



MHD free convection heat and mass transfer flow past a linearly accelerated vertical Porous plate with variable temperature and mass diffusion

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ABSTRACT

The objective of the present study is to investigate thermal diffusion and radiation effects on unsteady MHD flow past a linearly accelerated vertical plate with variable temperature and also with variable mass diffusion in presence of heat source or sink under the influence of applied transverse magnetic field. The fluid considered here is a gray, absorbing/ emitting radiation but a non-scattering medium. At time $t > 0$, the plate is linearly accelerated with a velocity $u = u_0 t$ in its own plane. And at the same time, plate temperature and concentration levels near the plate raised linearly with time t . The dimensionless governing equations involved in the present analysis are solved using the closed analytical method. The velocity, temperature, concentration, Skin-friction, Nusselt number and Sherwood number are discussed through graphs in terms of different physical parameters like magnetic field parameter (M), permeability parameter (K), radiation parameter (R), Schmidt parameter (Sc), Soret number (So), Heat source parameter (H), Prandtl number (Pr), thermal Grashof number (Gr), mass Grashof number (Gm) and time (t).

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Introduction

The study of magneto hydro-dynamics with mass and heat transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere, etc. In engineering we find its applications like in MHD pumps, MHD bearings, etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable on the solar surface. In free convection flow the study of effects of magnetic field play a major role in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydro magnetic problems with mass transfer have enormous applications. Radiative flows are encountered in many industrial and environment processes, e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

Chambre and Young [1] have presented a first order chemical reaction in the neighborhood of a horizontal plate. Dekha et al. [2] investigated the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past a vertical plate with a constant heat and mass transfer. Muthucumaraswamy [3] presented heat and mass transfer effects on a continuously moving isothermal vertical surface with uniform suction by taking into account the homogeneous chemical reaction of first order. Muthucumaraswamy and Meenakshisundaram [4] investigated theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature and mass diffusion.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the

boundary layer flow control and on the performance of many systems using electrically conducting fluids. Raptis et al. [5] analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Gribben [6] presented the boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of pressure gradient. He obtained solutions for large and small magnetic Prandtl number using the method of matched asymptotic expansion. Helmy [7] presented an unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by infinite vertical plane surface of constant temperature. Gregantopoulos et al. [8] studied two-dimensional unsteady free convection and mass transfer flow of an incompressible viscous dissipative and electrically conducting fluid past an infinite vertical porous plate. For some industrial applications such as glass production and furnace design, and in space technology applications such as cosmic flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar [9] analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Kim and Fedorov [10] analyzed transient mixed radiative convective flow of a micropolar fluid past a moving semi-infinite vertical porous plate. Muthuraj and Srinivas [11] studied the fully developed MHD flow of a micropolar and viscous fluid in a vertical porous space using HAM.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electric chips and

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semiconductor wafers. Seddeek [12] studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Patil and Kulkarni [13] studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Double-Diffusive Convection-Radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects was studied by Mohamed [14]. Radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium was studied by Ramachandraprasad et al. [15]. Satyanarayana et al [16] studied Hall current effect on magneto hydrodynamics free-convection flow past a semi-infinite vertical porous plate with mass transfer. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [17]. Dulal Pal et al [18] studied Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Recently, Ramana Reddy et al [19] have studied the mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption.

Formation of the Problem

We consider Thermal-diffusion and radiation effects on unsteady MHD flow past of a viscous incompressible, electrically conducting, radiating fluid past an impulsively started linearly accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of Heat source/sink under the influence of applied transverse magnetic field. The plate is taken along x' – axis in vertically upward direction and y' – axis is taken normal to the plate. Initially it is assumed that the plate and fluid are at the same temperature T'_∞ and concentration level C'_∞ in stationary condition for all the points. At time $t' > 0$, the plate is linearly accelerated with a velocity $u = u_0 t'$ in the vertical upward direction against to the gravitational field and at the same time the plate temperature is raised linearly with time t and also the mass is diffused from the plate to the fluid is linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The viscous dissipation and induced magnetic field are assumed to be negligible. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. Then under by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\nu u'}{K'} - \frac{\sigma}{\rho} B_0^2 u'$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q'_r}{\partial y'} - Q'(T'_\infty - T') \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - D_1 \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} t' \leq 0, \quad u' = 0, \quad T' = 0, \quad C' = 0 \quad \text{for all } y' \\ t' > 0 \begin{cases} u' = u_0 t', \quad T' = T'_\infty + \varepsilon(T'_w + T'_z) A t', \quad C' = C'_\infty + \varepsilon(C'_w - C'_z) A t' & \text{at } y' = 0 \\ u' = 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty & \text{as } y' \rightarrow \infty \end{cases} \end{aligned} \quad (4)$$

where $A = \frac{u_0^2}{\nu}$, u', v' are the velocity components in x', y' directions respectively, t' – the time, ρ – the fluid density, g – the acceleration due to gravity, β and β^* – the thermal and concentration expansion coefficients respectively, K' – the permeability of the porous medium, T' – the temperature of the fluid in the boundary layer, ν – the kinematic viscosity, σ – the electrical conductivity of the fluid, T'_∞ – the temperature of the fluid far away from the plate (in the free stream), C' – the species concentration in the boundary layer, C'_∞ – the species concentration in the fluid far away from the plate C in the free stream, B_0 – the magnetic induction, q'_r – the radiative heat flux, C_p – specific heat at constant, D – the coefficient of chemical molecular diffusivity, D_1 – the coefficient of thermal diffusivity. The term $Q'(T'_\infty - T')$ is assumed to be amount of heat source or sink per unit volume Q' is a constant.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q'_r}{\partial y'} = -4a * \sigma (T'^4_\infty - T'^4) \quad (5)$$

It is assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus we get

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (6)$$

From equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + 16a * \sigma T'^3_\infty (T'_\infty - T') \quad (7)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} u = \frac{u'}{u_0}, \quad y = \frac{y'}{\nu}, \quad t = \frac{t' u_0^2}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad K = \frac{K' u_0^2}{\nu^2}, \\ Pr = \frac{\rho C_p}{\kappa}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \quad Gr = \frac{\nu \beta g (T'_w - T'_\infty)}{u_0^3}, \quad Gm = \frac{\nu \beta^* g (C'_w - C'_\infty)}{u_0^3}, \quad (8) \\ H = \frac{\nu Q'}{\kappa u_0^2}, \quad So = \frac{D_1 (T'_w - T'_\infty)}{\nu (C'_w - C'_\infty)}, \quad R = \frac{16a * \nu^2 \sigma T'^3_\infty}{\kappa u_0^2}, \quad Q = \frac{\nu Q_0}{\rho C_p u_0^2} \end{aligned}$$

We get the following governing equations which are dimensionless

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K} \right) u - G_r \theta - G_m \phi \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{(R + H)}{Pr} \theta \quad (10)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + S_o \frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

Also, the corresponding boundary condition (6) reduces to:

$$t \leq 0: u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } y \quad (12)$$

$$t > 0: \begin{cases} u = t, & \theta = t, & C = t & \text{at } y = 0 \\ u \rightarrow 0, & \theta \rightarrow 0, & C \rightarrow 0, & \text{as } y \rightarrow \infty \end{cases}$$

Solution of the problem:

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we assume the trial solution for the velocity, temperature and concentration as:

$$u(y, t) = u_0(y) e^{i\omega t} \quad (13)$$

$$\theta(y, t) = \theta_0(y) e^{i\omega t} \quad (14)$$

$$\phi(y, t) = \phi_0(y) e^{i\omega t} \quad (15)$$

Substituting (13), (14) and (15) in equations (9)-(11), we obtain

$$u_0'' - \left(M + i\omega + \frac{1}{K} \right) u_0 = -[Gr\theta_0 + Gm\phi_0] \quad (16)$$

$$\theta_0'' - (R + H + i\omega Pr) \theta_0 = 0 \quad (17)$$

$$\phi_0'' - i\omega Sc \phi_0 = -Sc Sol \theta_0'' \quad (18)$$

In view of the above, the corresponding boundary conditions can be re written as

$$u_0 = t e^{-i\omega t}, \quad \theta_0 = t e^{-i\omega t}, \quad \phi_0 = t e^{-i\omega t} \quad \text{at } y = 0 \quad (19)$$

$$u_0 \rightarrow 0, \quad \theta_0 \rightarrow 0, \quad \phi_0 \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

The solutions of equations (16) – (18) satisfying the boundary conditions (19), the velocity, temperature and concentration are given by

$$u(y, t) = A_{10} \exp(-A_6 y) - A_{11} \exp(-A_1 y) - A_8 \exp(-A_2 y) \quad (20)$$

$$\theta(y, t) = t \exp(-A_1 y) \quad (21)$$

$$\phi(y, t) = A_4 \exp(-A_1 y) + A_5 \exp(-A_2 y) \quad (22)$$

Here the constants are not given due to shake of brevity.

Results and Discussion

In order to get the physical insight into the problem, we have plotted velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number for different values of the physical parameters like Radiation parameter (R), Magnetic parameter (M), permeability parameter (K), Heat source parameter (H), Soret number (So), Schmidt number (Sc), Thermal Grashof number (Gr), Mass Grashof number (Gm), time (t) and Prandtl number (Pr).

Fig.1. illustrates the effect of velocity profiles for different values of Soret number. It is seen that the velocity increases with an increases Soret number in the case cooling of the plate. The effect of the magnetic parameter M is shown in Fig.2. It is observed that the tangential velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the tangential velocity as the magnetic parameter M increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This

resistive force slows down the fluid velocity component as shown in Fig.2. For different values of thermal radiation (R) and heat source parameter (H) on the velocity profiles are shown in Figs.3 and 4 respectively. It is noticed that the thermal radiation and heat source parameter are increased with increases in the velocity profiles within the boundary layer. Fig.5 shows the effect of the permeability parameter K on the velocity distribution. It is found that the velocity increases with an increase in K .

The effect of the Schmidt number Sc on the velocity profiles is shown in Fig.6. As the Schmidt number increases, the velocity profiles increases and trend to get reversed. For various values of the thermal Grashof number Gr and Solutal Grashof number Gm , the velocity profiles are plotted in Figs. 7 and 8. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The Solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the Solutal Grashof number. The velocity profiles for different values of time t are shown in Fig.9. It is seen that as time t increases the velocity profiles is also increases gradually.

Fig.10 shows the plot of temperature of the flow field against for different values of Prandtl number (Pr) taking radiation parameter (R) as constant. It is observed that the temperature of the flow field decreases in magnitude as Pr increases. It is also observed that the temperature for air (Pr=0.71) is greater than that of water (Pr=7.0). This is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting decreases in thermal boundary layer. The temperature of the flow field is mainly affected by the flow parameters, namely, Radiation parameter (R) and the heat source parameter (H) keeping Prandtl number (Pr) as constant. The effects of these parameters on temperature of the flow field are shown in Fig.11 and 12 respectively. It is observed that as radiation parameter R or heat source parameter H increases the temperature of the flow field decreases at all the points in flow region.

The effect of the Schmidt number Sc on the concentration profiles are shown in Fig.13. As the Schmidt number increases, the concentration increases and trend get reversed. The concentration distributions of the flow field are displayed through Figs. 14 & 15. It is affected by three flow parameters, namely heat source parameter (H) and Soret number (So) respectively. From these figures it is clear that the concentration increases with increases in Soret number and heat source parameter.

The effect of magnetic parameter, while maintaining the constant other parameters has been examined on skin friction in Fig.16. It is seen that the skin friction increases with an increases in magnetic parameter. Nusselt number is presented in Fig.17 against time t . From this figure the Nusselt number is observed to increase with increase in R for both water (Pr=7.0) and air (Pr=0.71). It is also observed that Nusselt number for

water is higher than that of air ($Pr=0.71$). The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Pr , hence the rate of heat transfer is reduced. And finally, from Fig.18 it is observed that the Sherwood number decreases with an increasing Soret number (So).

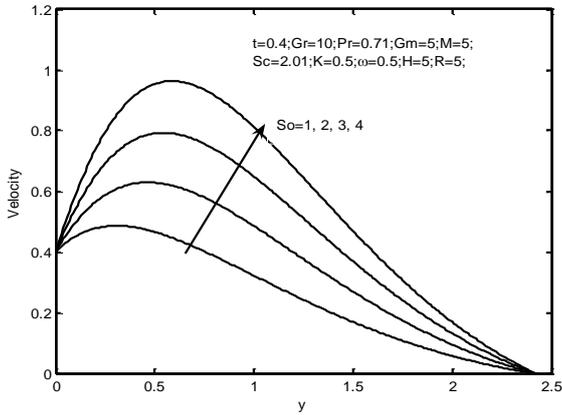


Fig.1. Effects of velocity profiles for different values of Soret number.

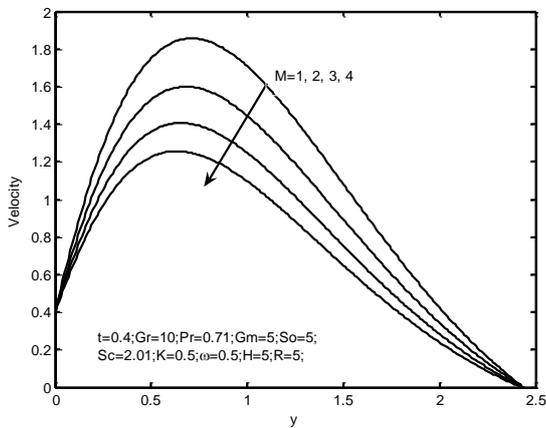


Fig.2. Effects of velocity profiles for different values of magnetic parameter.

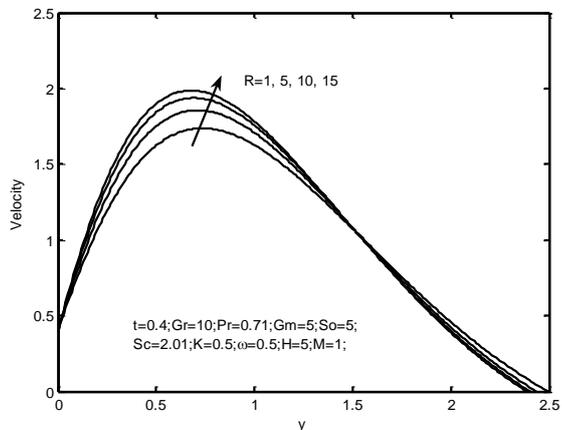


Fig.3. Effects of velocity profiles for different values of radiation parameter.

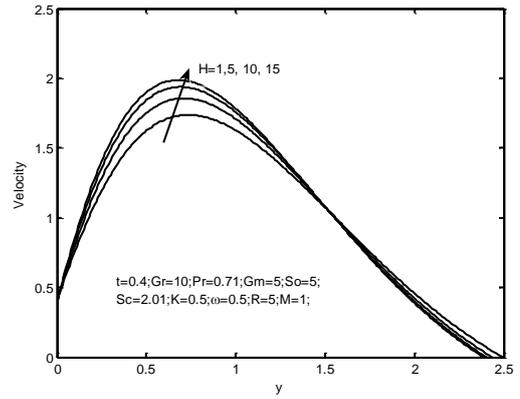


Fig.4. Effects of velocity profiles for different values of heat source parameter.

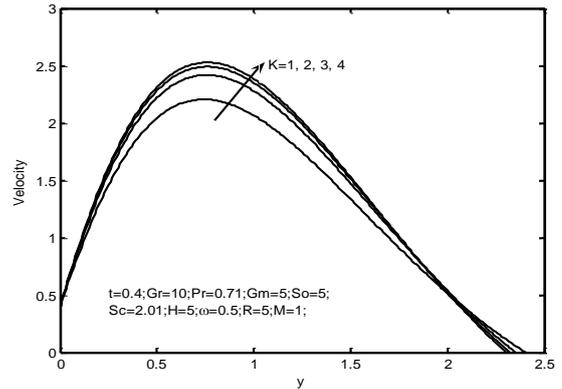


Fig.5. Effects of velocity profiles for different values of permeability parameter.

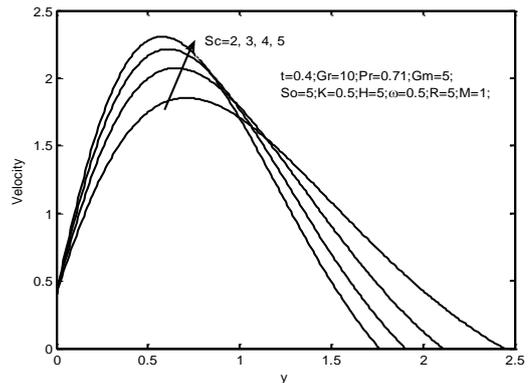


Fig.6. Effects of velocity profiles for different values of Schmidt number.

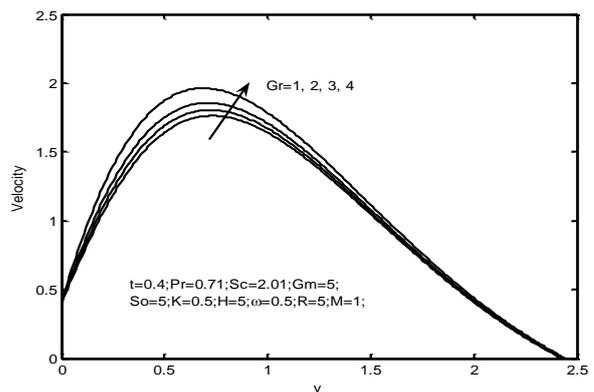


Fig.7. Effects of velocity profiles for different values of Grashof number.

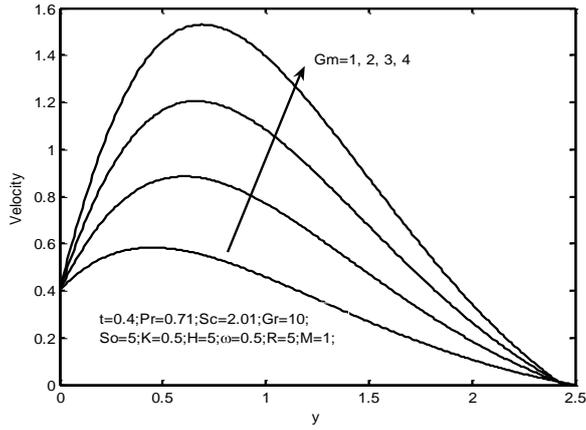


Fig.8. Effects of velocity profiles for different values of Solutal Grashof number.

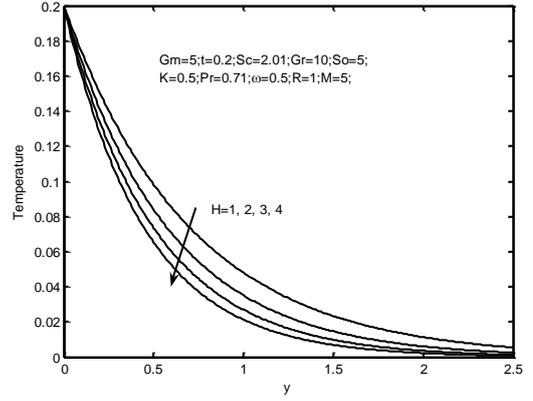


Fig.12. Effects of Temperature profiles for different values of heat source parameter.

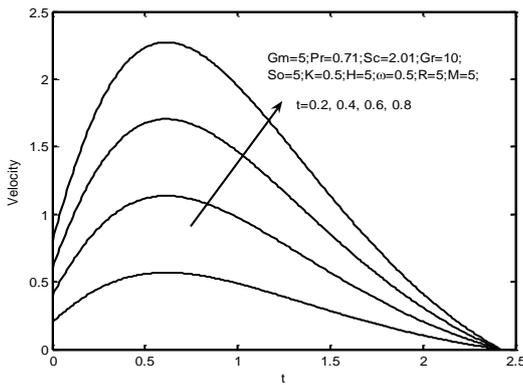


Fig.9. Effects of velocity profiles for different values of time.

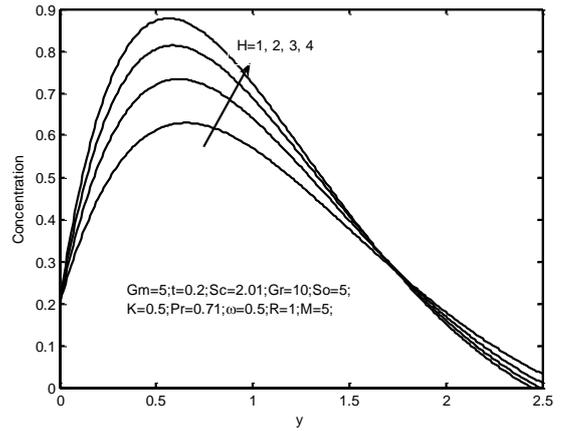


Fig.13. Effects of concentration profiles for different values of heat source parameter.

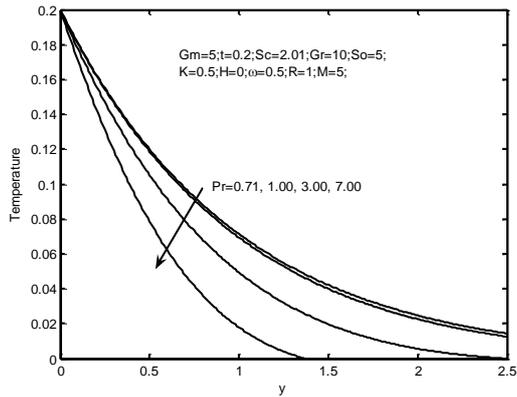


Fig.10. Effects of temperature profiles for different values of Prandtl number.

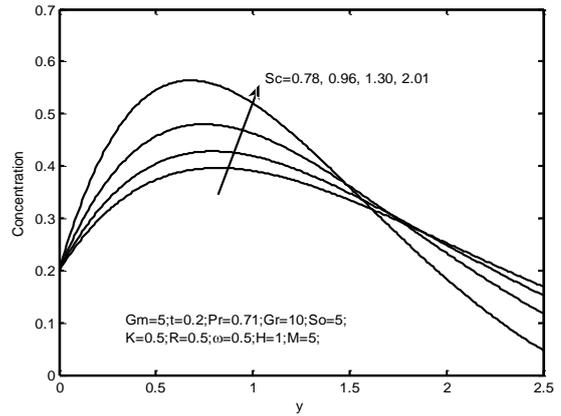


Fig.14. Effects of concentration profiles for different values of Schmidt number.

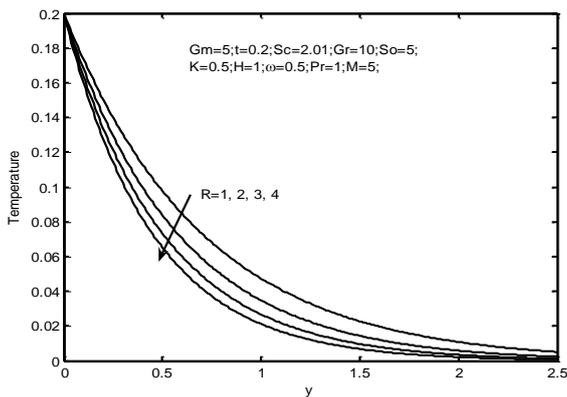


Fig.11. Effects of Temperature profiles for different values of radiation parameter.

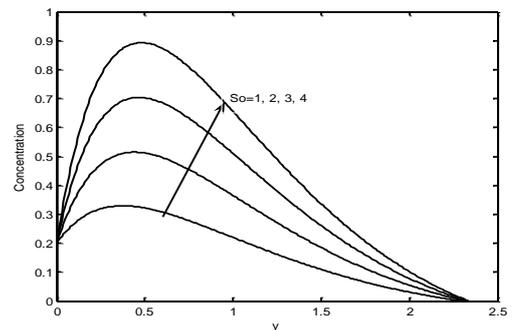


Fig.15. Effects of concentration profiles for different values of Soret number.

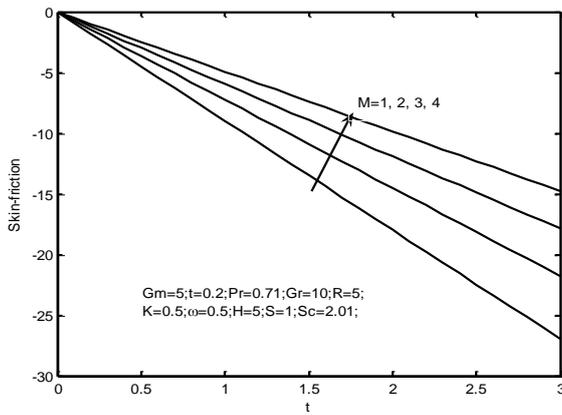


Fig.16. Effects of skin-friction for different values of magnetic parameter.

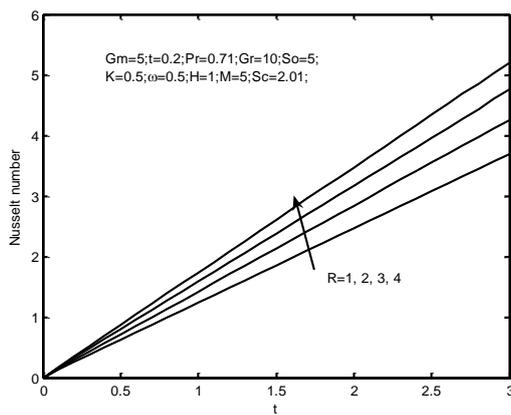


Fig.17. Effects of Nusselt number for different values of radiation parameter

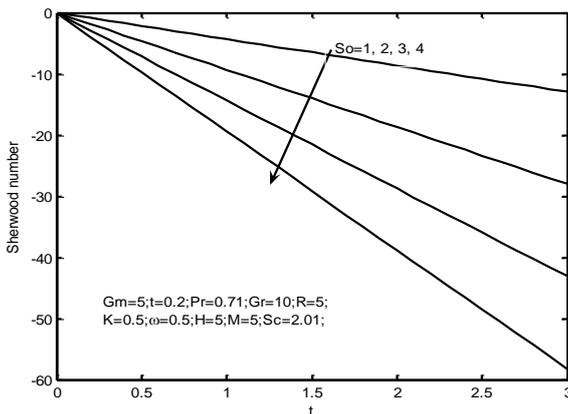


Fig.18. Effects of Sherwood number for different values of Soret number.

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