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Implementation of ZCT precoding based OFDM transmitter system using VHDL

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ABSTRACT

The large Peak to Average Power Ratio (PAPR) of Orthogonal Frequency Division Multiplexing (OFDM) system has undesirable effects on the system such as increase in the complexity of A/D convertor and reduction in the efficiency of power amplifier. In this paper, modified expression of Zadoff Chu sequence, generation of Zadoff Chu sequence using hardware efficient CORDIC algorithm and develop Zadoff Chu matrix transformation (ZCT) precoding based OFDM system to minimize PAPR in OFDM system. Zadoff Chu sequences are class of complex exponential polyphase sequences which has constant amplitude and these sequences posses' good correlation properties. In ZCT pre coding based OFDM system, ZCT is applied to the constellation symbols before the Inverse Fast Fourier Transform (IFFT). The proposed system is simulated with VHDL and hardware efficient CORDIC algorithm is used for the generation of Zadoff Chu sequence. The achieved simulation results confirm that the proposed methods are capable of reducing the PAPR significantly by 3 dB for N=64 subcarrier and 16-QAM modulation at clip rate of 10^{-2}.

Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a frequency division multiplexing (FDM) scheme utilized as a digital multicarrier modulation method [1]. A large number of closely spaced orthogonal subcarriers are used to carry data. The incoming serial bit stream is divided into several parallel streams of channels, one for each subcarrier. OFDM is attractive technique for high speed data transmission over multipath fading channels. It improves security, minimizes multipath effects, such as Inter symbol Interference (ISI) and Inter Carrier Interference (ICI). OFDM has been adopted by most wireless and wired digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrow band interference. This Multicarrier modulation system finds application in Digital data Transmission over the Telephone system, Digital audio broadcasting, Digital Television, and Wireless Local Area Networks. But presence of a large number of subcarriers with varying amplitude results in a high Peak to Average Power Ratio (PAPR) of transmitting OFDM signal. A High PAPR is still one of the major drawbacks of OFDM system, which require power amplifier with large linear ranges and also increases the complexity of the Analog to digital (A/D) and Digital to analog (D/A) converters and reduces the efficiency of the radio power amplifier [1].

There are a number of techniques to deal with the problem of PAPR. Some of them are "amplitude clipping", "clipping and filtering", "coding", "partial transmit sequence (PTS)", "selected mapping (SLM)" and "interleaving". These techniques achieve PAPR reductions at the expense of transmitted signal power, data rate and computational complexity.

Zadoff Chu matrix Transform (ZCT) pre coding scheme [3] does not require any side information and complex optimizations. The pre coding based technique is simple linear technique and hence easy to implement without the need of any side information. This scheme does not lose the data rate and spectral efficiency.

In this paper, development of ZCT pre coded OFDM system is discussed and improvement in the system performance is shown through simulation results. As an initial step toward hardware implementation, simulation of the transmitter using VHDL is carried out. Hardware efficient CORDIC algorithm is used for the generation of Zadoff Chu sequence.

The rest of the paper is organized as follows. Section II Introduction of OFDM system .Section III, Proposed model. Section IV, Zadoff-Chu sequences generation and implementing ZCT precoding based OFDM system. Section V presents simulation results. Section VI conclusion

OFDM System

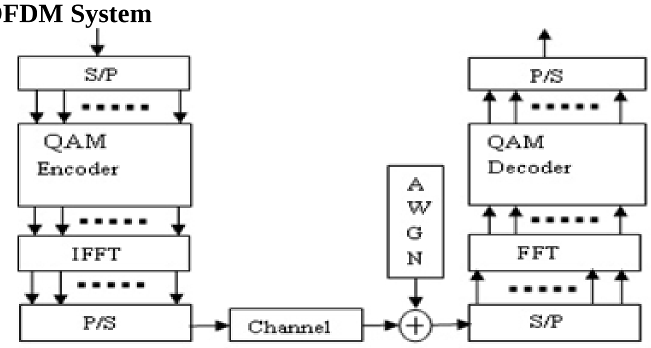

Figure 1 . Block diagram of OFDM system

Figure 1 shows the block diagram of OFDM system based on [1], the serial to parallel converter receive the serial bits to be transmitted, converts it to parallel form (symbol). The parallel data is passed through M-QAM encoder, each symbol is mapped by constellation modulator using Gray code which generates the complex valued constellation points. The complex vector of size N is represented as $X=[X_0, X_1, X_2 \ldots\ldots X_{N-1}]^T$. X is then passed

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through the IFFT block, mathematically modulating a waveform. The use of IFFT in OFDM eliminates the need for separate sinusoidal converters.

The complex data block to be transmitted is given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{k=N-1} X_k\, e^{\frac{j2\pi n}{N}k} \qquad n = 0,1 \ldots \ldots N-1 \quad (1)$$

The PAPR of the transmitted signal is defined as

$$PAPR = \frac{max\,[|x_n|^2]}{E\,[|x_n|^2]} \qquad (2)$$

Where $E[.]$ is expectation

Reducing $max[x_n]$ is the principle goal of PAPR reduction techniques. The Complementary Cumulative Distribution Function (CCDF) is a function used to find how often the random variable is *above* a particular level. For an OFDM signal CCDF is given by

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \qquad (3)$$

where $PAPR_0$ is the clipping level.

Proposed Model

Zadoff Chu sequence

This sequence is a complex valued mathematical sequence, which has constant amplitude and the autocorrelation of a prime length Zadoff–Chu sequence with a cyclically shifted version of itself has zero auto-correlation [2]

The Zadoff–Chu sequence with root index (r) is given by

$$Z_r(n) = \begin{cases} e^{j\frac{2\pi}{L}\cdot\frac{rn(n+2q)}{2}} & for\ L\ even \\ e^{j\cdot\frac{2\pi}{L}\cdot\frac{rn(n+1+2q)}{2}} & for\ L\ odd \end{cases} \qquad (4)$$

Modified expression of Zadoff chu sequence

Here we are considering only Zadoff-Chu sequence [3] with $q = 0$ and L is an odd length of Zadoff Chu sequence is written as

$$Z_r(n) = e^{j\frac{2\pi}{L}\cdot\frac{rn(n+1)}{2}} \qquad (5)$$

For prime length L, each root index value $r = 1 \ldots L-1$ can generate $L-1$ different Zadoff-Chu sequence. All cyclic time shifted and phase shifted sequence are considered to be same sequence they are not different sequences. Hence phase of a sequence is multiplication of the basic angle and multiplicative coefficient. Basic angle is given as $\theta_0 = \frac{2\pi}{L}$ which is constant and multiplicative coefficient is given by

$$y_r(n) = r.\frac{n(n+1)}{2} \qquad (6)$$

Now equation (5) can be written as

$$Z_r(n) = e^{j\theta_0 * y_r(n)} \qquad (7)$$

The multiplicative coefficients $y_r(n)$ should be modulo L and $Z_r(n)$ is periodic since $y_r(n)$ is periodic with period L and $e^{j\theta}$ is periodic with period 2π.

The multiplicative coefficient represented by equation (6) is modified by including frequency shift $k.n$ is written as

$$x_r(n) = r.\frac{n(n+1)}{2} + k.n = r.\frac{n(n+1+2r^{-1}.k)}{2} \qquad (8)$$

Where r -1 is the inverse of r modulo L. We observe that 1 + 2r-1·k is an odd integer since it's a odd prime length sequence and replacing this by K equation (8) becomes

$$x_r(n) \equiv r.\frac{n(n+K)}{2} \qquad (9)$$

K should be an odd integer and if K=1 this is similar to equation (6).Now we can represent modified expression for Zadoff Chu sequence using equation (9) and replacing K=L .So the new expression is

$$x_r(n) \equiv r.\frac{n(n+L)}{2} \qquad (10)$$

This modified expression is valid for odd length as well for even length Zadoff-Chu sequences. The modified expression of Zadoff Chu sequence is

$$Z_r(n) = e^{j\frac{2\pi}{L}\frac{rn(n+L)}{2}} \qquad (11)$$

Where $L =$ Length of sequence, $r =$ root index of sequence relatively prime to L, $q =$ arbitrary integer and

$n = 0,1 \ldots L-1$, $j = \sqrt{-1}$

ZCT precoding based OFDM (ZCT-OFDM) system

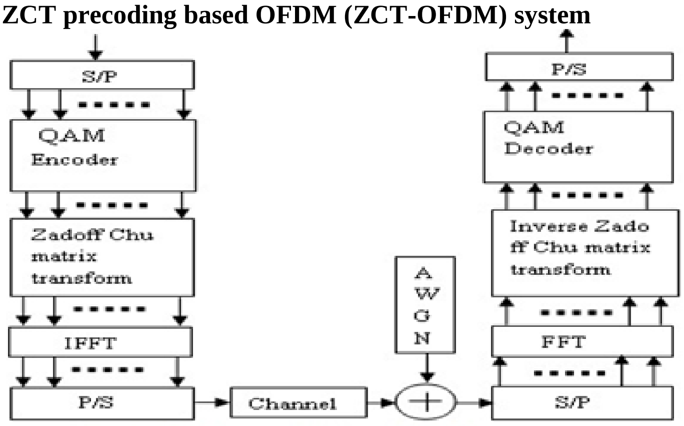

Figure 2 ZCT precoding based OFDM system

Figure 2 shows the Zadoff Chu matrix transform precoding based OFDM system based on [1]. Here we have implemented the pre coding matrix Z of size N ×N before the IFFT is taken, to reduce the PAPR. ZCT precoding is applied to QAM constellation points of complex vector $X=[X_0, X_1, X_2\ldots\ldots X_{N-1}]^T$, which transforms X into new vector $Y=ZX=[Y_0,Y_1,Y_2\ldots\ldots Y_{N-1}]^T$ [1]. By reordering $n = mN + l$ matrix Z with row wise reshaping is given by

$$Z = \begin{bmatrix} z_{00} & z_{01} & \cdots & z_{0(N-1)} \\ z_{10} & z_{11} & \cdots & z_{1(N-1)} \\ z_{(N-1)0} & z_{(N-1)1} & \cdots & z_{(N-1)(N-1)} \end{bmatrix} \qquad (12)$$

The Zadoff Chu sequence of odd length L generated using equation (11) .The Zadoff Chu sequence of length $L = N \times N$ fills the precoder matrix Z of size $N \times N$ row wise and ZCT is a complex orthogonal matrix.

Accordingly, precoding X gives rise to Y as follows: $Y = ZX$

$$y_m = \sum_{l=0}^{N-1} z_{m,l}\, x_l \qquad (13)$$

Where $m = 0,1,2 \ldots\ldots N-1$, $z_{m,l}$ mean m^{th} row l^{th}column of precoder matrix y_m represents the ZCT precoded constellation symbols. The complex baseband ZCT-OFDM signal with N subcarriers with precoding can be represented as

$$x_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{m=N-1} y_m\, e^{\frac{j2\pi n}{N}m} \quad (14) \quad n = 0,1 \ldots \ldots N-1$$

The PAPR of ZCT-OFDM signal can be written as

$$PAPR = \frac{max\,|x_n|^2}{E[|x_n|^2]} \qquad (15)$$

Where $E[.]$ is expectation

Zadoff Chu Sequence Generation And Implementing ZCT Based Precoding Of OFDM System Using VHDL

QAM Encoder

Incoming serial bit stream is converted to 2- parallel bits for 4-QAM and 4 parallel-bits for 16-QAM to form a symbol. These symbols are converted to gray code, then constellation of symbol is formed by splitting the symbol as Inphase and Quadrature phase and assigning the two intermediate levels of amplitude [4].

Zadoff Chu sequence generation using CORDIC algorithm

CORDIC (Coordinate Rotation Digital Computer) algorithm [5], iteratively solve trigonometric equations and including rotation transform, the rotation of a two-dimensional vector (*x*, *y*) in linear, circular and hyperbolic coordinates. CORDIC is a hardware efficient iterative algorithm for evaluating trigonometric function and many other functions using simple shift and add operations. The CORDIC algorithm performs a vector rotation as a sequence of successively smaller rotations, each of angle $atan(2^{-i})$, known as micro-rotations. The expression for the i^{th} iteration where *i* is the iteration index from *0* to *n* is given by

$$X_{i+1} = X_i - \alpha_i . Y_i . 2^{-i}$$
$$Y_{i+1} = Y_i + \alpha_i . X_i . 2^{-i}$$
$$Z_{i+1} = Z_i - \alpha_i . \tan^{-1} 2^{-i} \qquad (16)$$

Where $\alpha_i = +1 \; or -1$, α_i indicate the direction of rotation.

We have constructed the Zadoff Chu sequence given in equation (11) using CORDIC algorithm to compute the trigonometric functions with phase $\frac{2\pi}{L} * \frac{rn(n+L)}{2}$. The arguments are computed using floating point arithmetic. Here we used single precision floating point representation which is of 16-bit. In this representation MSB is sign bit that is (15^{th} bit), 5-bits are used for exponent (14^{th} to 10^{th} bit) and 10-bits are used for mantissa(9^{th} to 0^{th} bit) weights are shown below in table 1. If sign bit is 0 for positive numbers and 1 for negative numbers.

Table 1: Floating Point Representation

	Sign Bit15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	Decimal value
weights		2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}	2^{-10}	
Binary	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	3

Evaluated phase value is passed as angle to the CORDIC algorithm which gives the real and imaginary part of complex sequence [4] and phase is restricted between $-\pi \; to \; \pi$ later sign of sine and cosine adjusted.

ZCT Precoding

After generating Zadoff Chu sequence of length *L*, the elements of sequence are arranged into $N \times N$ matrix form. Hence length of the sequence L is chosen as $L = N \times N$. For precoding of OFDM transmitting symbol, the constellation symbol of QAM encoder output is multiplied with Zadoff Chu matrix of order $N \times N$ which reduces the PAPR of transmitting symbol.

IFFT

The OFDM signal is generated by implementing the Inverse Fast Fourier Transform (IFFT) function at the transmitter. Here we computed 8-points IFFT using floating point arithmetic which accepts 8 inputs of real binary bits [4].

After precoding, IFFT of precoded symbols are computed then transmitted through the channel.

Simulation Result

Zadoff Chu sequence using modified expression

The standard expression of zadoff chu sequence given in equation (4) is different for the even length and odd length sequence. Hence we can use modified expression to generate sequence which is valued for even as well as odd length sequence. Figure 3 shows the Zadoff chu sequence using standard and modified expression with $r = 1 \; and \; L = 257$, we observe that Zadoff chu sequence of modified expression is shifted version of Zadoff chu sequence of standard expression.

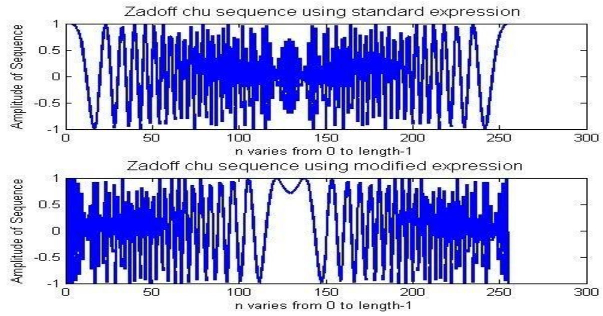

Figure 3: Zadoff chu sequence using standard and modified expression

QAM Encoder

For 16-QAM the gray code converted 4 bits are divided into 2 bits as Inphase and 2 bits as Quadrature phase. Two intermediate amplitude mapping is done using -3 for 00,-1 for 01, 3 for 10 and 1 for 11. Matlab and VHDL simulation of QAM are shown below in figure 4 and figure 5 and table 2 and 3 describes the input and output of QAM Encoder.

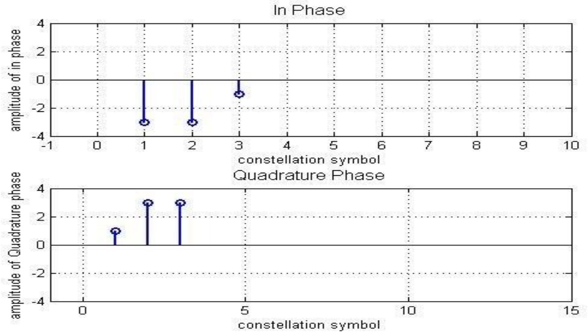

Figure 4 : Matlab simulation of QAM

Table 2: Matlab simulation results of QAM

Input	Inphase	Quadrature phase
0010	-3	1
0011	-3	3
0100	-1	3

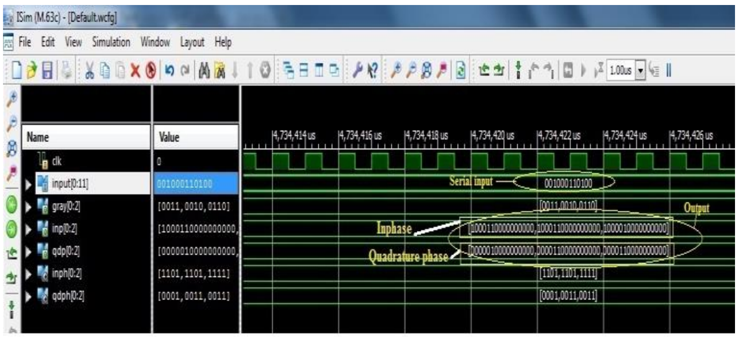

Figure 5: VHDL simulation of QAM

Table 3 :VHDL simulation results of QAM

Input	Simulation result of Inphase Binary values	Decimal values	Simulation result of Quadrature phase Binary values	Decimal Values
0010	1,00011,000000000	-3.0	0,00001,000000000	1.0
0011	1,00011,000000000	-3.0	0,00011,000000000	3.0
0010	1,00001,000000000	-1.0	0,00011,000000000	3.0

Zadoff Chu sequence generation

To generate Zadoff Chu sequence here we considered root index *r=5*, length *L=11* as input and output is evaluated phase and11 complex elements of sequence. Matlab and VHDL simulation of Zadoff Chu sequence are shown below in figure 6 and figure 7 and table 4 and 5 describes the output of Zadoff Chu Sequence generator.

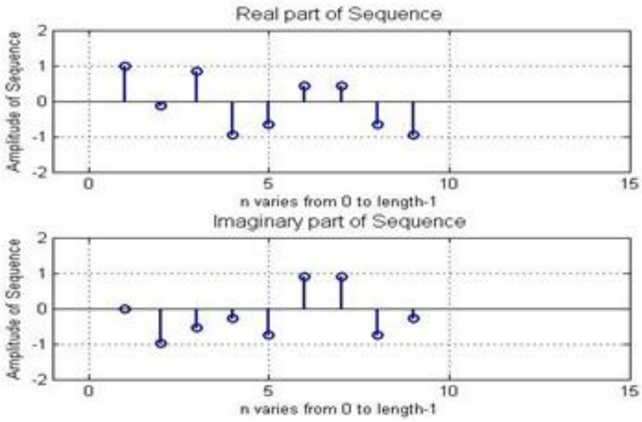

Figure 6: Matlab simulation of Zadoff Chu sequence

Table4: Matlab simulation result of Zadoff Chu sequence

phase	3.142	-1.428	-2.5704	-0.2856	-0.8568	-1.9992	-1.9992	-0.8568	-0.2856
Real part of sequence	1.0000	-0.1423	-0.9595	0.8413	-0.6549	0.4154	0.4154	-0.6549	-0.9595
Img part of sequence	0	-0.9898	-0.5406	-0.2817	-0.7557	0.9096	0.9096	-0.7557	-0.2817

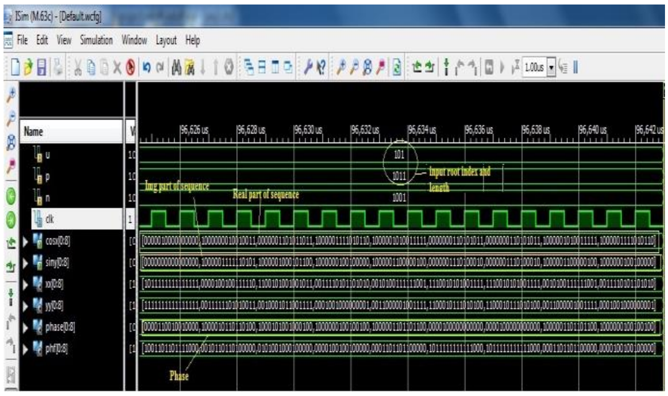

Figure 7: VHDL simulation of Zadoff Chu Sequence

Table 5: VHDL Simulation results of Zadoff chu sequence

Simulation result of phase Binary value	Decimal value	Simulation result of Real part of sequence Binary values	Decimal values	Simulation result of Imaginary part of sequence Binary values	Decimal Values
0,00011,0010010000	3.140	0,00001,000000000	1.0	0,00000,000000000	0.0
1,00001,0110110100	-1.4257	1,00000,0010010011	-0.1435	1,00000,1111110101	-0.9892
1,00010,1001000100	-2.5664	0,00000,1101011011	0.8388	0,00000,1000101100	0.5683
1,00000,0100100100	-0.2851	1,00000,1111010110	-0.9589	1,00000,0100100000	-0.2812
1,00000,1101101100	-0.8554	1,00000,1010011111	-0.6396	1,00000,1100000100	-0.7539
0,00010,0000000000	2.0	0,00000,0110101011	0.4169	1,00000,1110100010	-0.9414
0,00010,0000000000	2.0	0,00000,0110101011	0.4169	1,00000,1110100010	-0.9414
1,00000,1101101100	-0.8554	1,00000,1010011111	-0.6513	1,00000,1100000100	-0.7539
1,00000,0100100100	-0.2851	1,00000,1111010110	-0.9589	1,00000,0100100000	-0.2851

ZCT Precoding

The elements of Zadoff Chu sequence are rearranged in matrix form of size 3×3 which is a precoder matrix. Here we are considering 9 elements of sequence among 11 to for 3×3. Then this matrix is multiplied with QAM encoder generated constellation symbols which gives the precoded symbols. Matlab and VHDL simulation of precoded symbols are shown below in figure 9 and figure 10 and table 6 and 7 describes the output of precoder

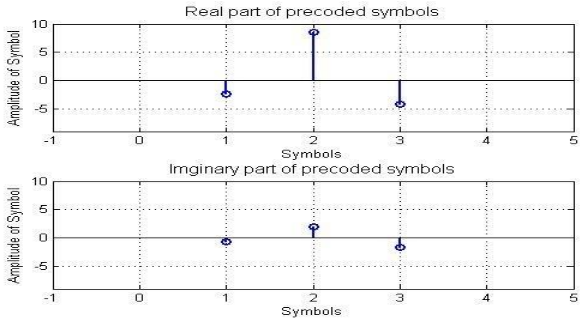

Figure8 :Matlab simulation of Precoding of Symbol

Table 6: Matlab simulation results of precoding

Real part of precoding	-2.4206	8.5707	-4.1536
Img part of precoding	-0.6967	1.9210	-1.6162

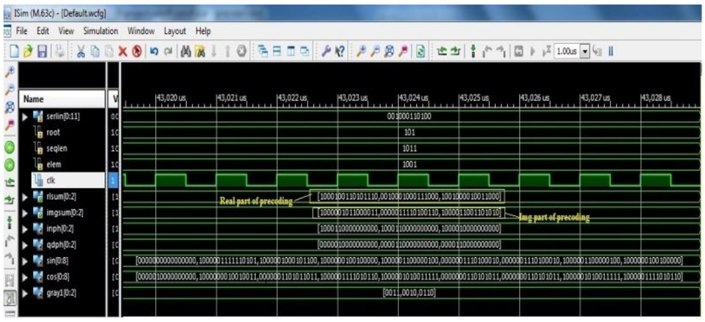

Figure 9: VHDL simulation of precoded symbols

Table 7 :VHDL simulation results of precoding

Simulation result of Real part of precoding Binary values	Decimal values	Simulation result of Imaginary part of Precoding Binary values	Decimal Values
1,00010,0110101110	-2.4199	0,00000,1011000011	-0.6904
0,01000,1000111000	8.5707	0,00001,1110100110	1.9121
1,00100,0010011000	-4.1484	1,00001,1001101010	-1.6035

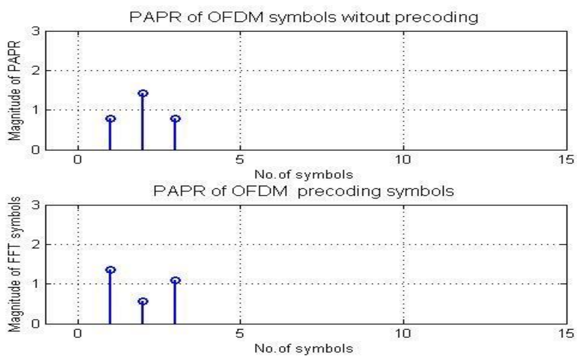

Figure 10: Matlab simulation of PAPR of OFDM symbol without and with precoding

Figure 10 shows the PAPR values of constellation symbol and precoded symbols.

IFFT

Here length of the IFFT considered is 8 point. Three Precoded symbols are given as input to IFFT which gives the 8 point as output. Matlab and VHDL simulation of precoded symbols are shown below in figure 11 and figure 12 and table 5 describes the output of IFFT.

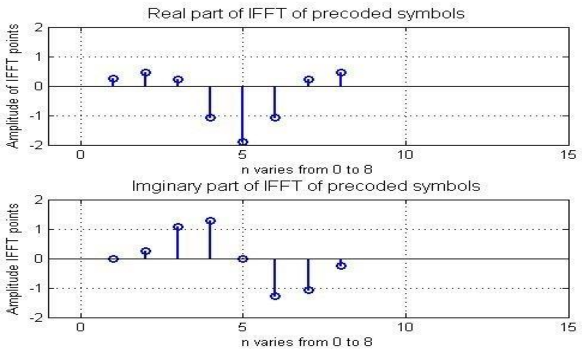

Figure1 1: Matlab simulation of IFFT of precoded Symbol

Table 8:Matlab simulation results of IFFT

Real part of IFFT	0.2496	0.4550	0.2166	-1.0601	-1.8931	0.2166	-1.0601	0.4550
Img part of IFFT	0	0.2383	1.0713	1.2767	0	-1.2767	-1.0713	-0.2383

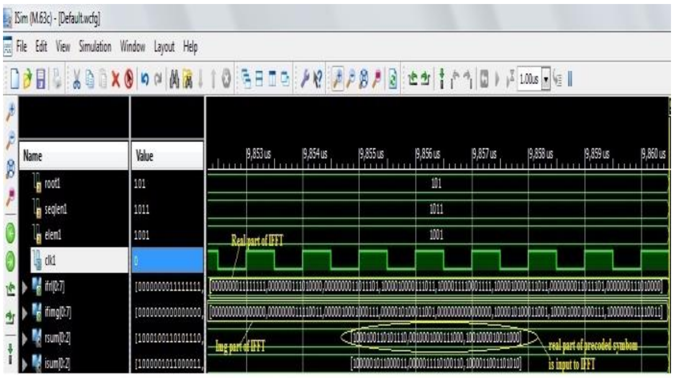

Figure 12 : VHDL simulation of IFFT of precoded symbols

Table 9:VHDL simulation results of IFFT

Simulation result of Real part of IFFT Binary values	Decimal values	Simulation result of Imaginary part of IFFT Binary values	Decimal Values
0,00000,0011111111	0.2490	0,00000,000000000	0.0
0,00000,0111001100	0.4492	0,00000,0011110011	0.2373
0,00000,0011011101	0.2158	0,00001,0001000111	1.0693
1,00001,0000110111	-1.0537	0,00001,0100011001	1.2705
1,00001,1110001111	-1.8896	0,0000000000000000	0.0
1,00001,0000110111	-1.0537	1,00001,0100011001	-1.2705
0,00000,0011011101	0.2158	1,00001,1000100111	-1.0693
0,00000,0111001100	0.4492	1,00000,0011110011	-0.2373

CCDF of PAPR

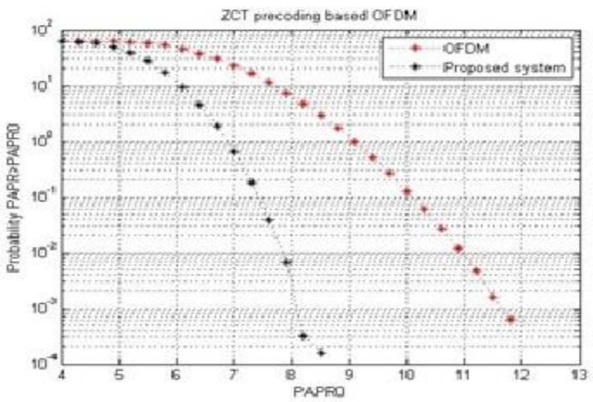

Figure 13 : CCDF plot of PAPR

The Figure 13 shows the Matlab simulation of CCDF comparison of PAPR of higher transmitting bits of OFDM system and Zadoff Chu matrix transformation precoding based OFDM .By observing Figure 11 we can conclude that PAPR of OFDM system is high compare to precoded OFDM system. OFDM systems with 16- QAM modulation clip rate of 10^{-2}, the PAPR is 11.0 dB and ZCT precoding based OFDM system is 8.9 dB

Conclusion

Here we proposed, PAPR reduction techniques namely Zadoff Chu matrix Transform (ZCT) precoding is used for OFDM system. Using ideal autocorrelation properties, reshaping of the Zadoff Chu matrix transform is carried out for precoding. Hence by observing Matlab simulation results, we can conclude that PAPR of conventional OFDM is high compare to ZCT precoding based OFDM system. VHDL implementation of precoding of OFDM transmitting symbol is done by generating Zadoff Chu sequence using hardware efficient CORDIC algorithm.

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