

$$B(t_{d5})=S_y^2\left\{\frac{m_1(m_1+1)}{2}\frac{(\delta_2-1)}{n}-m_1^2\frac{(\delta_6-1)}{n'}+\frac{m_1(m_1-1)}{2}\frac{(\delta_2-1)}{n'}\right.$$
$$\left.+\frac{m_2(m_2+1)}{2}\frac{(\delta_4-1)}{n'}-m_1\frac{(\delta_3-1)}{n}+m_1\frac{(\delta_3-1)}{n'}-m_2\frac{(\delta_5-1)}{n'}\right\} \tag{5.2}$$

From (5.1), we have

$$(t_{d5}-S_y^2)=S_y^2[e_0-m_1e_1+m_1e'_1-m_2e'_2] \tag{5.3}$$

Squaring both sides of (5.3) and then taking expectations, we get MSE of the estimator t_{d5}, up to the first order of approximation, as

$$MSE(t_{d5})=S_y^4\left[\frac{(\delta_1-1)}{n}+m_1^2f_1(\delta_2-1)+m_2^2\frac{(\delta_2-1)}{n'}+2m_1f_2(\delta_3-1)-2m_2(\delta_5-1)\right] \tag{5.4}$$

Now to obtain the bias and MSE of t_{d6} to the first order of approximation, we express equation (4.14) in terms of e's

$$t_{d6}=\overline{Y}(1+e_0)\exp\left(\frac{n_1e'_1}{2}\right)\exp\left(\frac{-n_1e_1}{2}\right)\exp\left(\frac{n_2e'_2}{2}\right) \tag{5.5}$$

Expanding the right hand side of equation (5.5) and retaining terms up to second power of e's, we have

$$t_{d6}=\overline{Y}\left(1+e_0+\frac{n_1e'_1}{2}-\frac{n_1e_1}{2}+\frac{n_1^2e_1^2}{4}+\frac{n_2e'_2}{2}+\frac{n_2e'^2_2}{4}+\frac{n_1e_0e'_1}{2}-\frac{n_1e_0e_1}{2}+\frac{n_2e_0e'_2}{2}\right) \tag{5.6}$$

subtracting $\overline{Y}$ from both sides of (5.6) and then taking expectations, we get the bias of the estimator t_{d6} up to the first order of approximation, as

$$B(t_{d6})=S_y^2\left\{\frac{n_1^2}{4}\frac{(\delta_2-1)}{n}+\frac{n_2^2}{4}\frac{(\delta_4-1)}{n'}+\frac{n_1}{2}f_2(\delta_3-1)+\frac{n_2}{2n'}(\delta_5-1)\right\} \tag{5.7}$$

From (5.6), we have

$$(t_{d6}-S_y^2)=S_y^2\left(e_0+\frac{n_1e'_1}{2}-\frac{n_1e_1}{2}+\frac{n_2e'_2}{2}\right) \tag{5.8}$$

squaring both sides of (5.8) and then taking expectations, we get the MSE of t_{d6} up to the first order of approximation, as

$$MSE(t_{d6}) = S_y^4\left[\frac{(\delta_1-1)}{n} + \frac{n_1^2 f_1(\delta_2-1)}{4} + n_2^2\frac{(\delta_4-1)}{n'} + 2n_1 f_2(\delta_3-1) + \frac{2n_2(\delta_5-1)}{n'}\right] \quad (5.9)$$

IMPROVED ESTIMATOR t_p IN TWO-PHASE SAMPLING

The estimator t_p in double sampling is written as

$$t_{pd} = h_0 s_y^2 + s_y^2\left(\frac{s'^2_x}{s_x^2}\right)^{m_1}\left(\frac{S_z^2}{s'^2_z}\right)^{m_2} + s_y^2 \exp\left(\frac{s'^2_x - s_x^2}{s'^2_x - s_x^2}\right)^{n_1}\exp\left(\frac{s'^2_z - S_z^2}{s'^2_z - S_z^2}\right)^{n_2} \quad (6.1)$$

Where, $m_1, m_2, n_1,$ and n_2 are real constants and $h_i\,(i = 0,1,2)$ are suitably chosen constants whose values are to be determined later.

Expressing (6.1) in terms of e's, we have

$$t_{pd} = h_0\bar{y} + h_1\bar{y}(1+e'_1)^{m_1}(1+e_1)^{-m_2}(1+e'_2)^{-m_2}$$
$$+ h_2\bar{y}\exp\left(\frac{n_1 e'_1}{2}\right)\exp\left(\frac{-n_1 e_1}{2}\right)\exp\left(\frac{n_2 e'_2}{2}\right) \quad (6.2)$$

Expanding the right hand side of (6.2) and retaining terms up to second power of e's as

$$t_{pd} = \bar{Y}\left[1 + e_0 + h_1\left(\frac{m_1(m_1+1)}{2}e_1^2 - m_1 e_1 - m_1 e_0 e_1 - m_2^2 e_1 e'_1 + \frac{m_1(m_1-1)2}{2}e'^2_1 + m_1 e'_1\right.\right.$$
$$\left. + m_1 e_0 e'_1 - m_2 e'_2 - m_2 e_0 e'_2 + \frac{m_2(m_2+1)}{2}e'^2_2\right)$$
$$\left. + h_2\left(\frac{n_1 e'_1}{2} - \frac{n_1 e_1}{2} + \frac{n_1^2 e_1^2}{4} + \frac{n_2 e'_2}{2} + \frac{n_2 e'^2_2}{4} + \frac{n_1 e_0 e'_1}{2} - \frac{n_1 e_0 e_1}{2} + \frac{n_2 e_0 e'_2}{2}\right)\right] \quad (6.3)$$

Subtracting $\bar{Y}$ from both the sides of (6.3) then taking expectations on both the sides, we get the bias of the estimator t_d up to the first order of approximation as

$$B(t_{pd}) = \bar{Y}\left[h_1 f_3 C_{p_1}^2\left(\frac{m_1^2}{2} + \frac{m_1}{2} + -m_1 K_{pb_1}\right) + h_1 f_2 C_{p_2}^2\left(\frac{m_2^2}{2} + \frac{m_2}{2} - m_2 k_{pb_2}\right)\right.$$
$$\left. + h_3 f_3 C_{p_1}^2\left(\frac{n_1^2}{8} + \frac{n_1}{8} - \frac{n_1}{2}K_{pb_1}\right) + h_3 f_2 C_{p_2}^2\left(\frac{n_2^2}{8} + \frac{n_2}{8} - \frac{n_2}{2}k_{pb_2}\right)\right] \quad (6.4)$$

From (6.3), we have

$$t_{pd} - \bar{Y} = \bar{Y}\left[e_0 + h_1\left(-m_1 e_1 + m_1 e'_1 - m_2 e'_2\right) + h_2\left(\frac{n_1 e'_1}{2} - \frac{n_1 e_1}{2} + \frac{n_2 e'_2}{2}\right)\right] \quad (6.5)$$

Squaring both sides of (6.5) and then taking expectations, we get MSE of the estimator t_p up to first order of approximation as

$$MSE(t_{pd}) = \frac{S_y^4}{n}\left[(\delta_1 - 1) + h_1^2 B_1 + h_2^2 B_2 - 2h_1 B_3 - h_2 B_4 + h_1 h_2 B_5\right] \quad (6.6)$$

where,

$$\left.\begin{aligned} h_1 &= \frac{4B_2B_3 - B_4B_5}{4B_1B_2 - B_5^2} \\ h_2 &= \frac{2B_1B_4 - 2B_3B_5}{4B_1B_2 - B_5^2} \end{aligned}\right\} \tag{6.7}$$

and

$$\left.\begin{aligned} B_1 &= m_2^2\frac{(\delta_4-1)}{n'} + m_1^2(\delta_2-1)\left(\frac{1}{n}-\frac{1}{n'}\right) \\ B_2 &= \frac{1}{4}\left[n_2^2\frac{(\delta_4-1)}{n'} + n_1^2(\delta_2-1)\left(\frac{1}{n}-\frac{1}{n'}\right)\right] \\ B_3 &= m_1\left(\frac{1}{n}-\frac{1}{n'}\right)(\delta_2-1) + m_2\frac{(\delta_4-1)}{n'} \\ B_4 &= n_1\left(\frac{1}{n}-\frac{1}{n'}\right)(\delta_2-1) - n_2\frac{(\delta_4-1)}{n'} \\ B_5 &= n_1m_1\left(\frac{1}{n}-\frac{1}{n'}\right)(\delta_2-1) - n_2m_2\frac{(\delta_4-1)}{n'} \end{aligned}\right\} \tag{6.8}$$

Empirical study

To illustrate the performance of various estimators of S_y^2, we consider the data given in Murthy (1967, p.-226). The variates are

Y: output, x: number of workers, z: fixed capital

N=80, n' =25, n=10

$\delta_1 = 2.2667$, δ_2 =3.65 , δ_3 =2.3377 , δ_4 =2.8664 , δ_5 =2.2208 , δ_6 =3.14

The percent relative efficiency (PRE) of various estimators of S_y^2 with respect to conventional estimator s_y^2 has been computed and displayed in the given table.

Table 7.1 PRE of different estimators of S_y^2 with respect to s_y^2

Choice of scalars								
w_0	w_1	w_2	α_1	α_2	β_1	β_2	**Estimator**	**PRE'S**
1	0	0					s_y^2	100
0	1	0	1	0			t_1	102.04
			0	1			t_2	183.18
			-1	1			t_5	25.61
0	0	1			1	0	t_3	214.15
					0	1	t_4	42.87
					1	-1	t_6	104.78
w_0	w_1	w_2	1	1	1	1	t_p	**282.39**

In table 7.2 PRE of various estimators of in two-phase sampling are displayed.

Table 7.2: PRE of different estimators of S_y^2 with respect to s_y^2

Choice of scalars								
h_0	h_1	h_2	m_1	m_2	n_1	n_2	**Estimator**	**PRE'S**
1	0	0					s_y^2	100
0	1	0	1	0			t_{d1}	1260.71
			0	1			t_{d2}	1033.90
			1	1			t_{d5}	1287.04
0	0	1			1	0	t_{d3}	1270.76
					0	1	t_{d4}	513.85
					1	1	t_{d6}	1380.62
h_0	h_1	h_2	1	1	1	1	t_{pd}	**1472.25**

Conclusion:

In this paper, we have suggested a class of estimators in single and double sampling for estimating population variance using two auxiliary variables. From Table 7.1 and Table 7.2, we infer that the proposed estimator t_p and its double sampling version t_{pd}, performs better than other estimators considered in this paper.

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