Exact traveling wave solutions of the (2+1)-dimensional modified Zakharov-Kuznetsov equation via new extended (G'/G) – expansion method

Md. Nur Alam[1,*], M. Ali Akbar[2], Kaniz Fatema[1] and Md. Golam Hafez[3]

[1]Department of Mathematics, Pabna University of Science and Technology, Bangladesh.

[2]Department of Applied Mathematics, University of Rajshahi, Bangladesh.

[3]Department of Mathematics, Chittagong University of Engineering and Technology, Bangladesh.

ABSTRACT

In this paper, the new extended (G'/G)-expansion method is used for constructing the new exact traveling wave solutions of nonlinear evolution equations arising in mathematical physics namely, the (2+1)-dimensional modified Zakharov-Kuznetsov equation. As a result, the traveling wave solutions are expressed in terms of hyperbolic, trigonometric and rational functions. Moreover, these methods could be more effectively used to deal with higher dimensional and higher order nonlinear evolution equations which frequently arise in many scientific real time application fields. It is shown that the method provides a powerful mathematical tool for solving nonlinear wave equations in applied mathematics, mathematical physics and engineering problems.

Introduction

The investigation of the travelling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In the past several decades, there have been significant improvements in the study of exact solutions. A variety of powerful methods, such as the Miura transformation method[1], the Jacobi elliptic function expansion method [2], the (G'/G)-expansion method [3-12], the modified simple equation method [13, 14], the method of bifurcation of planar dynamical systems [15, 16], the wave of translation method [17], the ansatz method [18, 19], the Darboux transformation method [20], the Cole-Hopf transformation method[21], the Exp-function method [22-24], the inverse scattering transform method[25], the new extended (G'/G)-expansion method [26] and so on.

The objective of this article is to be relevant the new extended (G'/G)-expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the (2+1)-dimensional modified Zakharov-Kuznetsov equation.

The article is prepared as follows: In Section 2, the new extended (G'/G)-expansion method is discussed. In Section 3, we apply this method to the nonlinear evolution equations pointed out above; in section 4, graphical representation of solutions; in section 5 conclusions are given.

Material and Method

Suppose we have a nonlinear partial differential equation for $u(x,t)$ in the form

$$P(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \cdots) = 0,$$
(1)

Tele:
E-mail addresses: nuralam.pstu23@gmail.com

where P is a polynomial in its arguments. The main steps of the method are as follows:

Step 1: By taking $u(x,t) = u(\xi)$, $\xi = x - Vt$, where V is the speed of the traveling wave . we look for traveling wave solutions of Eq. (1), and transform it to the ordinary differential equation

$$Q(u, u', u'', u''', \cdots) = 0,$$ (2)

Where prime denotes the derivative with respect to ξ.

Step 2: Introduce the solution $u(\xi)$ of Eq. (2) in the finite series form:

$$u(\xi) = \sum_{j=-n}^{n} \left\{ \frac{a_j (G'/G)^j}{[1 + \lambda(G'/G)]^j} + b_j (G'/G)^{j-1} \sqrt{\sigma[1 + \frac{1}{\mu}(G'/G)^2]} \right\},$$ (3)

Where $G = G(\xi)$ satisfying the equation

$$G'' + \mu G = 0$$ (4)

In which $a_j, b_j (j = -n, \ldots, n)$ and λ are constants to be determined later, and $\sigma = \pm 1, \mu \neq 0$. Fix n by balancing the highest-order derivative term with the nonlinear term in the reduced equation (2).

Step 3. Inserting Eq.(4) into Eq.(3) and making use of Eq.(5) and then extracting all terms of   powers of $(G'/G)^j$ and $(G'/G)^j \sqrt{\sigma[1 + (G'/G)^2 / \mu]}$ together set each coefficient of them to zero yield a over-determined system of algebraic equations and then solving this system of algebraic equations for $a_i, b_i (i = -n, \cdots, n)$ and λ, V, we obtain several sets of solutions.

Step 4. For the general solutions of Eq.(5), we have

$$\begin{cases} \mu < 0, \quad \dfrac{G'}{G} = \sqrt{-\mu} \left(\dfrac{A \sinh(\sqrt{-\mu}\xi) + B \cosh(\sqrt{-\mu}\xi)}{A \cosh(\sqrt{-\mu}\xi) + B \sinh(\sqrt{-\mu}\xi)} \right) = P_1(\xi) \\[4mm] \mu > 0, \quad \dfrac{G'}{G} = \sqrt{\mu} \left(\dfrac{A \cos(\sqrt{\mu}\xi) - B \sin(\sqrt{\mu}\xi)}{A \sin(\sqrt{\mu}\xi) + B \cos(\sqrt{\mu}\xi)} \right) = P_2(\xi) \end{cases}$$ (5)

where A, B are arbitrary constants. At last, inserting the values of $a_i, b_i (i = -n, \cdots, n)$, λ, V and (6) into Eq. (4) and obtain required traveling wave solutions of Eq.(1).

Application of the method

In this section, we employ the method to obtain some new and more general exact traveling wave solutions of the celebrated (2+1)-dimensional modified Zakharov-Kuznetsov equation. Let us consider the (2+1)-dimensional modified Zakharov-Kuznetsov equation,

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0.$$ (6)

We utilize the traveling wave variable $u(\eta) = u(x, y, z, t)$, $\xi = x + y - Vt$, Eq. (6) is carried to an ODE

$$-Vu' + u^2 u' + 2u''' = 0$$ (7)

Eq. (7) is integrable, therefore, integrating with respect to η once yields:

$$L - Vu + \frac{1}{3}u^3 + 2u'' = 0,$$ (8)

where L is an integration constant. Balancing the highest-order derivative term and nonlinear term in Eq.(8), therefore, we can write the form of the solutions of Eq.(8)

$$u(\xi) = a_0 + \frac{a_1(G'/G)}{1+\lambda(G'/G)} + \frac{a_{-1}[1+\lambda(G'/G)]}{(G'/G)} + b_0(G'/G)^{-1}\sqrt{\sigma\left[1+\frac{1}{\mu}(G'/G)^2\right]}$$

$$+b_1\sqrt{\sigma\left[1+\frac{1}{\mu}(G'/G)^2\right]} + b_{-1}(G'/G)^{-2}\sqrt{\sigma\left[1+\frac{1}{\mu}(G'/G)^2\right]},$$ (9)

where $G = G(\xi)$ satisfies Eq.(4). Substituting (9) along with Eq.(4) into Eq.(8), collecting all terms with the same powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+(G'/G)^2/\mu\right]}$, and setting them to zero, we get a over-determined system that consists of twenty-five algebraic equations.

Solving this over-determined system with the aid of Maple, we have the following results

Case 1:

$$L = 0, V = 4\mu, \lambda = \lambda, a_{-1} = 0, a_0 = \pm\frac{6\mu\lambda}{\sqrt{-3}}, a_1 = \pm2\sqrt{-3}(\lambda^2\mu+1), b_{-1} = 0, b_0 = 0, b_1 = 0$$

Case 2:

$$L = 0, V = 4\mu, \lambda = \lambda, a_{-1} = \pm2\sqrt{-3}\mu, a_0 = \pm2\sqrt{-3}\mu, a_1 = 0, b_{-1} = 0, b_0 = 0, b_1 = 0$$

Case 3:

$$L = 0, V = -8\mu, \lambda = 0, a_{-1} = \pm2\sqrt{-3}\mu, a_0 = 0, a_1 = \pm2\sqrt{-3}, b_{-1} = 0, b_0 = 0, b_1 = 0$$

Case 4:

$$L = \pm16\sqrt{-3}\mu^2\lambda(\lambda^2\mu+1), V = -4\mu(3\lambda^2\mu+2), \lambda = \lambda, a_{-1} = \pm2\sqrt{-3}\mu, a_0 = \pm2\sqrt{-3}\lambda\mu$$

$$a_1 = \pm2\sqrt{-3}(\lambda^2\mu+1), b_{-1} = 0, b_0 = 0, b_1 = 0$$

Case 5:

$$L = 0, V = -2\mu, \lambda = \lambda, a_{-1} = 0, a_0 = 0, a_1 = 0, b_{-1} = 0, b_0 = \pm2\sqrt{-3}, b_1 = 0$$

$$L = 0, V = -2\mu, \lambda = \lambda, a_{-1} = 0, a_0 = 0, a_1 = 0, b_{-1} = 0, b_0 = \pm2\sqrt{3}, b_1 = 0$$

Case 6:

$$L = 0, V = \mu, \lambda = 0, a_{-1} = 0, a_0 = 0, a_1 = \pm\sqrt{-3}, b_{-1} = 0, b_0 = 0, b_1 = \pm\sqrt{-3\mu}$$

$$L = 0, V = \mu, \lambda = 0, a_{-1} = 0, a_0 = 0, a_1 = \pm\sqrt{-3}, b_{-1} = 0, b_0 = 0, b_1 = \pm\sqrt{3\mu}$$

Case 7:

$$L = 0, V = -2\mu, \lambda = \lambda, a_{-1} = 0, a_0 = 0, a_1 = 0, b_{-1} = 0, b_0 = 0, b_1 = \pm2\sqrt{-3\mu}$$

$$L = 0, V = -2\mu, \lambda = \lambda, a_{-1} = 0, a_0 = 0, a_1 = 0, b_{-1} = 0, b_0 = 0, b_1 = \pm2\sqrt{3\mu}$$

Case 8:

$$L = 0, V = \mu, \lambda = \lambda, a_{-1} = \pm\sqrt{-3\mu}, a_0 = \pm\sqrt{-3}\lambda\mu, a_1 = 0, b_{-1} = 0, b_0 = \pm\sqrt{-3\mu}, b_1 = 0$$

$$L = 0, V = \mu, \lambda = \lambda, a_{-1} = \pm\sqrt{-3\mu}, a_0 = \pm\sqrt{-3}\lambda\mu, a_1 = 0, b_{-1} = 0, b_0 = \pm\sqrt{3\mu}, b_1 = 0$$

Now substituting Case 1- Case 8 and Eq. (4) into Eq.(9), we deduce abundant traveling wave solutions of Eq. (6) as follows.

When $\mu < 0$, we have the following hyperbolic function solutions:

Family 1:

$$u_1(\xi) = \frac{6\mu\lambda}{\pm\sqrt{-3}} \pm \frac{2\sqrt{-3}\left(\lambda^2\mu+1\right)(p_1)}{1+\lambda(p_1)},$$

where $\xi = x - 4\mu t$.

Family 2:

$$u_2(\xi) = \pm 2\sqrt{-3}\mu \pm \frac{2\sqrt{-3}\mu\left(1+\lambda(p_1)\right)}{(p_1)},$$

where $\xi = x - 4\mu t$.

Family 3:

$$u_3(\xi) = \pm 2\sqrt{-3}(p_1) \pm \frac{2\sqrt{-3}\mu}{(p_1)},$$

where $\xi = x - (-8\mu)t$.

Family 4:

$$u_4(\xi) = \pm 2\sqrt{-3}\lambda\mu \pm \frac{2\sqrt{-3}\left(\lambda^2\mu+1\right)(p_1)}{1+\lambda(p_1)} \pm \frac{2\sqrt{-3}\mu\left(1+\lambda(p_1)\right)}{(p_1)},$$

where $\xi = x - \left(-4\mu\left(3\lambda^2\mu+2\right)\right)t$.

Family 5:

$$u_5(\xi) = \pm 2\sqrt{-3}(p_1)^{-1}\sqrt{\left(1+\frac{1}{\mu}(p_1)^2\right)},$$

$$u_6(\xi) = \pm 2\sqrt{3}(p_1)^{-1}\sqrt{\left(-1-\frac{1}{\mu}(p_1)^2\right)};$$

where $\xi = x - (-2\mu)t$.

Family 6:

$$u_7(\xi) = \pm \frac{\sqrt{-3}(p_1)}{1+\lambda(p_1)} \pm \left(\sqrt{-3\mu}\right)\sqrt{\left(1+\frac{1}{\mu}(p_1)^2\right)},$$

$$u_8(\xi) = \pm \frac{\sqrt{-3}(p_1)}{1+\lambda(p_1)} \pm \left(\sqrt{3\mu}\right)\sqrt{\left(-1-\frac{1}{\mu}(p_1)^2\right)};$$

where $\xi = x - \mu t$.

Family 7:

$$u_9(\xi) = \pm 2\left(\sqrt{-2\mu}\right)\sqrt{\left(1+\frac{1}{\mu}(p_1)^2\right)},$$

$$u_{10}(\xi) = \pm 2\left(\sqrt{2\mu}\right)\sqrt{\left(-1-\frac{1}{\mu}(p_1)^2\right)};$$

where $\xi = x - (-2\mu)t$.

Family 8:

$$u_{11}(\xi) = \pm\sqrt{-3}\lambda\mu \pm \frac{\sqrt{-3}\mu(1+\lambda(p_1))}{(p_1)} \pm (\sqrt{-3})\mu(p_1)^{-1}\sqrt{\left(1+\frac{1}{\mu}(p_1)^2\right)} ,$$

$$u_{12}(\xi) = \pm\sqrt{-3}\lambda\mu \pm \frac{\sqrt{-3}\mu(1+\lambda(p_1))}{(p_1)} \pm (\sqrt{3})\mu(p_1)^{-1}\sqrt{\left(-1-\frac{1}{\mu}(p_1)^2\right)} ;$$

where $\xi = x - \mu t$.

Consequently, when $\mu > 0$, we obtain the following plane periodic solutions:

Family 9:

$$u_{13}(\xi) = \frac{6\mu\lambda}{\pm\sqrt{-3}} \pm \frac{2\sqrt{-3}(\lambda^2\mu+1)(p_2)}{1+\lambda(p_2)} ,$$

where $\xi = x - 4\mu t$.

Family 10:

$$u_{14}(\xi) = \pm 2\sqrt{-3}\mu \pm \frac{2\sqrt{-3}\mu(1+\lambda(p_2))}{(p_2)} ,$$

where $\xi = x - 4\mu t$.

Family 11:

$$u_{15}(\xi) = \pm 2\sqrt{-3}(p_2) \pm \frac{2\sqrt{-3}\mu}{(p_2)} ,$$

where $\xi = x - (-8\mu)t$.

Family 12:

$$u_{16}(\xi) = \pm 2\sqrt{-3}\lambda\mu \pm \frac{2\sqrt{-3}(\lambda^2\mu+1)(p_2)}{1+\lambda(p_2)} \pm \frac{2\sqrt{-3}\mu(1+\lambda(p_2))}{(p_2)} ,$$

where $\xi = x - (-4\mu(3\lambda^2\mu+2))t$.

Family 13:

$$u_{17}(\xi) = \pm 2\sqrt{-3}(p_2)^{-1}\sqrt{\left(1+\frac{1}{\mu}(p_2)^2\right)} ,$$

$$u_{18}(\xi) = \pm 2\sqrt{3}(p_2)^{-1}\sqrt{\left(-1-\frac{1}{\mu}(p_2)^2\right)} ;$$

where $\xi = x - (-2\mu)t$.

Family 14:

$$u_{19}(\xi) = \pm \frac{\sqrt{-3}(p_2)}{1+\lambda(p_2)} \pm (\sqrt{-3\mu})\sqrt{\left(1+\frac{1}{\mu}(p_2)^2\right)} ,$$

$$u_{20}(\xi) = \pm \frac{\sqrt{-3}(p_2)}{1+\lambda(p_2)} \pm \left(\sqrt{3\mu}\right)\sqrt{\left(-1-\frac{1}{\mu}(p_2)^2\right)} \; ;$$

where $\xi = x - \mu t$.

Family 15:

$$u_{21}(\xi) = \pm 2\left(\sqrt{-2\mu}\right)\sqrt{\left(1+\frac{1}{\mu}(p_2)^2\right)} \; ,$$

$$u_{22}(\xi) = \pm 2\left(\sqrt{2\mu}\right)\sqrt{\left(-1-\frac{1}{\mu}(p_2)^2\right)} \; ;$$

where $\xi = x - (-2\mu)t$.

Family 16:

$$u_{23}(\xi) = \pm\sqrt{-3}\lambda\mu \pm \frac{\sqrt{-3}\mu(1+\lambda(p_2))}{(p_2)} \pm (\sqrt{-3})\mu(p_2)^{-1}\sqrt{\left(1+\frac{1}{\mu}(p_2)^2\right)} \; ,$$

$$u_{24}(\xi) = \pm\sqrt{-3}\lambda\mu \pm \frac{\sqrt{-3}\mu(1+\lambda(p_2))}{(p_2)} \pm (\sqrt{3})\mu(p_2)^{-1}\sqrt{\left(-1-\frac{1}{\mu}(p_2)^2\right)} \; ;$$

where $\xi = x - \mu t$.

Graphical representation of the solutions

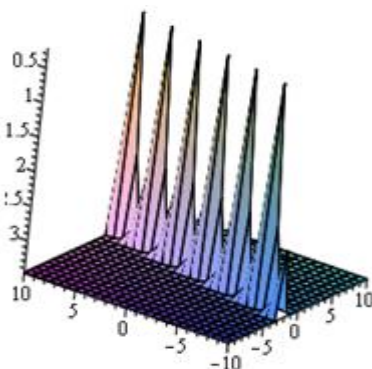The graphical illustrations of the solutions are given below in the figures with the aid of Maple.

Fig. 1: Multiple soliton solution, Shape of $u_1(\xi)$ **when** $A = 1, \; B = 2, \; \mu = -1, \; \lambda = 3$ **and** $-10 \le x, t \le 10$.
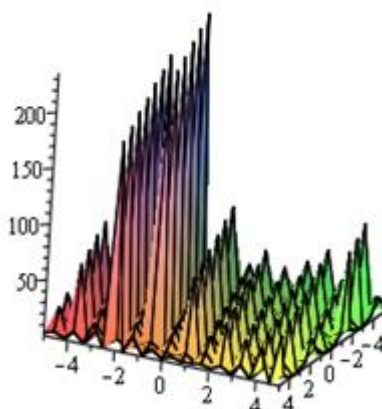

Fig. 2: Periodic solution, Shape of $u_{13}(\xi)$ **when** $A = 1, \; B = 2, \; \mu = 1, \; \lambda = 3$ **and** $-5 \le x, t \le 5$.
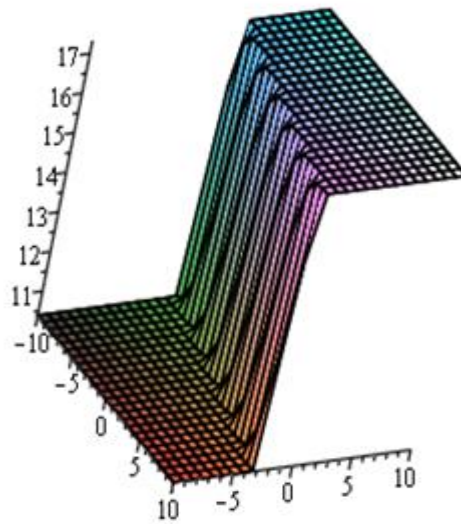
Fig. 3: Kink wave solutions, Shape of $u_2(\zeta)$ **when** $A=1,\ B=2,\ \mu=-1,\ \lambda=3$ **and** $-10\le x,t\le10.$
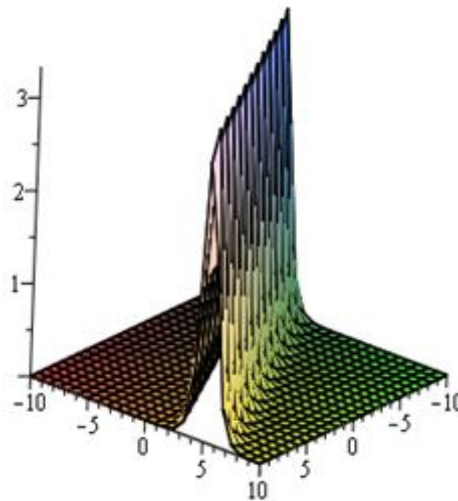

Fig. 4: Modulus plot of soliton wave solutions, Shape of $u_5(\xi)$ **when** $A=1,\ B=2,\ \mu=-1,\ \lambda=3$ **and** $-10\le x,t\le10.$
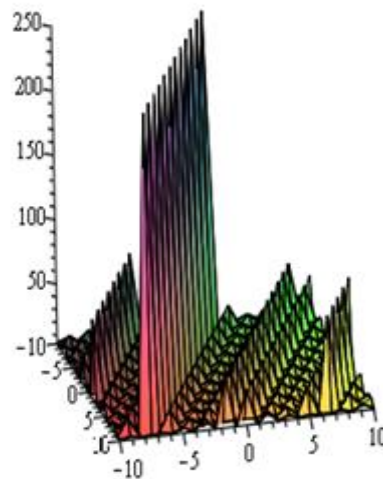

Fig. 5: Modulus plot of periodic wave solutions, Shape of $u_{17}(\xi)$ **when** $A=1,\ B=2,\ \mu=1,\ \lambda=3$ **and** $-10\le x,t\le10.$
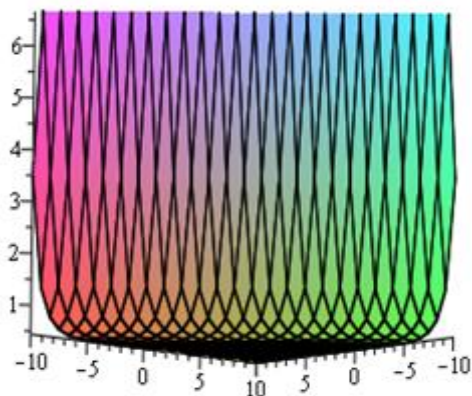
Fig. 6: Soliton wave solutions, Shape of $u_7(\xi)$ **when** $A = 1, \ B = 2, \ \mu = -1, \ \lambda = 3$ **and** $-10 \le x, t \le 10.$
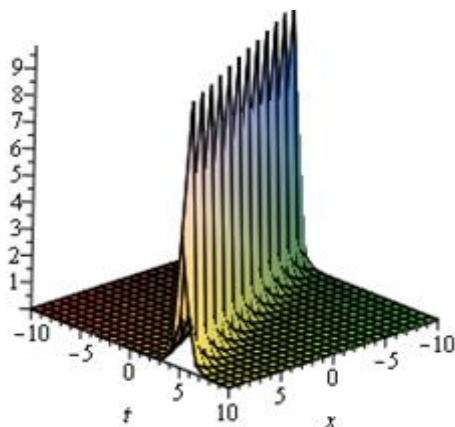

Fig. 7: Modulus plot of soliton wave solutions, Shape of $u_9(\xi)$ **when** $A = 1, \ B = 2, \ \mu = -1, \ \lambda = 3$ **and** $-10 \le x, t \le 10.$
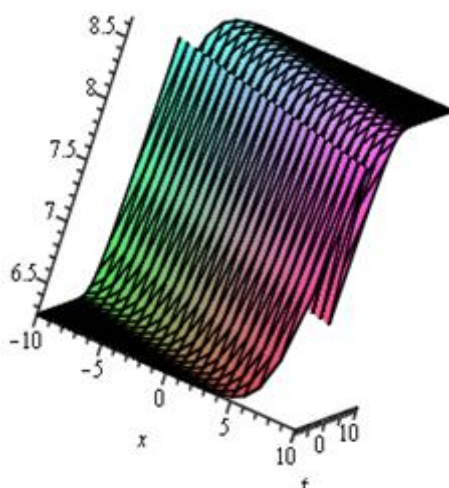

Fig. 8: Modulus plot of singular Kink wave solutions, Shape of $u_{11}(\xi)$ **when** $A = 1, \ B = 2, \ \mu = -1, \ \lambda = 3$ **and**

$$-10 \le x, t \le 10$$

Conclusion

The new extended (G'/G)-expansion method is successfully used to establish travelling wave solutions to the (2+1)-dimensional modified Zakharov-Kuznetsov equation. As a result, we obtained plentiful new exact solutions. The solutions are in the form of trigonometric, rational and hyperbolic. It is shown that the performance this method is productive, effective and well-built mathematical tool for solving nonlinear partial differential equations. Therefore, the method could be applied to solve different nonlinear PDEs which frequently arise in mathematical physics, engineering and many scientific real time application fields.

References

1. T.L. Bock and M.D. Kruskal, A two-parameter Miura transformation of the Benjamin-One equation, Phys. Lett. A 74 (1979) 173-176.

2. Y. Chen and Q. Wang, Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic functions solutions to (1+1)-dimensional dispersive long wave equation, Chaos solitons Fractals, 24 (2005) 745-57.

3. M.L. Wang, X.Z. Li and J. Zhang, The (G'/G)-expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A 372 (2008) 417-423.

4. M.N. Alam and M.A. Akbar, Exact traveling wave solutions of the KP-BBM equation by using the new generalized (G'/G)-expansion method, SpringerPlus, 2(1), 2013, 617. DOI: 10.1186/2193-1801-2-617.

5. M.N. Alam, M.A. Akbar and S.T. Mohyud-Din, A novel (G'/G)-expansion method and its application to the Boussinesq equation, Chin. Phys. B, Vol. 23(2), 2014, 020203-020210, DOI: 10.1088/1674-1056/23/2/020203.

6. M. Song and Y. Ge, Application of the (G'/G)-expansion method to (3+1)-dimensional nonlinear evolution equations, Computers and Mathematics with Applications 60 (2010) 1220-1227.

7. Q. Feng and B. Zheng, Traveling wave solutions for three nonlinear equations by (G'/G)-expansion method, Wseas Transactions on Computers, Vol. 9, Issue 3, March 2010, 1109-2750.

8. A. Bekir, Application of the (G'/G)-expansion method for nonlinear evolution equations, Phys. Lett. A 372 (2008) 3400-3406.

9. E.M.E. Zayed, The (G'/G)-expansion method and its applications to some nonlinear evolution equations in the mathematical physics, J. Appl. Math. Comput., 30 (2009) 89-103.

10. S. Zhang, J. Tong and W. Wang, A generalized (G'/G)-expansion method for the mKdV equation with variable coefficients, Phys. Lett. A 372 (2008) 2254-2257.

11. M.N. Alam, M.A. Akbar and S.T. Mohyud-Din, General traveling wave solutions of the strain wave equation in microstructured solids via the new approach of generalized (G'/G)-Expansion method, Alexandria Eng. J., (2014) 53, 233-241, http://dx.doi.org/10.1016/j.aej.2014.01.002.

12. J. Zhang, F. Jiang and X. Zhao, An improved (G'/G)-expansion method for solving nonlinear evolution equations, Int. J. Com. Math., 87(8) (2010) 1716-1725.

13. A.J.M. Jawad, M.D. Petkovic and A. Biswas, Modified simple equation method for nonlinear evolution equations, Appl. Math. Comput., 217 (2010) 869-877.

14. K. Khan, M.A. Akbar and M.N. Alam, Traveling wave solutions of the nonlinear Drinfel'd-Sokolov-Wilson equation and modified Benjamin-Bona-Mahony equations, J. Egyptian Math. Soc., 21, 2013, 233-240, http://dx.doi.org/10.1016/j.joems.2013.04.010.

15. J.B. Li and Z.R. Liu, Smooth and non-smooth traveling waves in a nonlinearly dispersive equation, Appl. Math. Model. 25 (2000) 41-56.

16. Z.R. Liu and T.F. Qian, Peakons and their bifurcation in a generalized Camassa-Holm equation, Int. J. Bifur. Chaos 11 (2001) 781-792.

17. J.S. Russell, Report on waves, in proceedings of the 14th Meeting of the British Association for the Advancement of Science, 1844.

18. J. L. Hu, Explicit solutions to three nonlinear physical models. Phys. Lett. A, 287: 81-89, 2001.

19. J. L. Hu, A new method for finding exact traveling wave solutions to nonlinear partial differential equations. Phys. Lett. A, 286: 175-179, 2001.

20. V.B. Matveev and M.A. Salle, Darboux transformation and solitons, Springer, Berlin, 1991.

21. A.H. Salas and C.A. Gomez, Application of the Cole-Hopf transformation for finding exact solutions to several forms of the seventh-order KdV equation, Mathematical Problems in Engineering, Art. ID 194329 (2010) 14 pages.

22. J.H. He and X.H. Wu, Exp-function method for nonlinear wave equations. Chaos, Solitons and Fract., 30: 700-708, 2006.

23. M.M. Kabir, A. Khajeh, New explicit solutions for the Vakhnenko and a generalized form of the nonlinear heat conduction equations via Exp-function method, Int. J. Nonlinear Sci. Numer. Simul., 10(10): 1307–1318, 2009.

24. S.A. El-Wakil, M.A. Abdou, A. Hendi, New periodic wave solutions via Exp-function method. Phys. Lett. A, 372: 830–840, 2008.

25. M.J. Ablowitz and P.A. Clarkson, Soliton, nonlinear evolution equations and inverse scattering method. Cambridge University Press, New York, 1991.

26. H.O. Roshid, M.A. Akbar, M.N. Alam , M.F. Hoque and N. Rahman, New extended (G'/G)-expansion method to solve nonlinear evolution equation: The (3+1)-dimensional potential-YTSF equation, Springer Plus 2014, 3:122 doi:10.1186/2193-1801-3-122.