Mass transfer and thermal radiation effects on MHD flow near a stagnation point on a linear stretching sheet through porous medium with variable thermal conductivity, heat generation and viscous dissipation

B. Lavanya[1] and A. Leela Ratnam[2]

[1]Department of Mathematics, Priyadarshini College of Engineering & Technology, Nellore.
[2]Department of Applied Mathematics, Sri Padmavathi Mahila Visva Vidyalayam, Tirupathi.

ABSTRACT

A numerical analysis is carried out to study the effects of mass transfer and thermal radiation on a steady boundary layer flow of a viscous incompressible electrically conducting fluid with uniform transverse magnetic field and variable free stream near a stagnation point on a non-conducting stretching sheet through a porous medium in presence of variable thermal conductivity, heat generation and viscous dissipation. The equations of continuity, momentum, energy and mass are transformed into ordinary differential equations and solved numerically using shooting method. The velocity, temperature and concentration distributions are discussed numerically and presented through graphs. Skin-friction coefficient, Nusselt number and Sherwood number at the sheet are derived, discussed numerically and their numerical values for various values of physical parameters are presented through tables.

Introduction

The field of boundary layer flow problem over a stretching sheet has many industrial applications such as polymer sheet or filament extrusion from a dye or long thread between feed roll or wind-up roll, glass fiber and paper production, drawing of plastic films, liquid films in condensation process. Due to the high applicability of this problem in such industrial phenomena, it has attracted the attentions of many researchers.

Sakiadis [1, 2] was the first to study boundary layer flow over a stretching surface moving with a constant velocity in an ambient fluid. He employed a similarity transformation and obtained a numerical solution for the problem. Erickson *et al.* [3] extended the work of Sakiadis [1] to account for mass transfer at the stretching surface. Tsou *et al.* [4] presented a combined analytical and experimental study of the flow and temperature field in the boundary layer on a continuous moving surface, in which the flow is caused by an elastic sheet moving in its own plane with a velocity varying linearly with the distance from a fixed point studied by Crane [5]. The study of stretching surface and the several combinations of additional effects on the stretching problems are important in many practical application because the production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. In the manufacture of the latter, the material is in a molten phase when thrust through an extrusion die and then cools and solidifies some distance away from the die before arriving at the collecting stage. The quality of the resulting sheeting material, as well as the cost of production, is affected by the speed of collection and the heat transfer rate, and knowledge of the flow properties of the ambient fluid is clearly desirable. On the other hand, it should be pointed out that the very important practical problems of the thermal processing of sheet like materials which is a necessary operation in the production of paper, linoleum, polymeric sheets, roofing shingles, insulating materials, fine-fiber matt's are described in the excellent papers by Sparrow and Abraham [6] and Abraham and Sparrow [7]. Other very important applications of the stretching sheets are described in the papers by Lakshmisha et al.   [8].

The stagnation point flows are classic problems in the field of fluid dynamics and have been investigated by many researches. These flows can be viscous or inviscid, steady or unsteady, two-dimensional or three –dimensional, normal or oblique, and forward or reverse. The steady flow in the neighborhood of a stagnation-point was first studied by Hiemenz [9], who used a similarity transformation to reduce the Navier-Stokes equations to nonlinear ordinary differential equations. This problem has been extended by

Tele:
E-mail addresses: ibrahimsvu@gmail.com

Homann [10] to the case of axisymmetric stagnation-point flow. Later the problem of stagnation point flow either in the two or three-dimensional cases has been extended in numerous ways to include various physical effects. The results of these studies are of great technical importance, for example in the prediction of skin friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. Mahapatra and Gupta [11] and Nazar *et al.* [12] studied the heat transfer in the steady two dimensional stagnation-point flow of a viscous fluid by taking into account different aspects.

Stagnation point flows have been discussed by Pai [13], Schlichting [14] and Bansal   [15] etc. Kay [16] proposed that thermal conductivity of liquids with low Prandtl number varies linearly with temperature in range of 0^0 F to 400^0 F. Arunachalam and Rajappa [17] considered forced convection in liquid metals (i.e fluid with low Prandtl number) with variable thermal conductivity and capacity in potential flow and derived explicit closed form of analytical solution. Fluid flow and heat transfer characteristics on stretching sheet with variable temperature conditions have been investigated by Grubha and Bobba [18].

MHD(magnetohydrodynamic) stagnation point flows with thermal effects have applications in many manufacturing processes in industry and engineering. These applications include the aerodynamic extrusion of plastic sheets, boundary layers along material handling conveyers, the cooling of an infinite metallic plate in a cooling bath, blood flow problems and textile and paper industries. The two-dimensional MHD boundary layer flow in the region of the stagnation point on a stretching sheet has been investigated by several authors, such as Ding and Zhang [19], Ishak et al [20], Mahapatra and Gupta [11] and the references cited therein.

Recently, a considerable amount of interest has been evinced in the study of MHD (magnetohydrodynamic) stagnation point flow with heat and mass transfer in presence of porous medium fluid towards a stretching surface in view of its various industrial applications such as extrusion of polymer sheets, continuous stretching, manufacturing plastic films, and artificial fibres. In a melt-spinning process, the extradite from the die is usually drawn and simultaneously stretched into a sheet which is then solidified through quenching or gradual cooling by direct contact with water. The qualities of the final product depend on the rate of heat transfer at the stretching surface. It is, therefore, of great importance to know the flow behavior over a stretching surface which determines the rate of cooling. Wu et al [21] investigated the importance of Stagnation point flows in porous medium.

The steady two dimensional laminar MHD mixed convection stagnation point flow with mass and heat transfer over a surface was examined by Abdelkhalek [22] by using perturbation technique. Rashidi and Erfani [23] proposed that a new analytical study of MHD flow near a stagnation point flow through a porous medium with heat transfer. MHD stagnation point ferrofluid flow and heat transfer towards a stretching sheet was proposed by Khan et al. [24]

Watanabe [25] discussed stability of boundary layer and effect of suction/injection in MHD flow under pressure gradient.

A new dimension is added to the study of flow and heat transfer over a stretching sheet by consisting the effect of thermal radiation. It is well known that the effect of thermal radiation is important in space technology and high temperature processes. Thermal radiation also plays an important role in controlling heat transfer process in polymer processing industry. Pop et al [26] discussed the flow over stretching sheet near a stagnation point taking thermal radiation effect. The effect of radiation on heat transfer problems has been studied by Hossain and Takhar [27]. Pal [28] has investigated heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Vyas and Srivastava [29] present a numerical study for the steady two-dimensional radiative MHD boundary-layer flow of an incompressible, viscous, electrically conducting fluid caused by a non-isothernal linearly stretching sheet placed at the bottom of fluid saturated porous medium.

Possible heat generation effect may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semi conduction wafers. The steady hydromagnetic laminar stagnation point flow of an incompressible viscous fluid impinging on a permeable stretching surface with heat generation or absorption has been analyzed by Attia [30]. Chaim [31] discussed the heat transfer in fluid flow on stretching sheet at stagnation point in presence of heat source/sink and stress with constant fluid properties. Sharma and Singh [32] investigated the effects of variable thermal conductivity and heat source/sink on flow near a stagnation point on a non- conducting stretching sheet. The effects of variable thermal conductivity and heat source/sink on

steady two-dimensional radiative MHD boundary-layer flow of a viscous, incompressible, electrically conducting fluid in presence of variable free steam near a stagnation point on a non-conducting stretching sheet has been studied by Al-Sudais [33].

In all these investigations, the viscous dissipation is neglected. The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in a high gravitational field. Gebhart [34] was the first who studies the problem taking into account the viscous dissipation. Vajravelu and Hadjinicalaous [35] studied the heat transfer characteristic in the laminar boundary layer of a viscous fluid over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Tania and Samad [36] investigated the effect of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Viscous dissipation and thermal radiation effects on steady MHD stagnation point flow towards a stretching sheet in presence of induced magnetic field was formulated by Ali et al [37]

However the interaction of thermal radiation effect of an electrically conducting fluid past near a stagnation point on a linear stretching sheet has received little attention. Hence an attempt is made to investigate the thermal radiation and mass transfer effects on a steady MHD flow near a stagnation point on a linear stretching sheet in presence of variable thermal conductivity, heat generation and viscous disspation. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and analyzed in detail.

Formulation of the problem

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity in the vicinity of a stagnation point on a non-conducting stretching sheet in the presence of transverse magnetic field and volumetric rate of heat generation. The stretching sheet has uniform temperature T_w, linear velocity $u_w(x)$. It is assumed that external field is zero, the electric field owing to polarization of charges and Hall effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x- axis is taken along the sheet as shown in the Figure A. The fluid occupies the upper half plane i.e. $y > 0$.
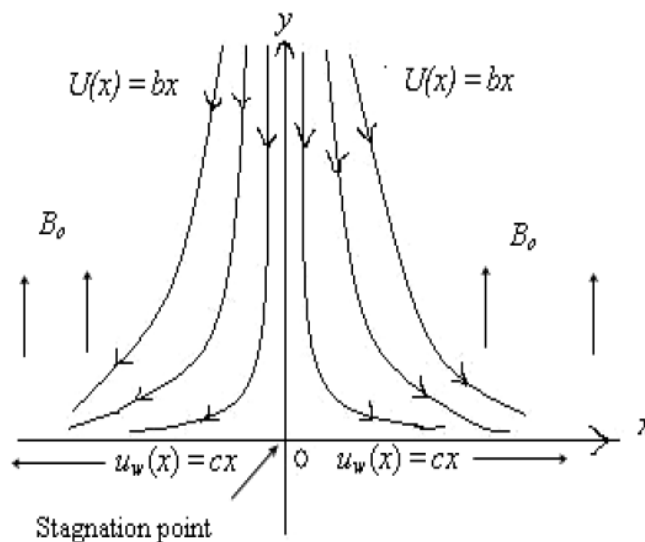
Figure A. Sketch of the physical model.

The governing equations of continuity, momentum, energy and concentration under the influence of externally imposed transverse magnetic field (Bansal [15]) with variable thermal conductivity in the boundary layer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{v}{K^*}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right)-\frac{\partial q_r}{\partial y}+Q^*(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Where x and y represents the coordinate axes along the continuous stretching surface in the direction of motion and normal to it, respectively; u and v are the velocity components along the x and y axes respectively, p is the pressure of the fluid, V is the kinematics viscosity, μ is the dynamic viscosity, σ is electrical conductivity, B_0 is the magnetic field intensity, ρ is the fluid density, K^* is the permeability of the porous medium, c_p is the specific heat at constant pressure, q_r is the radiation heat flux, T is the fluid temperature, T_∞ is the fluid free steam temperature, T_w is the fluid temperature of stretching sheet, C is the fluid concentration, C_∞ is the fluid free stream concentration, C_w is the fluid concentration of stretching sheet, k^* is the variable thermal conductivity, Q^* is the volumetric rate of heat generation, D is the molecular diffusivity of the species concentration.

The second derivatives of u and T with respect to x have been eliminated on the basis of magnitude analysis considering that Reynolds number is high. Hence the Navier-Stokes equation modifies into Prandtl's boundary layer equation.

In the free stream $u=U(x)=bx,$ the equation (2) reduces to

$$U\frac{dU}{dx}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2}{\rho}U \tag{5}$$

Eliminating $\frac{\partial p}{\partial x}$ between the equations (2) and (5), we obtain

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(u-U)-\frac{v}{K^*}u \tag{6}$$

The boundary conditions for the present problem are:

$$u=u_w(x)=cx,\quad v=0,\quad T=T_w,\quad C=C_w\quad\text{at}\quad y=0,$$

$$u=U(x)=bx,\quad T=T_\infty,\quad C=C_\infty\quad as\quad y\rightarrow\infty. \tag{7}$$

Using the Rosseland approximation for radiation (Brewster [38]), the radiative heat flux q_r could be expressed by:

$$q_r=-\frac{4\sigma^*}{3k_0}\frac{\partial T^4}{\partial y} \tag{8}$$

Where the σ^* represents the Stefan-Boltzman constant and k_0 is the Rosseland mean absorption coefficient. Assuming the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher orders we get:

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{9}$$

Following Arunachalam and Rajappa [17] and Chaim [31], the thermal conductivity k^* is taken of the form as given below

$$k^*=k(1+\varepsilon\theta) \tag{10}$$

The continuity equation (1) is satisfied by introducing the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}.$$

We introduce the following non-dimensional variables:

$$\psi(x, y) = (cv)^{\frac{1}{2}} x f(\eta), \quad \eta = \left(\frac{c}{v}\right)^{\frac{1}{2}} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$$\lambda = \frac{b}{c}, \quad K = \frac{v}{K^* c}, \quad M = \frac{\sigma B_0^2}{\rho c}, \quad R = \frac{16\sigma^* T_\infty^3}{3 k_0 k}, \quad \Pr = \frac{\mu c_p}{k}, \quad Q = \frac{Q^*}{\rho c_p c}, \quad Ec = \frac{c^2 x}{c_p (T_w - T_\infty)}, \quad Sc = \frac{v}{D}.$$

$$(11)$$

Using equations (8) – (11) into the equations (6), (3) and (4) we get the following ordinary differential equations:

$$f''' + ff'' - f'^2 - M(f' - \lambda) - Kf' + \lambda^2 = 0, \tag{12}$$

$$(1 + R + \varepsilon\theta)\theta'' + \varepsilon\theta'^2 + \Pr f\theta' + \Pr Q\theta + \Pr Ecf''^2 = 0 \tag{13}$$

$$\phi'' + Scf\phi' = 0 \tag{14}$$

where the primes denote the differentiation with respect to η, λ is the ratio of free stream velocity parameter to stretching sheet parameter, b is the free stream velocity parameter, c is the stretching sheet parameter, M is the magnetic parameter, K is the permeability parameter, $\Pr$ is the Prandtl number, R is the thermal radiation parameter, ε is perturbation parameter, Q is the heat generation parameter, Ec is the Eckert number and Sc is the Schmidt number.

The corresponding boundary conditions are reduced to

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1$$

$$f'(\infty) = \lambda, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \tag{15}$$

Skin-friction coefficient, Nusselt number and Sherwood number

In practical applications, three quantities of physical interest are to be determined, such as, surface shear stress, the rate of heat transfer and rate of mass transfer at the surface. These may be obtained in terms of the skin friction coefficient,

$$C_f = \frac{\tau_w}{\rho c \sqrt{cv}} = xf''(0) \tag{16}$$

The local Nusselt number,

$$Nu = \left(\frac{v}{c}\right)^{\frac{1}{2}} \frac{q_w}{k^*(T_w - T_\infty)} = -\theta'(0) \tag{17}$$

and the local Sherwood number

$$Sh = \left(\frac{v}{c}\right)^{\frac{1}{2}} \frac{m_w}{D(C_w - C_\infty)} = -\phi'(0) \tag{18}$$

where, $\tau_w = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$ is the shear-stress along the sheet,

$q_w = -k^* \left(\frac{\partial T}{\partial y}\right)_{y=0}$ is the surface heat transfer rate, and

$$m_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0}$$ is the surface mass transfer rate.

Method of Solution

The governing boundary layer , thermal and concentration boundary layer equations (12) – (14) with the boundary conditions (15) are solved using Runge-Kutta fourth order technique along with shooting method (Conte and Boor [39]). First of all, higher order non-linear differential equations (12)- (14) are converted into simultaneous linear differential equations of order first and they are further transformed into initial value problem applying the shooting technique. Once the problem is reduced to initial value problem, then it is solved using Runge-Kutta fourth order technique (Jain [40], Jain et al. [41], Krishnamurthy and Sen [42]).

Results and Discussion

The system of non-linear ordinary differential equations (12) – (14) are solved numerically using Shooting method for different values of Magnetic field parameter *M*, permeability of the porous medium *K,* ratio of free stream velocity parameter to stretching sheet parameter λ , thermal radiation parameter *R,* Prandtl number *Pr,* heat generation parameter *Q,* perturbation parameter ε , and Schmidt number *Sc.*

In order to assess the accuracy of the numerical method, results for $f''(0)$ in the absence of Magnetic field parameter (*M = 0*) was compared with those of Pop et al. [26], Mahapatra and Gupta [11] and Al-sudais [33] . Former have used Runge-Kutta fourth order method and shooting technique, while the second have used finite difference technique and Thomas algorithm, the later have used Runge-Kutta fourth order method along with shooting technique and found to be in good agreement. These comparisons are shown in Table 1. It is seen from Table 2 that the numerical values of $-\theta'(0)$ in the present paper when *M = K = R = Q = Sc = ε = 0* and *Pr* = 0.05 are agreement with those obtained by Pop et al. [26], Mahapatra and Gupta [11]and Al-sudais [33].

To analyze the results, numerical computations has been carried out for variations in the governing parameters such as the of Magnetic field parameter *M*, permeability of the porous medium *K,* ratio of free stream velocity parameter to stretching sheet parameter λ , thermal radiation parameter *R,* Prandtl number *Pr,* heat generation parameter *Q,* perturbation parameter ε , and Schmidt number *Sc.*

In the present study following default parameter values are adopted for computations: *M = 0.1, K = 0.1, Pr = 0.71, R = 1.0, Q = 0.1,* $\lambda = 0.1$, $\varepsilon = 0.1$, *Ec = 0.1, Sc = 0.22*. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

Figure 1 represents the importance of magnetic field on the velocity profiles. The presence of transverse magnetic field parameter *M* sets in Lorentz force, which results in retarding force on the velocity filed and therefore as magnetic field parameter increases, so does the retarding force and hence the velocity profiles decrease. This is shown in Figure 1. Figure 2 and Figure 3 display on temperature and concentration profiles with magnetic field parameter respectively. From these figures, it is observed that both temperature and concentration profiles increases as magnetic field parameter increase.

The effect of the permeability of porous medium parameter *K* on velocity, temperature and concentration profiles is shown in Figures 4, 5, 6 respectively. It can be observed the velocity reduces as permeability of porous medium *K* increases, while temperature and concentration enhances as permeability of porous medium *K* increases which imply that the resistance of the medium decreases. This is due to the increased restriction resulting from decreasing the porosity of porous medium.

It is observed from Figure 7 that with the increase in Prandtl number *Pr*, temperature profile decreases. This is because of the fact that with the increase in Prandtl number *Pr,* thermal boundary layer thickness reduces. The effect is even more pronounced for small Prandtl number *Pr* because the thermal boundary layer thickness is comparatively large.

Figure 8 predicts the influence of the radiation parameter *R* on the temperature field. The radiation parameter *R* defines the relative contribution of conduction heat transfer to theramal radiation transfer. It is obvious that an increase in the radiation parameter results in increasing temperature within the boundary layer.

Figure 9 shows the influence of the heat generation parameter Q on temperature profiles within the thermal boundary layer. From the Figure 9 it is observed that the temperature increases with an increase in the heat generation parameter Q.

The influence of viscous dissipation parameter i.e., the Eckert number Ec on the temperature profiles is shown in Figure 10. The Eckert number Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature this behavior is evident from Figure 10.

Figure 11 represents the temperature profiles for some values of the perturbation parameter ε and for fixed values of all other. Figure 11 shows that with the increase in the value of ε, temperature profiles increase hence considering the thermal conductivity constant would lead to lower approximation of the temperature profile.

For different values of the Schmidt number Sc, the concentration profile is plotted in Figure 12. The Schmidt number Sc embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass transfer (concentration) boundary layer. As the Schmidt number Sc increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reduction in the concentration profiles is accompanied by simultaneous reductions in the concentration boundary layers, which is evident from Figure 12.

Figure 13 depicts the ratio of free steam velocity parameter to stretching sheet parameter λ on velocity profiles. It is clear that the velocity profiles increase with an increase in the values of parameter λ. Figures 14 and 15 show the temperature and concentration profiles for different values of the ratio of free steam velocity parameter to stretching sheet parameter λ. It is seen Figures 14 and 15 that fluid temperature and concentration decreases due to increase in λ.

The effects of various governing parameters on the skin-friction coefficient C_f, the Nusselt number Nu and the Sherwood number Sh are shown in Tables 3 and 4. From Tables, it is noticed that as magnetic parameter M or Permeability parameter K increases, the skin-friction coefficient increases and both Nusselt number and Sherwood number decreases. As λ increases skin-friction coefficient reduces, while Nusselt number and Sherwood number increase. The Nusselt number increases as Prandlt number increase, while Nusselt number reduces as thermal radiation parameter R or heat generation parameter Q or Eckert number Ec or perturbation parameter ε increases. It is seen that, the Sherwood number increases as an increase in the Schmidt number Sc.
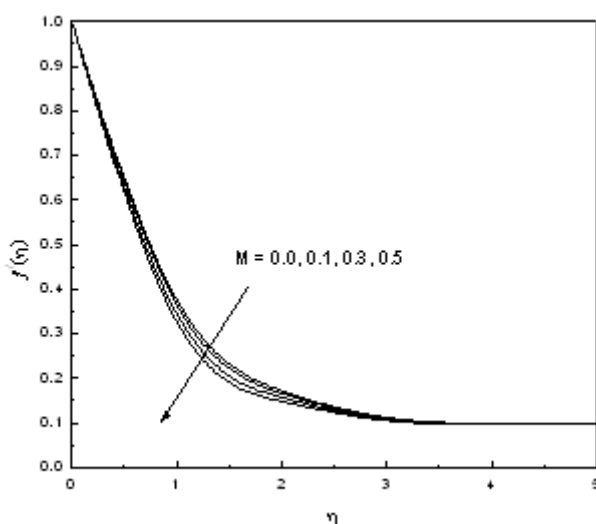

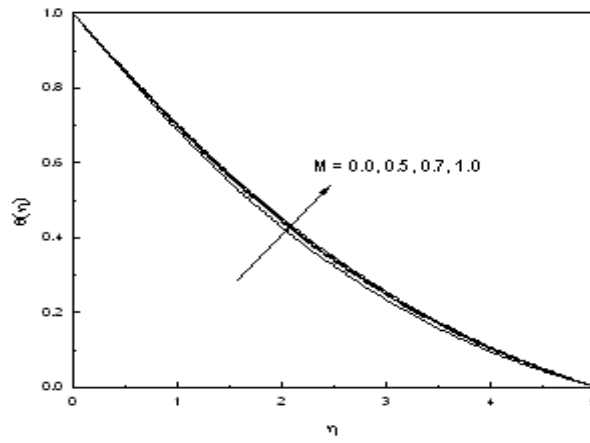**Figure 1. Velocity profiles for different values of *M***
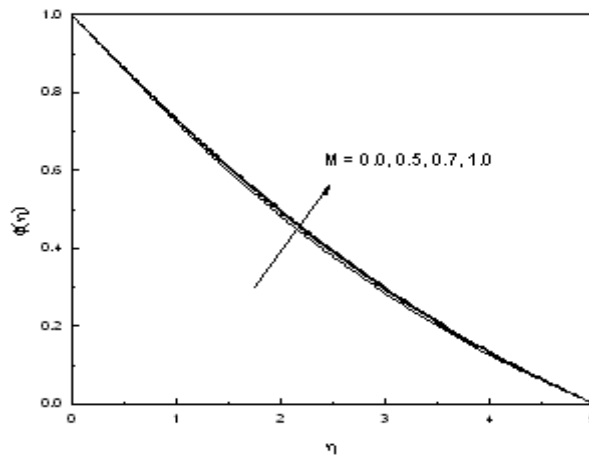
Figure 2. Temperature profiles for different values of *M*

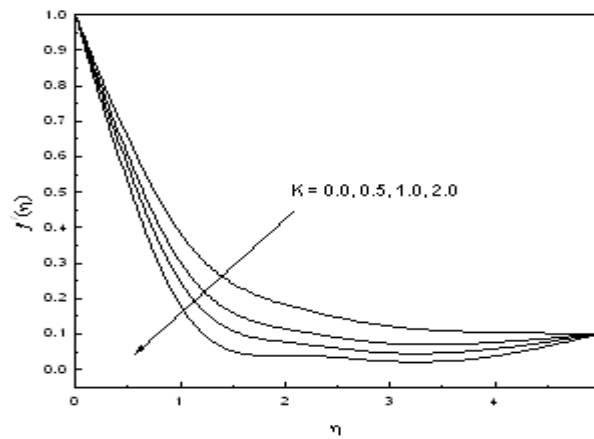**Figure 3. Concentration profiles for different values of *M***

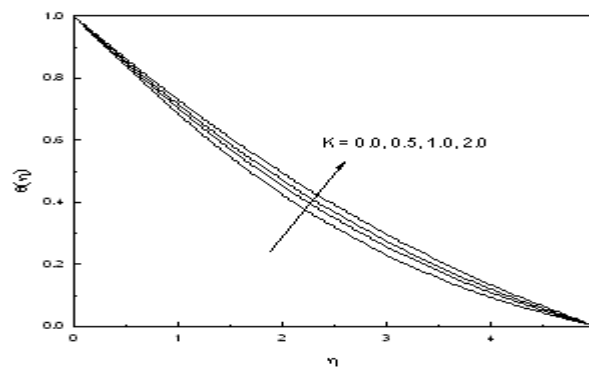**Figure 4. Velocity profiles for different values of *K***

Figure 5. Temperature profiles for different values of *K*
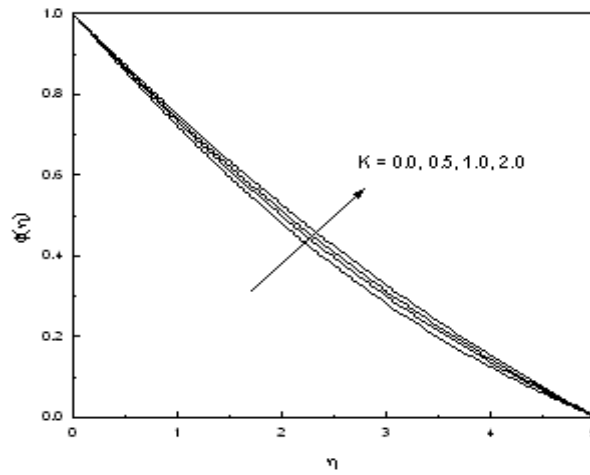
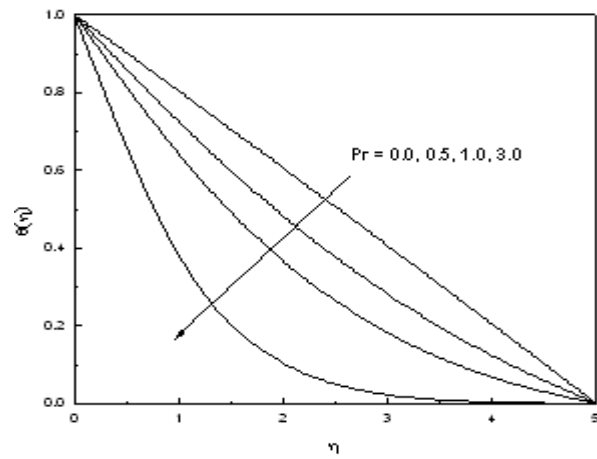**Figure 6. Concentration profiles for different values of *K***

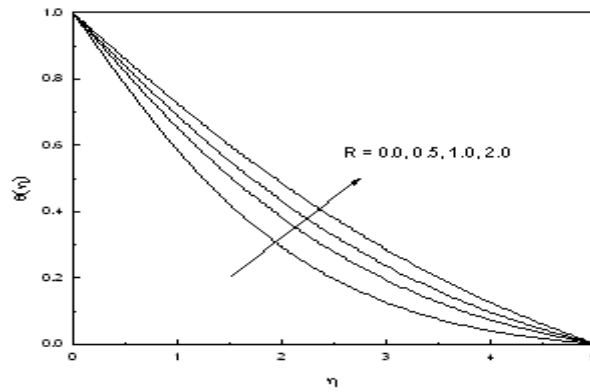**Figure 7. Temperature profiles for different values of *Pr***

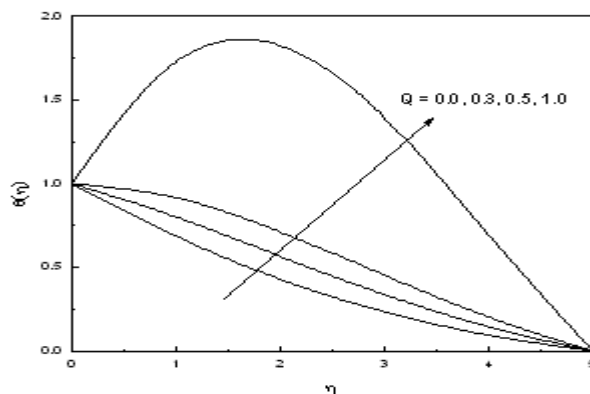**Figure 8. Temperature profiles for different values of *R***

Figure 9. Temperature profiles for different values of *Q*
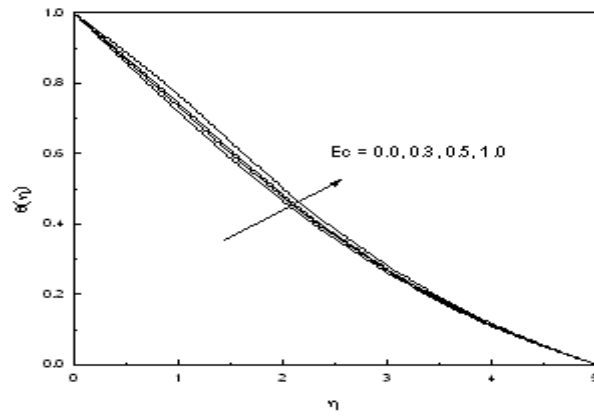
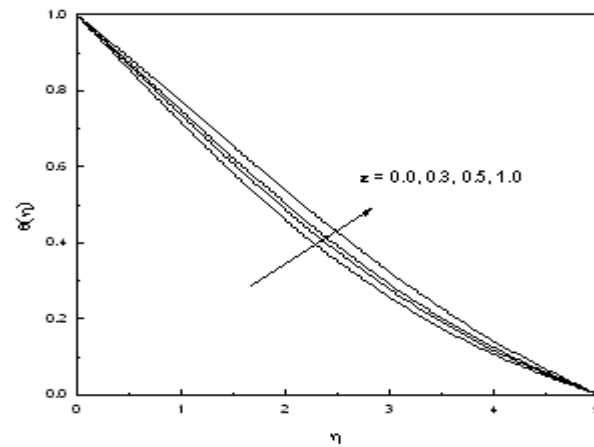**Figure 10. Temperature profiles for different values of *Ec***

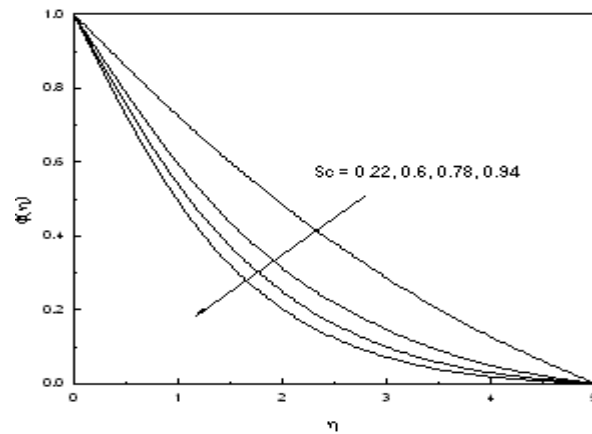**Figure 11 Temperature profiles for different values of ε**

Figure 12 Concentration profiles for different values of *Sc*
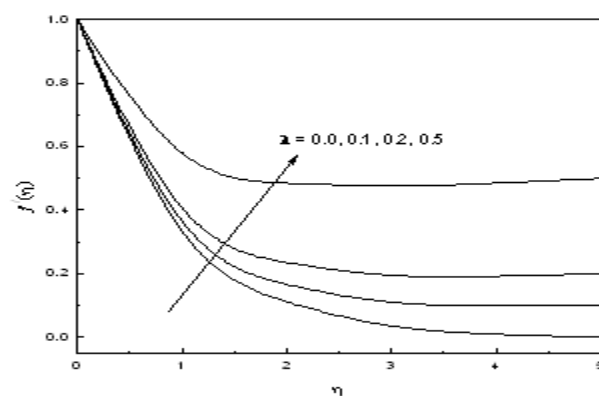

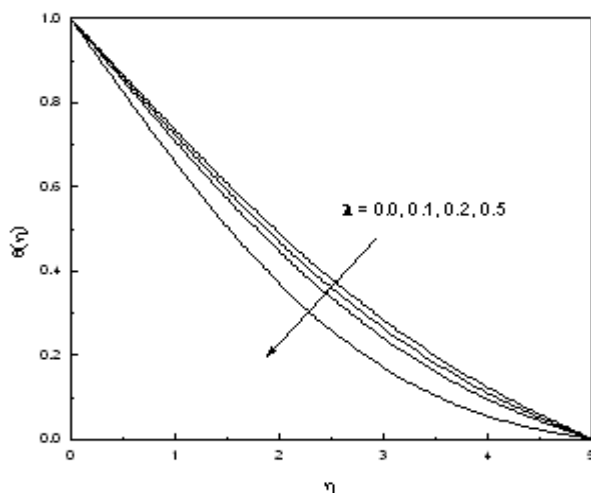**Figure 13 Velocity profiles for different values of λ**

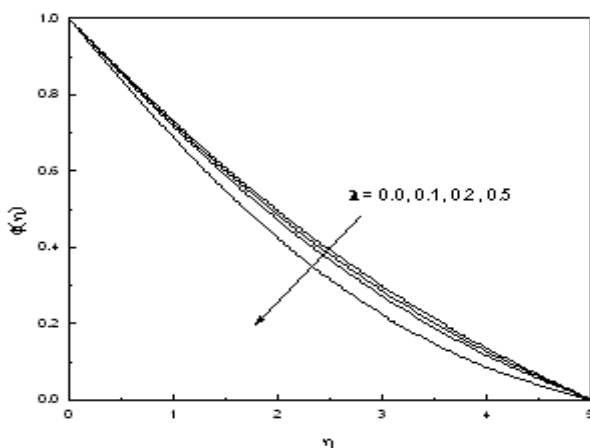**Figure 14 Temperature profiles for different values of** λ

Figure 15 Concentration profiles for different values of λ

Table 1. A comparison of skin-friction coefficient $f''(0)$ **for different values of** λ **and** *M = 0.0.*

λ	$f''(0)$			
	Pop et al. [26]	Mahapatra and Gupta [11]	Al-sudais [33]	Present paper
0.1	-0.9694	-0.9694	-0.969705	-0.969385
0.2	-0.9181	-0.9181	-0.918349	-0.9181065
0.5	-0.6673	-0.6673	-0.6674611	-0.667261
2.0	2.0174	2.0175	2.01728138	2.0174904

Table 2: A comparison of Nusselt number $-\theta'(0)$ **for different values of** λ **when** $M = Q = R = Ec = \varepsilon = 0.0, \Pr = 0.05 \cdot$

λ	$-\theta'(0)$			
	Pop et al. [26]	Mahapatra and Gupta [11]	Al-sudais [33]	Present paper
0.1	0.081	0.081	0.080547	0.081241
0.5	0.135	0.136	0.135358	0.135575
2.0	0.241	0.241	0.241025	0.241029

Table 3 Numerical values of the skin-friction coefficient, Nusselt number and Sherwood number for *Pr = 0.71, R = 1.0, Q = 0.1, Ec = 0.1,* $\varepsilon = 0.1$ *, Sc = 0.22.*

M	K	λ	C_f	Nu	Sh
0.1	0.1	0.1	-1.0602	0.260115	0.281977
0.3	0.1	0.1	-1.13321	0.254331	0.279277
0.5	0.1	0.1	-1.20198	0.249174	0.276908
0.1	0.3	0.1	-1.15376	0.249468	0.276929
0.1	0.5	0.1	-1.2398	0.240366	0.272698

0.1	0.1	0.3	-0.934202	0.294526	0.299526
0.1	0.1	0.5	-0.747837	0.332768	0.320372

Table 4. Numerical values of the skin-friction coefficient, Nusselt number and Sherwood number for *M = 0.1, K = 0.1*,

$$\lambda = 0.1.$$

Pr	*R*	*Q*	*Ec*	*ε*	*Sc*	C_f	*Nu*	*Sh*
0.71	1.0	0.1	0.1	0.1	0.22	-1.0602	0.260115	0.281977
1.0	1.0	0.1	0.1	0.1	0.22	-1.0602	0.29574	0.281977
2.0	1.0	0.1	0.1	0.1	0.22	-1.0602	0.433854	0.281977
0.71	2.0	0.1	0.1	0.1	0.22	-1.0602	0.236797	0.281977
0.71	3.0	0.1	0.1	0.1	0.22	-1.0602	0.226233	0.281977
0.71	1.0	0.3	0.1	0.1	0.22	-1.0602	0.142917	0.281977
0.71	1.0	0.5	0.1	0.1	0.22	-1.0602	0.0713254	0.281977
0.71	1.0	0.1	0.3	0.1	0.22	-1.0602	0.229629	0.281977
0.71	1.0	0.1	0.5	0.1	0.22	-1.0602	0.199143	0.281977
0.71	1.0	0.1	0.1	0.3	0.22	-1.0602	0.244796	0.281977
0.71	1.0	0.1	0.1	0.5	0.22	-1.0602	0.231904	0.281977
0.71	1.0	0.1	0.1	0.1	0.6	-1.0602	0.260115	0.434922
0.71	1.0	0.1	0.1	0.1	0.94	-1.0602	0.260115	0.566934

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